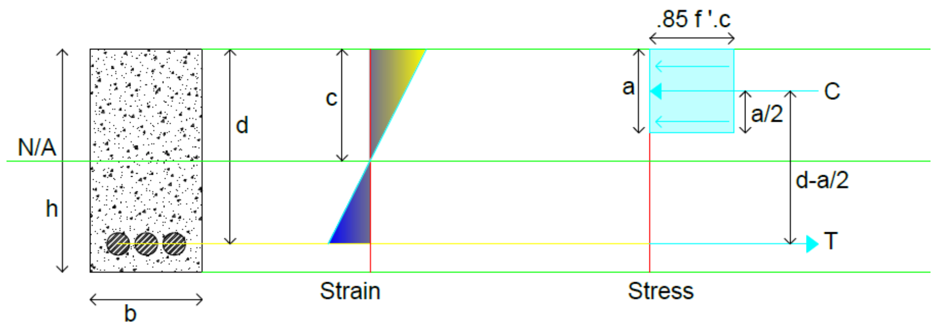
Singly Reinforced Beam Section



Assumptions: Steel only in tension
Steel Yields before the concrete reaches the maximum compression
Uses ACI 318 - 08 ; CODE: 10.2.7
Equivalent Concrete Stress Distribution

Variable Definitions

A_s = Area of Tension Reinforcement (steel)

A'_s = Area of Compression

b = Width of Compression Zone

h = Height

d = Distance from the extreme/top fiber in compression to the centroid of tension steel

f'_c = Compressive Strength of Concrete

C = Compressive Force

T = Tensile Force

a = Distance from the top fiber in compression to the bottom of the equivalent compressive stress distribution

c = Distance from the top fiber in compression to the neutral axis

β_1 = Constant from the ACI 318 - 08 manual that is used to calculate, c or a

f_y = Yielding Stress of Steel Rebar

E = Young's Modulus

ε_s = Steel Strain

ε_y = Yielding Steel Strain

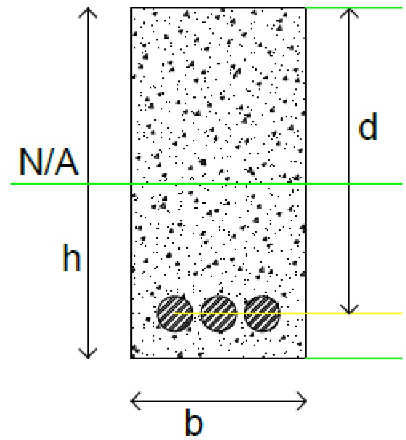
M_n = Nominal Moment

ε_{cu} = Ultimate Compressive Strain of Concrete

Given:

- Geometric structure of the beam

$h := 24 \cdot in$ $b := 14 \cdot in$ $d := 21 \cdot in$



- Properties of Concrete

Compressive Strength of the Concrete $f'_c := 3 \cdot ksi$

- Properties of Steel

$Rebar_{number} := 9$ $Rebar_{amount} := 3$

$Rebar_{grade} := 60$

Yielding Stress of Steel Rebar $f_y := Rebar_{grade} \cdot ksi$ $f_y = 60\,ksi$

Data := [redacted]

Area of steel $A_S := \left(Rebar_{amount} \cdot Data\right)_{match\left(Rebar_{number}, Data^{\langle 0 \rangle}\right)_{0}, 2} in^2$ $A_S = 3 \cdot in^2$

Strategy

Step 1: Check that $A_s > A_{smin}$ (code) **NEEDS TO BE BUILT IN ONCE I KNOW THE CODE**

Step 2: Compute " a " based on the assumption that the steel is yielding before the concrete
Step 3: verify that the steel is yielding
Step 4: Calculate the nominal moment " M_n "

Step 2:

Computation of " a "

C is equal to T, and each is just the summation of the forces for the whole object.

