Flexible Sled

$ORIGIN := 1$

Basic Data

$XZ :=$

	1	2
1	500	100
2	498.997	...

$$XZ := \frac{XZ \cdot ft}{m}$$

These distances are supplied in feet, but it's easier to deal with Odesolve later if everything is unitless. Because this worksheet has SI units as its default, I've divided by metres to get the correct dimensionless distances.

Extract the seperate x and z values.

$$X := XZ_{1,1} - XZ^{\langle 1 \rangle}$$

Reset the x distances so they are measured from zero.

$$Z := XZ^{\langle 2 \rangle}$$

Calculate distances along surface at data points

$$i := 2 .. \text{last}(X)$$

$$S_1 := 0 \qquad S_i := \sqrt{\left(X_i - X_{i-1}\right)^2 + \left(Z_i - Z_{i-1}\right)^2} + S_{i-1}$$

Calculate angles to the horizontal of the surface between succesive data points (assuming a linear change between them).

$$j := 1 .. \text{last}(X) - 1$$

$$\Theta_j := \text{atan}\left(\frac{Z_{j+1} - Z_j}{X_{j+1} - X_j}\right) \qquad \Theta_{\text{last}(X)} := 0$$

Gravitational acceleration

$g := 9.807$

Mass of vehicle

$mass := \frac{600lb}{kg}$ $mass = 272.155$ Again we remove units

Length of vehicle

$L := 11.5 \frac{ft}{m}$ $L = 3.505$ Units removed

Friction coefficient

$\mu := 0.08$

Initial velocity

$v0 := \frac{20 \frac{ft}{s}}{\frac{m}{s}}$ $v0 = 6.096$ with units removed

Basic Data

Useful Functions

Function to obtain z value at any x, by linear interpolation

$$z1(x) := \left| \begin{array}{l} \text{return } Z_1 \text{ if } x < 0 \\ \text{linterp}(X, Z, x) \end{array} \right.$$

Function to obtain x value at any s, by linear interpolation

$$x(s) := \begin{array}{|l} \text{return } 0 \text{ if } s < 0 \\ \text{linterp}(S, X, s) \end{array}$$

Function to obtain θ at any x by linear interpolation

$$\theta(x) := \begin{array}{|l} \text{return } 0 \text{ if } x < 0 \\ \text{linterp}(X, \Theta, x) \end{array}$$

Gradient function g1

$$g1(x) := \tan(\theta(x))$$

Function g2 to obtain rate of change of gradient g1 wrt x (i.e. dg1(x)/dx)

$$g2(x) := \left(\tan(\theta(x))^2 + 1\right) \cdot \frac{d}{dx}\theta(x)$$

Radius of curvature of surface function: R
(see http://en.wikipedia.org/wiki/Radius_of_curvature_(mathematics) for the basic expression).
There is a sign associated with the radius in order to get the right sign for the centripetal force on the mass - see the Normal force function below.

$$R(x) := \begin{array}{|l} dg1dx \leftarrow g2(x) \\ \text{return } \infty \text{ if } |dg1dx| < 10^{-6} \\ \dfrac{\left(1 + g1(x)^2\right)^{\frac{3}{2}}}{dg1dx} \end{array}$$

Normal force function N averaged over length of vehicle. When radius is positive the centripetal force increases the normal force; when negative it reduces it.

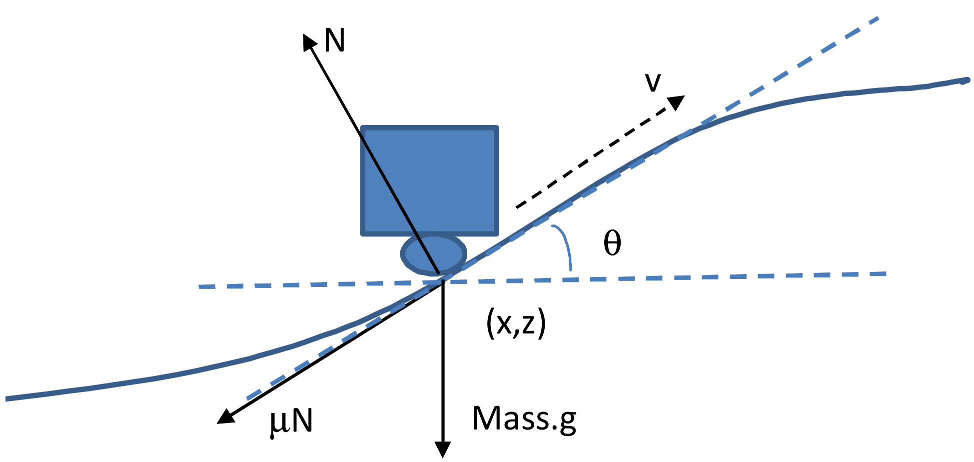
$$N(s,v) := \max\left[\frac{mass}{L}\cdot g\cdot\left(\int_{s-L}^{s} \cos(\theta(x(s))) + \frac{v^2}{g\cdot R(x(s))}\,ds\right), 0\right]$$

Useful Functions

Extended single point model

Here we still consider the whole mass of the sled to be concentrated in a point when calculating movement, but now we base the forces on a continuous stretch of the vehicle covering the surface.

The picture below shows the forces (weight, normal and friction) at an arbitrary point (x,z).



