$a_1(t) := J_1 \cdot t \rightarrow J_1 \cdot t \quad [0; t_1]$

$a_2(t) := J_1 \cdot t_1 \quad [t_1; t_2]$

assuming symmetry ($t_2 = t_3 - t_1$)

$$v\_max(t_1, t_3, J_1) := 2 \cdot \left(\int_0^{t_1} a_1(t)\,dt + \int_{t_1}^{\frac{t_3}{2}} a_2(t)\,dt \right) \rightarrow J_1 \cdot t_1^2 + J_1 \cdot t_1 \cdot (t_3 - 2 \cdot t_1)$$

$$a\_avg(t_1, t_3, J_1) := \frac{2}{t_3} \cdot \left(\int_0^{t_1} a_1(t)\,dt + \int_{t_1}^{\frac{t_3}{2}} a_2(t)\,dt \right) \rightarrow \frac{2 \cdot \left[\frac{J_1 \cdot t_1^2}{2} + \frac{J_1 \cdot t_1 \cdot (t_3 - 2 \cdot t_1)}{2} \right]}{t_3}$$

$$j\_pct(t_1, t_3) := \frac{2 \cdot t_1}{t_3} \rightarrow \frac{2 \cdot t_1}{t_3}$$

$$v_1(t) := \int_0^{t} a_1(t\_)\,dt\_ \rightarrow \frac{J_1 \cdot t^2}{2} \qquad v_1(t) \rightarrow \frac{J_1 \cdot t^2}{2}$$

$$v_2(t) := \int_{t_1}^{t} a_2(t\_)\,dt\_ + v_1(t_1) \rightarrow \qquad v_2(t) \text{ expand } \rightarrow J_1 \cdot t \cdot t_1 - \frac{J_1 \cdot t_1^2}{2}$$

$$v_3(t) := v_1(t_1) - v_1(t_3 - t) + v_2(t_3 - t_1) \qquad v_3(t) \text{ simplify } \rightarrow -\frac{J_1 \cdot (t^2 - 2 \cdot t \cdot t_3 + 2 \cdot t_1^2 - 2 \cdot t_1 \cdot t_3 + t_3^2)}{2}$$

$$v_4(t) := v_3(t_3)$$

$$d\_end(t_1, t_3, t_4, J_1) := 2 \cdot \left(\int_0^{t_1} v_1(t)\,dt + \int_{t_1}^{t_3 - t_1} v_2(t)\,dt + \int_{t_3 - t_1}^{t_3} v_3(t)\,dt + \int_{t_3}^{t_4} v_4(t)\,dt \right) \rightarrow \frac{J_1 \cdot t_1^3}{3} - \frac{J_1 \cdot t_1^2 \cdot (7 \cdot t_1 - 6 \cdot t_3}{3}$$

$$Solution := \begin{pmatrix} j\_pct(t_1, t_3) = j_{pct} \\ a\_avg(t_1, t_3, J_1) = a_{avg} \\ v\_max(t_1, t_3, J_1) = v_{max} \\ d\_end(t_1, t_3, t_4, J_1) = d_{end} \end{pmatrix} \left| \begin{array}{l} \text{solve}, \begin{pmatrix} t_1 \\ t_3 \\ J_1 \\ t_4 \end{pmatrix} \\ \text{simplify} \end{array} \right. \rightarrow \left[\frac{j_{pct} \cdot v_{max}}{2 \cdot a_{avg}} \quad \frac{v_{max}}{a_{avg}} \quad -\frac{4 \cdot a_{avg}^2}{j_{pct} \cdot v_{max} \cdot (j_{pct} - 2)} \quad \frac{v_{max}}{2 \cdot a_{avg}} + \frac{d_{end}}{2 \cdot v_{max}} \right]$$