C = T = .85 * f'_c * a * b = A_s * f_y

$$a := \frac{A_S \cdot f_y}{.85 \cdot f'_c \cdot b} \qquad \frac{in^2 \cdot \frac{lbf}{in^2}}{\left(\frac{lbf}{in^2}\right) \cdot in} = 1\,in$$

$a = 5.04202\,in$

Computation of " β_1 " **(ACI 10.2.7.3)**

"For f'_c between 2500 and 4000 psi, β_1 shall be taken as 0.85. For f'_c above 4000 psi, β_1, shall be reduced linearly at a rate of .05 for each 1000 psi of strength in excess of 4000 psi, but β_1 shall not be taken less than 0.65."

$$\beta_1 := \text{if}\left[f'_c \le 4 \cdot ksi, 0.85, \max\left[0.85 - .05 \cdot \left(\frac{f'_c - 4ksi}{1 \cdot ksi}\right), 0.65\right]\right]$$

$\beta_1 = 0.85$

Computation of " c " **(ACI 10.2.7.1)**

"Concrete stress of .85 *f'_c shall be assumed uniformly distributed over an equivalent compression zone bounded by edges of the cross section and a straight line located parallel to the neutral axis at a distance a = c * β_1 from the fiber of maximum compressive strain. "

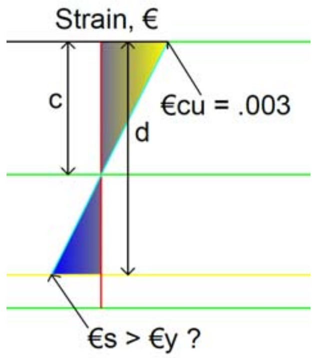
c = a / β_1

$$c := \frac{a}{\beta_1}$$

$c = 5.93178\,in$

Step 3:

The ultimate compressive strain of the concrete is a constant and known to be .0035. By calculating " c " in Step 2, and knowing the value of " d ", we are able to use simple triangular geometry to calculate ε_s. We can then check to see if ε_s is actually yielding by comparing it to ε_y ,the yielding strain of the steel that is derived by using the Young's modulus and the yielding stress.



E = Young's Modulus = σ/ε = stress / strain

*Using the young's modulus assumes elastic behavior and obey hooke's law.
*E is a property of the material and is constant.

E_{steel} = 29,000,000 psi (29,000 ksi)

ε_{cu} = ultimate compressive strain of concrete = .0035

Computation of " ε_y "

$E := 29000 \cdot ksi$ $\quad$ $f_y = 60\,ksi$

$\varepsilon_y = \sigma/E = f_y / E$

Yielding Strain of Steel Rebar $\quad$ $\varepsilon_y := \frac{f_y}{E}$

$\varepsilon_y = 0.00207$

Computation of " ε_s " and comparison to " ε_y "

$\varepsilon_{cu} := .003$

$$\varepsilon_s := \frac{(d - c) \cdot \varepsilon_{cu}}{c}$$

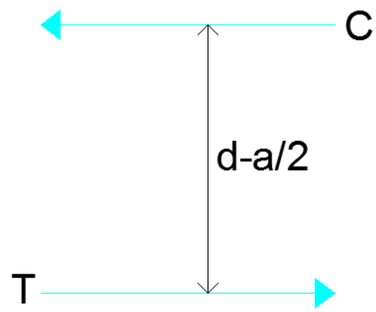
$\varepsilon_s = 0.00762$

***If " ε_s" is less than "ε_y" then reevaluation is required.**

$\varepsilon_s = 0.00762$ $\quad$ $\varepsilon_y = 0.00207$

$\varepsilon_{ten} := \text{if}\left(\varepsilon_s \geq \varepsilon_y, \text{"OK"}, \text{"NG"}\right)$ $\quad$ $\varepsilon_{ten} = \text{"OK"}$

Step 4:



Calculation of the nominal moment is easily computed by multiplying either the compressive force, C, or tension force, T, by the moment arm. Either method is effective.

M_n = T (d-a/2)

The Tension force can be calculated by Knowing the tensile stress that is applied to the steel. Fundamentally, stress is a force divided by the area the force is applied to. The entire tension force is being applied through the cross sectional area of the steel rebar, which is a known quantity.

σ = F/A

Tensile Stress = Yielding Stress assumed at the beginning of the analysis.

$f_y = 60\,\text{ksi}$

The Area of the Steel was calculated previously.

$A_S = 3\,\text{in}^2$

$T := f_y \cdot A_S$

$T = 180\,\text{kip}$

$M_n := T \cdot \left(d - \frac{a}{2}\right)$

$M_n = 1.28421 \times 10^9 \frac{\text{lb} \cdot \text{in}^2}{\text{s}^2}$