The scenario is two vessels connected in series. The first vessel is initially full of water while the second vessel is empty. At time = 0, a flow of a second fluid starts into vessel 1. The overflow from vessel 1 goes into vessel 2. When vessel 2 is filled, then the overflow from vessel 2 goes to a drain.

I want to model the volume fraction of the second fluid in each vessel as a function of time. Assume the vessels are instantaneously perfectly mixed.

The equations are:

$$\frac{d}{dt}F_1(t) = Q_0 - F_1(t) \cdot \frac{Q_0}{V_1}$$

$$\frac{d}{dt}F_2(t) = F_1(t) \cdot \frac{Q_0}{A_2 \cdot h_2(t)}$$ when vessel 2 is not full

$$\frac{d}{dt}F_2(t) = F_1(t) \cdot \frac{Q_0}{A_2 \cdot h_2(t)} - F_2(t) \cdot \frac{Q_0}{V_2}$$ when vessel 2 is full

$$h_2(t) = \frac{Q_0}{A_2} \cdot t$$ when vessel 2 is not full

$$h_2(t) = \frac{V_2}{A_2}$$ when vessel 2 is full

Subscripts 1 and 2 refer to vessel 1 and 2.
F is the volume of the second fluid in each vessel.
Q_0 is the flow rate of the second fluid into vessel 1.
V is the vessel volume, A is the area of the vessel base.
h_2 is the depth of the mixture in vessel 2.
t is time.

The question is how to solve this when the equations change at a certain point. It seems the easiest would be an IF test in the solve block to check when vessel 2 is filled. However, it seems an IF test is not allowed in a solve block. The only solution I've come up with is to use the step function, $\Phi(x)$. I've found that this can be made to work, but it has some odd behavior that must be accounted for. $\Phi(x)$ = 1/2 when x = 0. I've given two solve blocks with the step function slightly changed in the second so that the solution seems to work. The solution in the first block results in a nonsensical answer. It seems to be due to the step function randomly (?) varying between 0, 0.5, and 1 where it should be 0.

Any suggestions for a more clean way to handle this?

Set parameter values:

$H := 1 \cdot m$

$A_2 := H^2 = 1\ m^2$ $V_1 := H^3 = 1\ m^3$ $V_2 := V_1 = 1\ m^3$

$Q_0 := 1 \cdot \frac{m^3}{hr}$

$t_0 := 0 \cdot hr$ $t_{end} := 10 \cdot \frac{V_1}{Q_0} = 10\ hr$ $nsteps := 1000$

Solution block 1: Step function handled the way it looks like it should be.

Guess Values

Constraints

$F_1(t_0) = 0 \cdot m^3$ $F_2(t_0) = 0 \cdot m^3$ $h_2(t_0) = 0.0001 \cdot \frac{V_2}{A_2}$

$$\frac{d}{dt}F_1(t) = Q_0 - F_1(t) \cdot \frac{Q_0}{V_1}$$

$$h_2(t) = \min\left(\frac{V_2}{A_2}, \frac{Q_0 \cdot t}{A_2}\right)$$

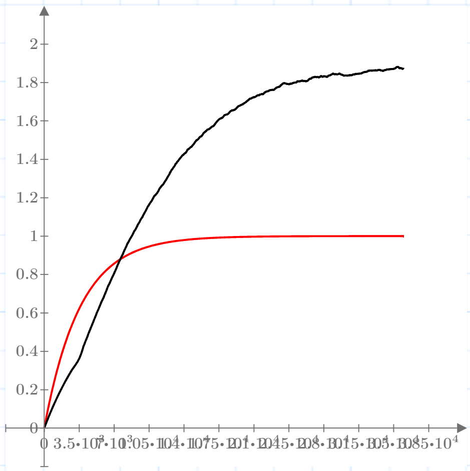
$$\frac{d}{dt}F_2(t) = F_1(t) \cdot \frac{Q_0}{V_1} - F_2(t) \cdot \frac{Q_0 \cdot \left(1 - \Phi\left(\frac{V_2}{A_2} - h_2(t)\right)\right)}{A_2 \cdot h_2(t)}$$

Solver

$$\begin{bmatrix} F_1 \\ F_2 \\ h_2 \end{bmatrix} := \mathbf{odesolve}\left(\begin{bmatrix} F_1(t) \\ F_2(t) \\ h_2(t) \end{bmatrix}, t_{end}, nsteps\right)$$