Solve ODEs

Initially assume a long end time (seconds)

$t_{end} := 6$

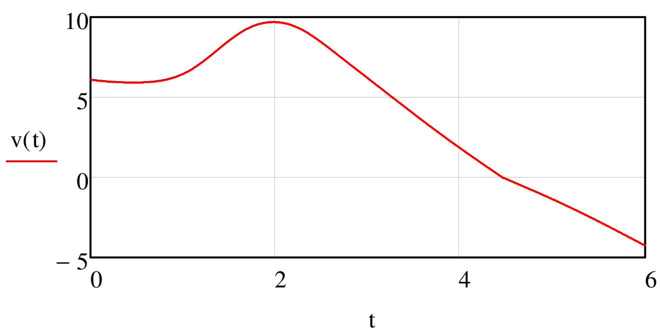
Given

$$\frac{d}{dt}s(t) = v(t) \qquad s(0) = 0$$

$$\frac{d}{dt}v(t) = \frac{-g}{L}\cdot\int_{s(t)-L}^{s(t)} \sin(\theta(x(s)))\,ds - \frac{\mu}{mass}\cdot N[(s(t)), v(t)]\cdot \text{sign}(v(t)) \qquad v(0) = v0$$

$$\begin{pmatrix} s \\ v \end{pmatrix} := \text{Odesolve}\left[\begin{pmatrix} s \\ v \end{pmatrix}, t, t_{end}\right]$$

Plot velocity against time and look for first time at which velocity goes to zero.



Find end time of forward movement (which will occur just after 4 seconds)

$\tau := 5$

Given $\quad v(\tau) = 0 \quad \tau := \text{Find}(\tau)$

$\tau = 4.454$

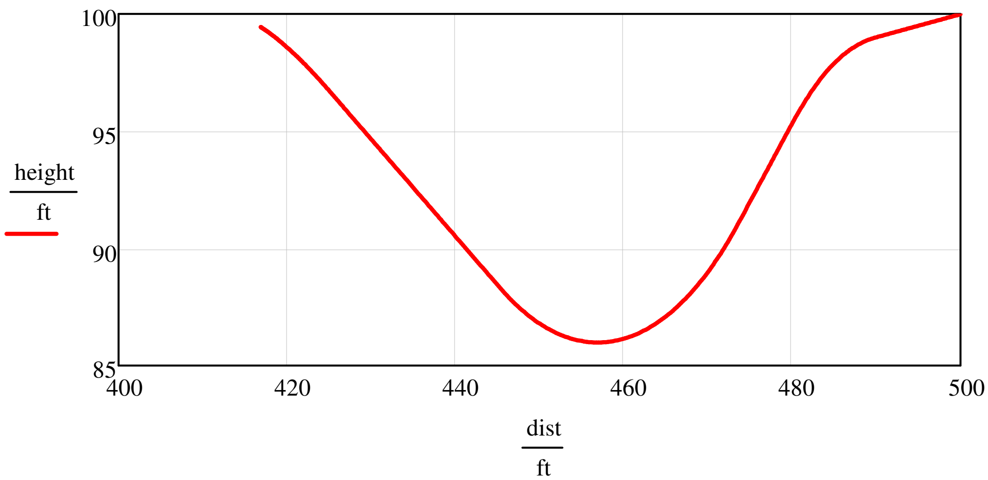
Find values of x and z at time τ

$x(s(\tau)) = 25.337$ metres $\qquad z1(x(s(\tau))) = 30.314$ metres

Plot surface profile from time 0 to time τ (this represents the position of the front of the vehicle).

$npts := 500 \qquad k := 1 .. npts \qquad t_k := \tau\cdot\frac{k-1}{npts-1}$

$$\text{height}_k := z1(x(s(t_k)))\cdot m \qquad \text{dist}_k := 500ft - x(s(t_k))\cdot m$$



Validity checks

Need to check that normal force is always positive as model is invalid otherwise.

tt is the time at which the normal force is closest to 0.

$tt := 0.1$ initial guess

Given

$N(s(tt), v(tt)) = 0$

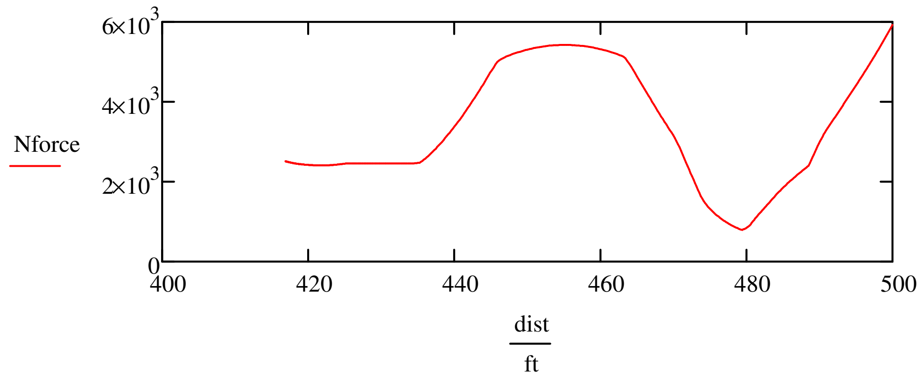
$tt := \text{Minerr}(tt)$

$tt = 1.1$

$N(s(tt), v(tt)) = 818.016$ smallest value of averaged normal force

Plot averaged normal force as a function of horizontal distance

$$\text{Nforce}_k := N(s(t_k), v(t_k))$$



Validity checks