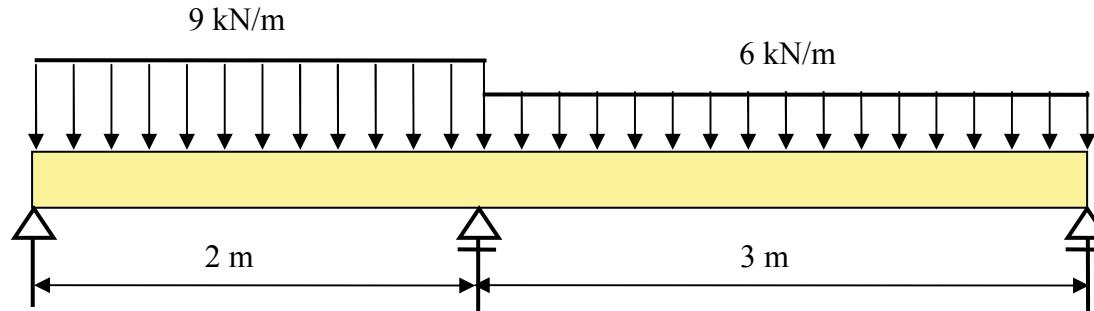


SA2 - Assignment 1: Analysis of beam structure

Consider a 2-D beam structure exposed to a given load, as shown below (the prescribed load includes self-weight of the beam and all other loads).



The beam is made of timber and has a rectangular cross-section of 0.3×0.15 m (height x width). For the sake of simplicity, assume that the material has isotropic elastic properties with Young modulus of 12 GPa and Poisson ratio of 0.35. Use the Timoshenko beam theory to solve the following problems:

- (a) Plot the deflection (w), slope of the deformed center line due to bending (ψ) and that due to shearing (β), shear force (V) and bending moment (M) along the beam axis. Identify the locations and values of extremes of w , M , V .
- (b) Plot the cross-sectional distributions of:
 - normal and shear stress at the sections immediately to the left and to the right of the intermediate support,
 - normal stress at the location of extreme bending moment within one of the spans (away from the supports). Select the span, where the magnitude of the moment is higher).

Include numerical values of extremes in the plots.

Before we begin the analysis, the Maxima environment must be properly set up and the required libraries loaded:

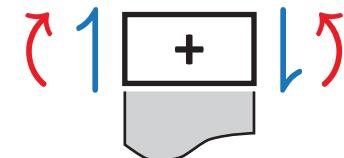
```
(%i1) kill(all)$  
ratprint:false$  
fpprintprec:4$  
load(pw)$
```

1 Definitions

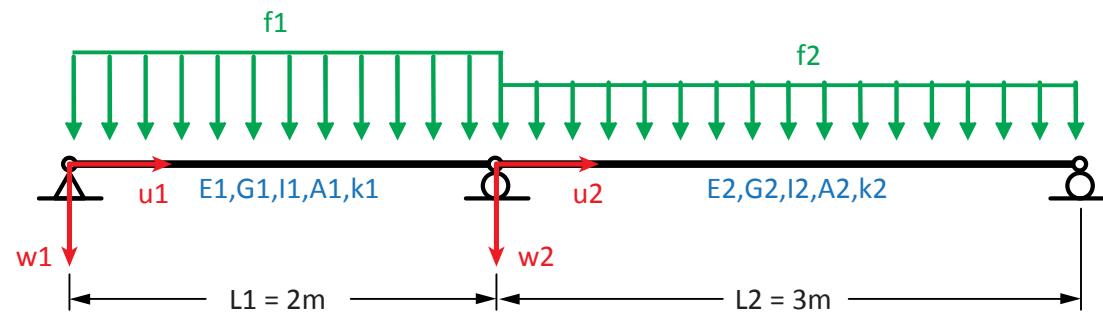
1.1 Sign convention and model setup

For the purpose of this analysis, the following sign convention has been adopted:

- i) Positive shear force spins an element clockwise (i.e. up on the left, and down on the right),
- ii) Positive bending moment warps the element concave upward (i.e. clockwise on the left, and counterclockwise on the right)
- iii) Bending moment diagram shall be drawn on the tension side of the beam.



