Please use a friction of 0.08 and assume no drag. To be more descriptive the vehicle is about twelve feet long on a smooth resin surface with an average weight of 600lbs.

The vehicle is actuall 11' 6" long. It is a flat inner tube, no wheels and it is in contact with the surface.

Additional information:
Vehicle width: 46"

$k_f = 0.08 \qquad Wgt := 600lbf$

Friction is defined (traditionally) as k_f x Wgt; the force opposes motion and is proportional to the normal force to the surface.

Initial speed $\qquad V_o := 20\frac{ft}{sec}$

Drag review. Flat plate drag $C_D = 1.5$ $\qquad F_{drag} = C_D \cdot \frac{\rho \cdot V^2}{2} \cdot Area$

for air near the earth's surface

$\rho_{air} := 0.0765\frac{lb}{ft^3} \qquad Area := 1ft \cdot 46in = 3.833 \cdot ft^2$ (no height given.)

$$1.5 \cdot \frac{\rho_{air} \cdot \left(20\frac{ft}{sec}\right)^2}{2} \cdot Area = 2.734 \cdot lbf$$

$$\frac{1.5 \cdot \frac{\rho_{air} \cdot \left(20\frac{ft}{sec}\right)^2}{2} \cdot Area}{k_f \cdot Wgt} = 5.697 \cdot \%$$

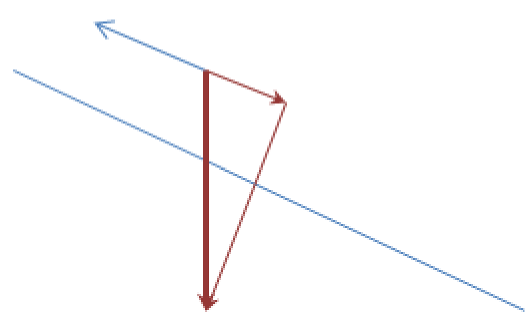
Probably can ignore drag for initial pass.

Slope where friction and gravity cancel. (If the hill is steep enough, the acceleration due to gravity will equal the retarding force of friction, and the sled will move at constant speed.

Retarding force: $\qquad F_{fric} = k_f \cdot Wgt \cdot \cos(\theta)$

Gravitational Acceleration: $F_{grav} = Wgt \cdot \begin{pmatrix} \sin(\theta) \\ \cos(\theta) \end{pmatrix}$

$$k_f \cdot Wgt \cdot \cos(\theta) = Wgt \cdot \sin(\theta)$$



$k_f = \dfrac{\sin(\theta)}{\cos(\theta)} = \tan(\theta)$ $\quad\quad$ $\operatorname{atan}(k_f) = 4.574 \cdot \text{deg}$

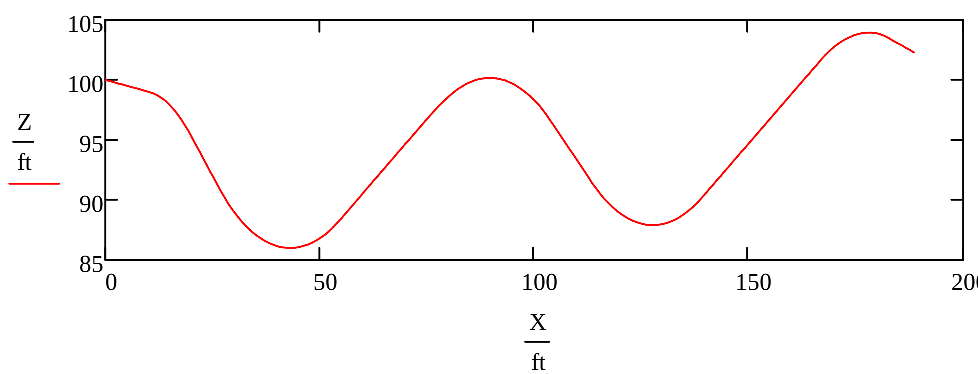
Profile

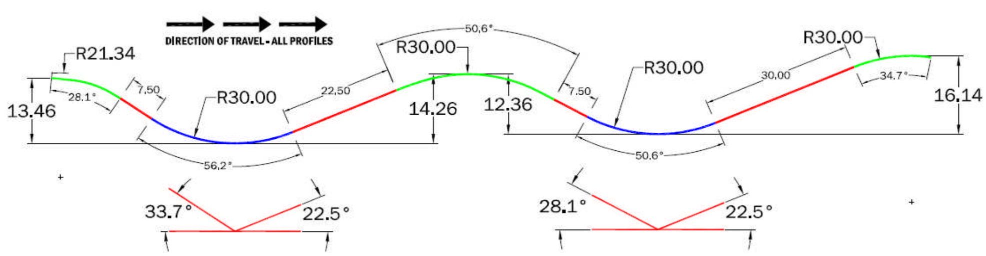
data :=

	0	1
0	500	400
1	498.997	...