$i := 0 .. 1000$

$tt_i := i \cdot \frac{t_{end}}{nsteps}$



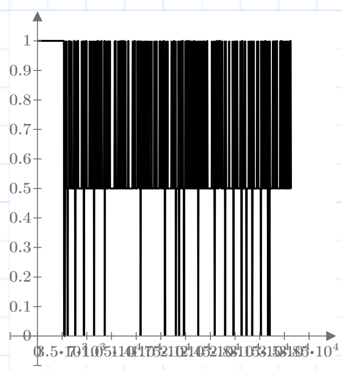
$$\frac{F_1\left(tt_i\right)}{V_1}$$

$$\frac{F_2\left(tt_i\right)}{A_2 \cdot h_2\left(tt_i\right)}$$

$tt_i \ (s)$

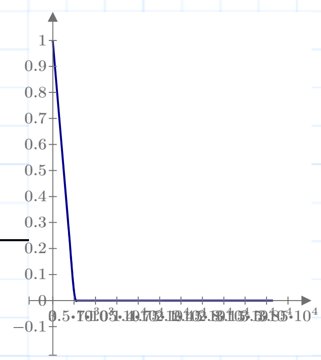
This result in nonsensical.

The step function is not behaving as expected. It should be 0 in the area where it fluctuates.



$$\Phi\left(\frac{V_2}{A_2} - h_2\left(tt_i\right)\right)$$

$tt_i \ (s)$



$$\frac{V_2}{A_2} - h_2\left(tt_i\right) \ (m)$$

$tt_i \ (s)$

Solve block 2 with the step function modified to force it to be 0 when it should be.

Guess Values

$F_{11}(t_0) = 0 \cdot m^3 \qquad F_{22}(t_0) = 0 \cdot m^3 \qquad h_{22}(t_0) = 0.0001 \cdot m$

Constraints

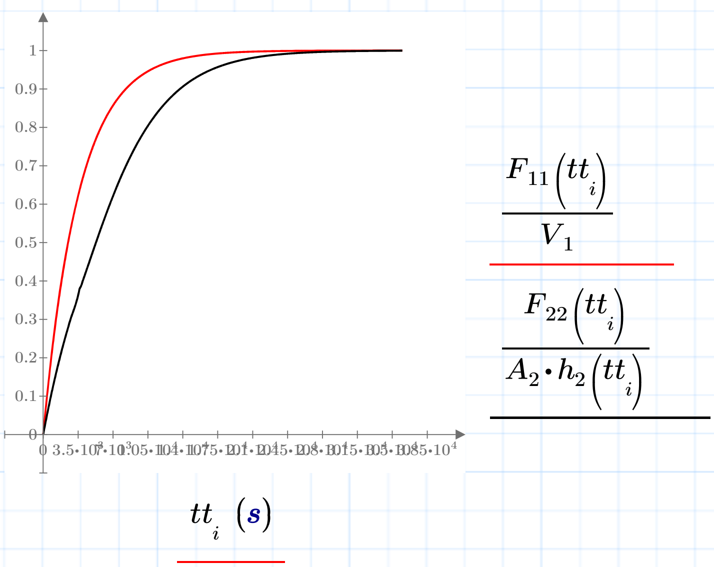
$$\frac{d}{dt} F_{11}(t) = Q_0 - F_{11}(t) \cdot \frac{Q_0}{V_1}$$

$$h_{22}(t) = \min\left(\frac{V_2}{A_2}, \frac{Q_0 \cdot t}{A_2}\right)$$

$$\frac{d}{dt} F_{22}(t) = F_{11}(t) \cdot \frac{Q_0}{V_1} - F_{22}(t) \cdot \frac{Q_0 \cdot \left(1 - \Phi\left(.99 \cdot \frac{V_2}{A_2} - h_2(t)\right)\right)}{A_2 \cdot h_2(t)}$$

Solver

$$\begin{bmatrix} F_{11} \\ F_{22} \\ h_{22} \end{bmatrix} := \mathbf{odesolve}\left(\begin{bmatrix} F_{11}(t) \\ F_{22}(t) \\ h_{22}(t) \end{bmatrix}, t_{end}, nsteps\right)$$



This result seems sensible.

The step function behaves as expected.