The structure is divided into two beam elements (elements 1 and 2) as follows:



1.2 Analysis parameters

The following parameters are assumed throughout the calculations. They describe the geometry, material, cross section and loading parameters of the analysis. Please note that the units used throughout this document are in kN, m, kN/m, kPa, and rad.

```
(%i4) par:[  
E1=12e6,G1=12e6/(2*(1+0.35)),I1=1/12*0.15*0.3^3,A1=0.15*0.3,L1=2,f1=9,k1=(10*(1+0.35))/(12+(11*0.35)),nu1=0.35,  
E2=12e6,G2=12e6/(2*(1+0.35)),I2=1/12*0.15*0.3^3,A2=0.15*0.3,L2=3,f2=6,k2=(10*(1+0.35))/(12+(11*0.35)),nu2=0.35]$
```

□ **2 General Solution of Governing Equations of Bending Response**

□ **2.1 Governing differential equations**

From equilibrium analysis of the infinitesimal section, we were able to derive two differential equations:

$$\downarrow \sum F_z \quad \frac{d}{dx} kGA\beta(x) = -f_z \\ \frac{d}{dx} \beta(x) = \frac{-f_z}{kGA}$$

$$\zeta \sum M \quad EI \frac{d^2}{dx^2} \Psi(x) + kGA\beta(x) = 0 \\ \frac{d^2}{dx^2} \Psi(x) = \frac{-kGA\beta(x)}{EI}$$

Also recall that:

$$\frac{d}{dx} w(x) = \beta(x) + \Psi(x)$$

Thus for the two elements we shall have the following differential equations to solve:

(%i5) eqbeta1 : diff(beta1(x),x,1)=-f1/(k1*G1*A1);
eqpsi1 : diff(psi1(x),x,2)=(-k1*G1*A1*beta1(x))/(E1*I1);
eqw1 : diff(w1(x),x,1)=beta1(x)+psi1(x);
eqbeta2 : diff(beta2(x),x,1)=-f2/(k2*G2*A2);
eqpsi2 : diff(psi2(x),x,2)=(-k2*G2*A2*beta2(x))/(E2*I2);
eqw2 : diff(w2(x),x,1)=beta2(x)+psi2(x);

$$(%o5) \frac{d}{dx} \text{beta1}(x) = -\frac{f1}{k1 A1 G1}$$

$$(%o6) \frac{d^2}{dx^2} \text{psi1}(x) = -\frac{k1 \text{beta1}(x) A1 G1}{E1 I1}$$

$$(%o7) \frac{d}{dx} w1(x) = \text{psi1}(x) + \text{beta1}(x)$$

$$(%o8) \frac{d}{dx} \text{beta2}(x) = -\frac{f2}{k2 A2 G2}$$

$$(%o9) \frac{d^2}{dx^2} \text{psi2}(x) = -\frac{k2 \text{beta2}(x) A2 G2}{E2 I2}$$

$$(%o10) \frac{d}{dx} w2(x) = \text{psi2}(x) + \text{beta2}(x)$$

Note that "x" stands for local coordinate along each beam element throughout the analysis, unless otherwise noted.

□ 2.2 Derivation of the general solution of the differential equations

□ In order to derive the equation for beta(x), psi(x) and w(x), a method of successive integration and substitution will be employed. We shall begin with finding beta(x):

□ (%i11) `reset(integration_constant_counter)$`

□ (%i12) `sbeta1:integrate(eqbeta1,x);
sbeta2:integrate(eqbeta2,x);`

$$(\%o12) \text{beta1}(x) = \%c1 - \frac{f1 x}{k1 A1 G1}$$

$$(\%o13) \text{beta2}(x) = \%c2 - \frac{f2 x}{k2 A2 G2}$$

□ We will then substitute the equation derived for beta(x) into the second differential equation and through successive integration, compute the general solution for psi(x):

□ (%i14) `spsi1:integrate(integrate(subst(rhs(sbeta1),beta1(x),eqpsi1),x),x);
spsi2:integrate(integrate(subst(rhs(sbeta2),beta2(x),eqpsi2),x),x);`

$$(\%o14) \text{psi1}(x) = -\frac{k1 A1 \left(\frac{\%c1 x^2}{2} - \frac{f1 x^3}{6 k1 A1 G1} \right) G1}{E1 I1} + \%c3 x + \%c4$$

$$(\%o15) \text{psi2}(x) = -\frac{k2 A2 \left(\frac{\%c2 x^2}{2} - \frac{f2 x^3}{6 k2 A2 G2} \right) G2}{E2 I2} + \%c5 x + \%c6$$

