Flexible Sled

$ORIGIN := 1$

Basic Data

$XZ :=$

	1	2
1	500	100
2	498.997	...

$$XZ := \frac{XZ \cdot ft}{m}$$

These distances are supplied in feet, but it's easier to deal with Odesolve later if everything is unitless. Because this worksheet has SI units as its default, I've divided by metres to get the correct dimensionless distances.

Extract the seperate x and z values.

$$X := XZ_{1,1} - XZ^{\langle 1 \rangle}$$

Reset the x distances so they are measured from zero.

$$Z := XZ^{\langle 2 \rangle}$$

Gravitational acceleration

$$g := 9.807$$

Mass of vehicle

$$mass := \frac{600lb}{kg} \qquad mass = 272.155$$

Again we remove units

Friction coefficient

$$\mu := 0.08$$

Initial velocity

$$v0 := \frac{20\frac{ft}{s}}{\frac{m}{s}} \qquad v0 = 6.096$$

with units removed

Basic Data

Useful Functions

Function to obtain z value at any x, by linear interpolation

$$z(x) := \text{linterp}(X, Z, x)$$

Gradient function g1 from linearly interpolating gradients obtained by numerical differentiation at supplied data points.

$$gX := \overrightarrow{\frac{d}{dX} z(X)}$$

$$g1(x) := \text{linterp}(X, gX, x)$$

Rate of change of gradient function g2

$$g2(x) := \frac{d}{dx} g1(x)$$

Angle of curve to horizontal function θ

$$\theta(x) := \text{atan}(g1(x))$$

Radius of curvature of surface function: Radius
(see http://en.wikipedia.org/wiki/Radius_of_curvature_(mathematics) for the basic expression)
I've assumed that any radius greater than 50m is essentially infinite (i.e. it's a straight section).
There is a sign associated with the radius in order to get the right sign for the centripetal force on the mass - see the Normal force function below.

$$\text{Radius}(dzdx, d2zdx2) := \left| \begin{array}{l} \text{Rmag} \leftarrow \text{NaN} \quad \text{on error} \left| \dfrac{\left(1 + dzdx^2\right)^{\frac{3}{2}}}{d2zdx2} \right| \\ \text{return } \infty \text{ if } \text{IsNaN}(\text{Rmag}) \vee |\text{Rmag}| > 50 \\ \text{Rmag} \cdot \text{sign}(d2zdx2) \end{array} \right.$$

Function R to obtain radius at any point by interpolating between radii at supplied data points.

$$j := 1 .. \text{last}(X) \qquad RX_j := \text{Radius}\left(gX_j, g2\left(X_j\right)\right)$$

$$R(x) := \text{linterp}(X, RX, x)$$

Normal force function N. When radius is positive the centripetal force increases the normal force; when negative it reduces it.

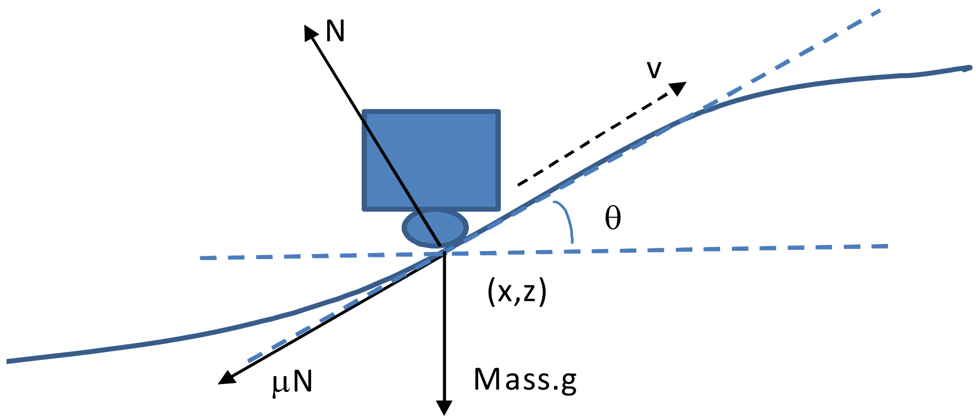
$$N(x, v) := \text{mass} \cdot \left(g \cdot \cos(\theta(x)) + \frac{v^2}{R(x)} \right)$$

Useful Functions

Single point model

Let's start by considering a single point model. That is, the whole mass of the sled is considered to be concentrated in a point. This is not going to be a very accurate model for the flexible extended sled, but it will provide a first step to a more complicated representation.

The picture below shows the forces (weight, normal and friction) at an arbitrary point (x,y).



Solve ODEs

Initially assume a long end time (seconds)

$t_{end} := 5$

Given

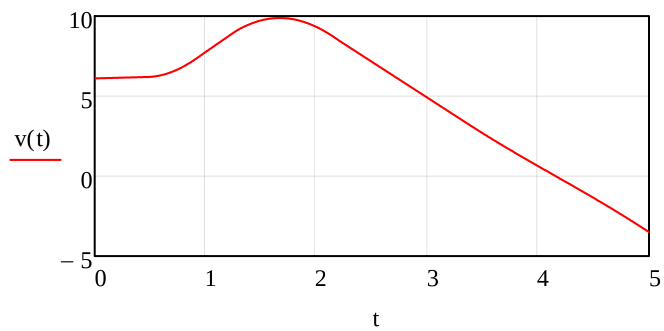
$$\frac{d}{dt}x(t) = \frac{v(t)}{\sqrt{1 + g1(x(t))^2}} \qquad x(0) = 0$$

$$\frac{d}{dt}v(t) = -g \cdot \sin(\theta(x(t))) - \frac{\mu}{mass} \cdot N(x(t), v(t)) \qquad v(0) = v0$$

$$\begin{pmatrix} x \\ v \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} x \\ v \end{pmatrix}, t, t_{end}\right]$$

v is ds/dt where s is distance measured along the surface. $ds^2 = dx^2 + dz^2$ so $ds/dt = dx/dt.(1+(dz/dx)^2)^{0.5}$. This is where the term in the denominator of the RHS of the equation for dx/dt comes from.

Plot velocity against time and look for first time at which velocity goes to zero.



Find end time of forward movement (which will occur just after 4 seconds)

$\tau := 4$

Given $\quad v(\tau) = 0 \quad \tau := \mathrm{Find}(\tau)$

$\tau = 4.155$

Find values of x and y at time τ

$x(\tau) = 23.995$ metres $\quad z(x(\tau)) = 29.921$ metres

Plot surface profile from time 0 to time τ

$npts := 500 \quad k := 1 .. npts \quad t_k := t_{end} \cdot \frac{k}{npts}$

$height_k := z\left(x\left(t_k\right)\right) \cdot m \quad dist_k := 500ft - x\left(t_k\right) \cdot m$