$X := \text{data}^{\langle 0 \rangle} \text{ft}$ $\quad\quad$ $Z := \text{data}^{\langle 2 \rangle} \text{ft}$

Let's turn the X coordinate around $\quad X := -(X - X_0)$





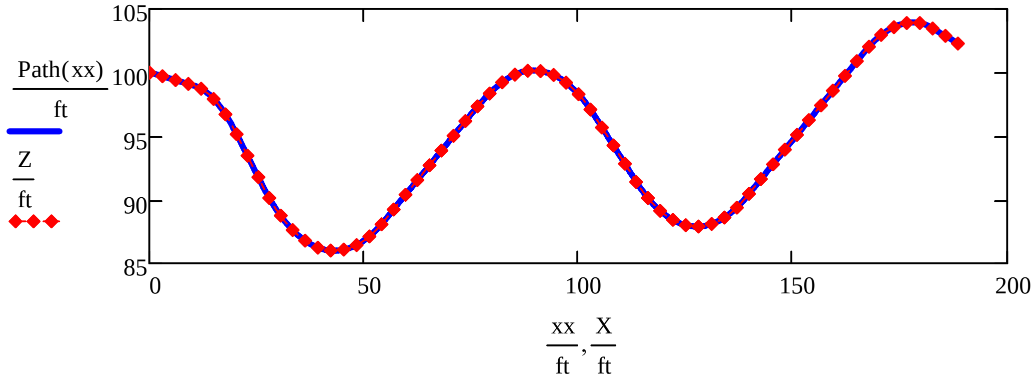
Yes the units are feet. The vehicle (inner Tube) travels from right to left on a wet surface entering the profile at 20 fts.

Now traveling left to right since X was reversed!

Build a function expression for the profile

$vs := \text{lspline}(X, Z)$ $\quad\quad$ $\text{Path}(x) := \text{interp}(vs, X, Z, x)$

$xx := 0\text{ft}, 6\text{in} .. 189\text{ft}$



slope of the profile is the derivative

$$slp(x) := \frac{d}{dx}Path(x) \qquad slp(0ft) = -5.639 \cdot deg$$

Now, gravity always points down, friction always points back the path.

$$F_{grav}(x) := Wgt \cdot \begin{pmatrix} 0 \\ 0 \\ -1 \end{pmatrix} \qquad F_{fric}(x) := (k_f \cdot Wgt) \begin{pmatrix} -\left|\sin(slp(x))\right| \\ 0 \\ \cos(slp(x)) \end{pmatrix} \qquad F_{fric}(0ft) = \begin{pmatrix} -4.717 \\ 0 \\ 47.768 \end{pmatrix} \cdot lbf$$

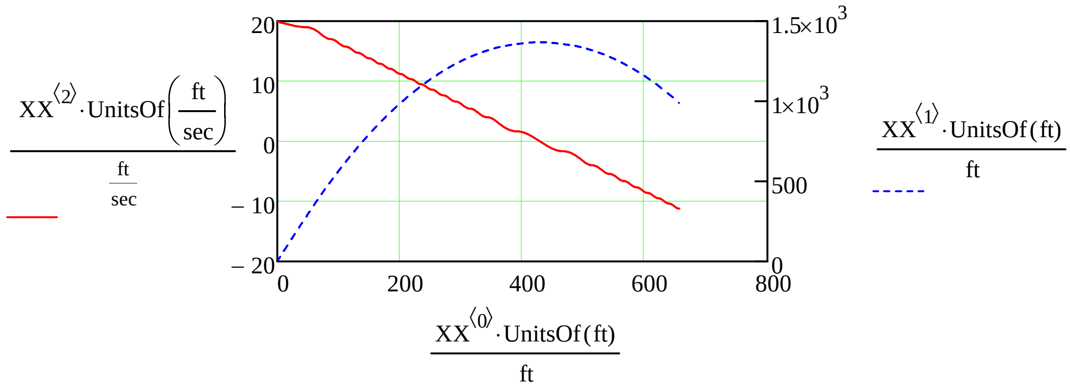
$$Acc(x) := \frac{F_{grav}(x) + F_{fric}(x)}{Wgt} \cdot g \qquad Acc(70ft) = \begin{pmatrix} -0.032 \\ 0 \\ -0.927 \end{pmatrix} \cdot g$$

$$V_o := 20\frac{ft}{sec} \cdot \begin{pmatrix} \cos(slp(0ft)) \\ 0 \\ \sin(slp(0ft)) \end{pmatrix} \qquad V_o = \begin{pmatrix} 19.903 \\ 0 \\ -1.965 \end{pmatrix} \cdot \frac{ft}{sec}$$

We set this problem up in full coordinates, but we only are interested in the X direction.

$$D(t, U) := \begin{bmatrix} U_1 \\ \dfrac{\left(F_{grav}\left(U_0 \cdot UnitsOf(ft)\right) + F_{fric}\left(U_0 \cdot UnitsOf(ft)\right)\right)_0}{Wgt} \end{bmatrix}$$

$$XX := rkfixed\left[\begin{pmatrix} 0 \\ \dfrac{V_{o_0}}{UnitsOf\left(\dfrac{ft}{sec}\right)} \end{pmatrix}, 0, 200, 1000, D\right]$$



Change friction factor to see effect: $k_f \equiv 0.08$