$$Solution \rightarrow \left[\frac{j_{pct} \cdot v_{max}}{2 \cdot a_{avg}} \quad \frac{v_{max}}{a_{avg}} \quad -\frac{4 \cdot a_{avg}^2}{j_{pct} \cdot v_{max} \cdot (j_{pct} - 2)} \quad \frac{v_{max}}{2 \cdot a_{avg}} + \frac{d_{end}}{2 \cdot v_{max}} \right]$$

$$t1(v_{max}, a_{avg}, j_{pct}, d_{end}) := (Solution^T)_0 \rightarrow \frac{j_{pct} \cdot v_{max}}{2 \cdot a_{avg}}$$

$$t3(v_{max}, a_{avg}, j_{pct}, d_{end}) := (Solution^T)_1 \rightarrow \frac{v_{max}}{a_{avg}}$$

$$t4(v_{max}, a_{avg}, j_{pct}, d_{end}) := \left(Solution^T\right)_3 \rightarrow \frac{v_{max}}{2 \cdot a_{avg}} + \frac{d_{end}}{2 \cdot v_{max}}$$

$$J1(v_{max}, a_{avg}, j_{pct}, d_{end}) := \left(Solution^T\right)_2 \rightarrow -\frac{4 \cdot a_{avg}^2}{j_{pct} \cdot v_{max} \cdot (j_{pct} - 2)}$$

End Position: $d_{end} := 1000$

Peak Speed: $v_{max} := 1400$

Avg. Accel: $a_{avg} := 2600$

% of Time Jerk $j_{pct} := 50\%$

$$J_1 := J1(v_{max}, a_{avg}, j_{pct}, d_{end}) = 25752.380952$$

$t_1 := t1(v_{max}, a_{avg}, j_{pct}, d_{end}) = 0.134615$

$t_2 := t_3 - t_1 = 0.403846$

$t_3 := t3(v_{max}, a_{avg}, j_{pct}, d_{end}) = 0.538462$

$t_4 := t4(v_{max}, a_{avg}, j_{pct}, d_{end}) = 0.626374$

$$a\_(t) := \begin{cases} J_1 \cdot t & \text{if } 0 \le t \le t_1 \\ \text{otherwise} & \begin{cases} J_1 \cdot t_1 & \text{if } 0 \le t \le t_2 \\ \text{otherwise} & \begin{cases} J_1 \cdot (t_3 - t) & \text{if } 0 \le t \le t_3 \\ 0 & \text{if } t \le 2 \cdot t_4 - t_3 \quad \text{otherwise} \end{cases} \end{cases} \end{cases}$$

$$a(t) := \begin{cases} a\_(t) & \text{if } 0 \le t \le t_4 \\ -a\_(2 \cdot t_4 - t) & \text{otherwise} \end{cases}$$

TIME — Accel Time: $t_3 = 0.53846$

End Time: $2 \cdot t_4 = 1.25275$

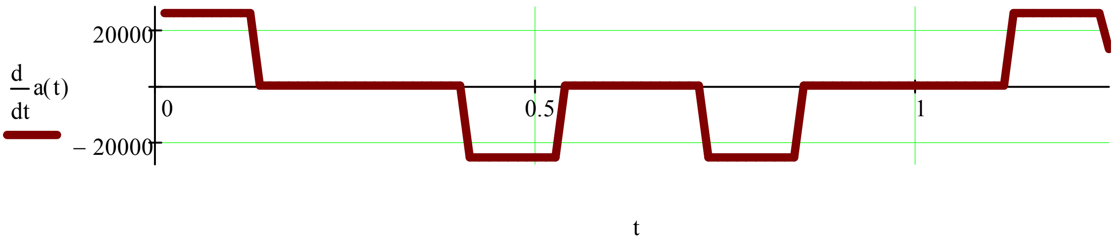
SPEED — Avg. Speed: $\frac{d_{end}}{2 \cdot t_4} = 798.246$

