The scenario is two vessels connected in series. The first vessel is initially full of water while the second vessel is empty. At time = 0, a flow of a second fluid starts into vessel 1. The overflow from vessel 1 goes into vessel 2. When vessel 2 is filled, then the overflow from vessel 2 goes to a drain.

I want to model the volume fraction of the second fluid in each vessel as a function of time. Assume the vessels are instantaneously perfectly mixed.

The equations are:

\[ \frac{d}{dt} F_1(t) = Q_0 - F_1(t) \cdot \frac{Q_0}{V_1} \]

\[ \frac{d}{dt} F_2(t) = F_1(t) \cdot \frac{Q_0}{A_2 \cdot h_2(t)} \] when vessel 2 is not full

\[ \frac{d}{dt} F_2(t) = F_1(t) \cdot \frac{Q_0}{A_2 \cdot h_2(t)} - F_2(t) \cdot \frac{Q_0}{V_2} \] when vessel 2 is full

\[ h_2(t) = \frac{Q_0}{A_2} \cdot t \] when vessel 2 is not full

\[ h_2(t) = \frac{V_2}{A_2} \] when vessel 2 is full

Subscripts 1 and 2 refer to vessel 1 and 2.
F is the volume of the second fluid in each vessel.
Q_0 is the flow rate of the second fluid into vessel 1.
V is the vessel volume, A is the area of the vessel base.
h_2 is the depth of the mixture in vessel 2.
t is time.

The question is how to solve this when the equations change at a certain point. It seems the easiest would be an IF test in the solve block to check when vessel 2 is filled. However, it seems an IF test is not allowed in a solve block. The only solution I’ve come up with is to use the step function, Φ(x). I’ve found that this can be made to work, but it has some odd behavior that must be accounted for. Φ(x) = 1/2 when x = 0. I’ve given two solve blocks with the step function slightly changed in the second so that the solution seems to work. The solution in the first block results in a nonsensical answer. It seems to be due to the step function randomly (?) varying between 0, 0.5, and 1 where it should be 0.

Any suggestions for a more clean way to handle this?
Set parameter values:

\[ H := 1 \cdot m \]

\[ A_2 := H^2 = 1 \ m^2 \quad V_1 := H^3 = 1 \ m^3 \quad V_2 := V_1 = 1 \ m^3 \]

\[ Q_0 := 1 \cdot \frac{m^3}{hr} \]

\[ t_0 := 0 \cdot hr \quad t_{end} := 10 \cdot \frac{V_1}{Q_0} = 10 \ hr \quad nsteps := 1000 \]

Solution block 1: Step function handled the way it looks like it should be.

\[
F_1(t_0) = 0 \cdot m^3 \quad F_2(t_0) = 0 \cdot m^3 \quad h_2(t_0) = 0.0001 \cdot \frac{V_2}{A_2}
\]

\[
\frac{d}{dt} F_1(t) = Q_0 - F_1(t) \cdot \frac{Q_0}{V_1}
\]

\[
h_2(t) = \min \left( \frac{V_2}{A_2}, \frac{Q_0 \cdot t}{A_2} \right)
\]

\[
\frac{d}{dt} F_2(t) = F_1(t) \cdot \frac{Q_0}{V_1} - F_2(t) \cdot Q_0 \cdot \left(1 - \Phi \left(\frac{V_2}{A_2} - h_2(t)\right)\right)
\]

\[
\begin{bmatrix} F_1 \\ F_2 \\ h_2 \end{bmatrix} := \text{odesolve} \left( \begin{bmatrix} F_1(t) \\ F_2(t) \\ h_2(t) \end{bmatrix}, t_{end}, nsteps \right)
\]

\[ i := 0 \ldots 1000 \]

\[ tt_i := i \cdot \frac{t_{end}}{nsteps} \]
This result in nonsensical.

The step function is not behaving as expected. It should be 0 in the area where it fluctuates.
Solve block 2 with the step function modified to force it to be 0 when it should be.

\[ F_{11}(t_0) = 0 \cdot m^3 \quad F_{22}(t_0) = 0 \cdot m^3 \quad h_{22}(t_0) = 0.0001 \cdot m \]

\[ \frac{d}{dt} F_{11}(t) = Q_0 - F_{11}(t) \cdot \frac{Q_0}{V_1} \]

\[ h_{22}(t) = \min \left( \frac{V_2}{A_2}, \frac{Q_0 \cdot t}{A_2} \right) \]

\[ \frac{d}{dt} F_{22}(t) = F_{11}(t) \cdot \frac{Q_0}{V_1} - F_{22}(t) \cdot \frac{Q_0 \cdot \left( 1 - \Phi \left( \frac{0.99 \cdot V_2}{A_2} \cdot h_2(t) \right) \right) \cdot A_2 \cdot h_2(t)}{A_2} \]

\[
\begin{bmatrix}
F_{11} \\
F_{22} \\
h_{22}
\end{bmatrix}
= \text{odesolve}
\begin{bmatrix}
F_{11}(t) \\
F_{22}(t) \\
h_{22}(t)
\end{bmatrix}, t_{\text{end}}, n_{\text{steps}}
\]

This result seems sensible.
The step function behaves as expected.

\[ \Phi \left( \frac{0.99 \cdot V \cdot (tt_i)}{A_2 - h_2(tt_i)} \right) \]

\[ tt_i (s) \]