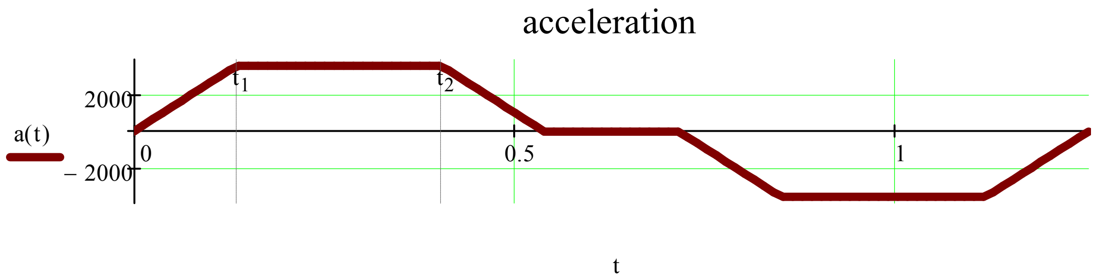
ACCEL — Peak Accel: $a(t_1) = 3466.67$

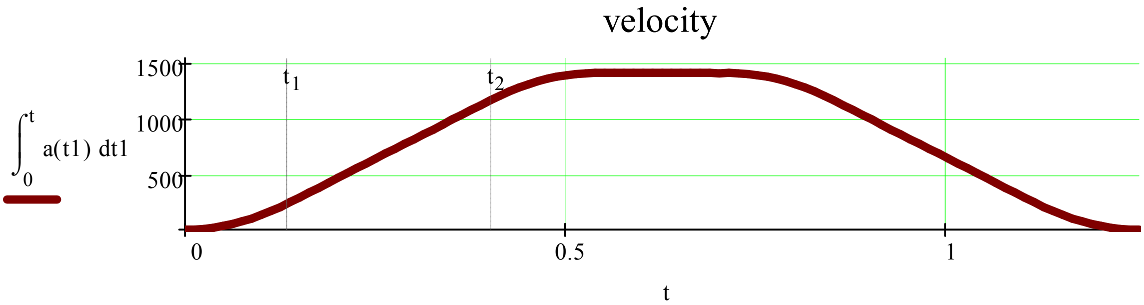
JERK — Peak Jerk: $J_1 = 25752.4$

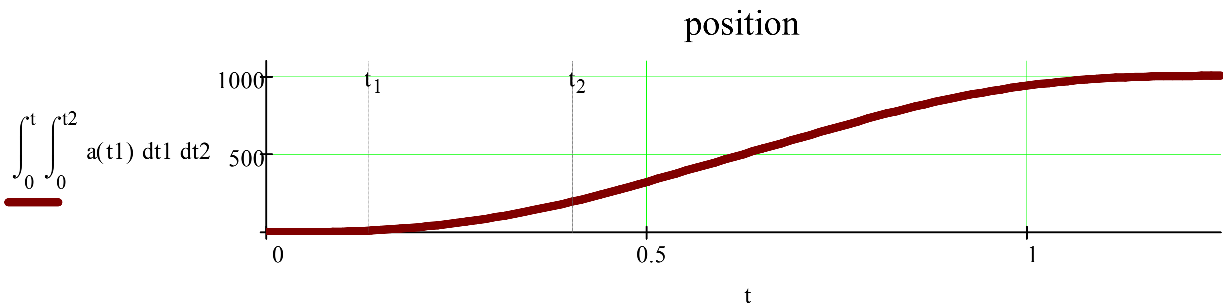
$nr\_of\_points := 100$ $\qquad t := 0, \frac{2 \cdot t_4}{nr\_of\_points} .. 2 \cdot t_4$

jerk




acceleration
a(t)
2000
− 2000
0
0.5
1
t
t_1
t_2


velocity
$\int_0^t a(t1)\,dt1$
1500
1000
500
0
0.5
1
t
t_1
t_2


position
$\int_0^t \int_0^{t2} a(t1)\,dt1\,dt2$
1000
500
0
0.5
1
t
t_1
t_2