□ Finally, using both beta(x) and psi(x), we are now able to express the general form of the vertical displacement, w(x):

```
(%i16) sw1:integrate(subst(rhs(spsi1),psi1(x),subst(rhs(sbeta1),beta1(x),eqw1)),x);
      sw2:integrate(subst(rhs(spsi2),psi2(x),subst(rhs(sbeta2),beta2(x),eqw2)),x);

(%o16) w1(x)=-
$$\frac{k_1 A_1 \left(\frac{\%c_1 x^3}{6} - \frac{f_1 x^4}{24 k_1 A_1 G_1}\right) G_1}{E_1 I_1} - \frac{f_1 x^2}{2 k_1 A_1 G_1} + \frac{\%c_3 x^2}{2} + \%c_4 x + \%c_1 x + \%c_7$$

(%o17) w2(x)=-
$$\frac{k_2 A_2 \left(\frac{\%c_2 x^3}{6} - \frac{f_2 x^4}{24 k_2 A_2 G_2}\right) G_2}{E_2 I_2} - \frac{f_2 x^2}{2 k_2 A_2 G_2} + \frac{\%c_5 x^2}{2} + \%c_6 x + \%c_2 x + \%c_8$$

```

For convenience we re-assign the solutions found above to the functions:

```
(%i18) define(beta1(x),rhs(sbeta1))$  
define(psi1(x),rhs(spsi1))$  
define(w1(x),rhs(sw1))$  
define(beta2(x),rhs(sbeta2))$  
define(psi2(x),rhs(spsi2))$  
define(w2(x),rhs(sw2))$
```

It is important to note that the general solution of the differential equations involves eight integration constants (%c1 - %c8). These constants will be solved for using the boundary and continuity conditions, outline in Section 3.1.

Given the above solutions of the differential equations it is now possible to also derive the general solution of the bending moment, M(x), and shear force, V(x). These functions may be very useful for the computation of the constants using the boundary conditions.

```

(%i24) define(M1(x),-E1*I1*diff(psi1(x),x,1));
define(M2(x),-E2*I2*diff(psi2(x),x,1));
define(V1(x),k1*G1*A1*beta1(x));
define(V2(x),k2*G2*A2*beta2(x));

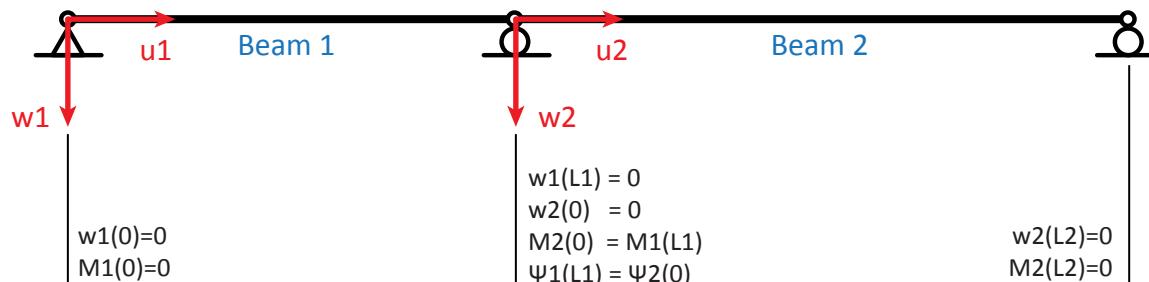
(%o24) M1(x):=-E1 \left( \frac{k1 A1 \left( \frac{f1 x^2}{2 k1 A1 G1} \right) G1}{E1 I1} \right) I1
(%o25) M2(x):=-E2 \left( \frac{k2 A2 \left( \frac{f2 x^2}{2 k2 A2 G2} \right) G2}{E2 I2} \right) I2
(%o26) V1(x):=k1 A1 \left( \frac{f1 x}{k1 A1 G1} \right) G1
(%o27) V2(x):=k2 A2 \left( \frac{f2 x}{k2 A2 G2} \right) G2

```

3 Specific Solution of the Boundary Value Problem

3.1 Definition of boundary and continuity conditions

In order to solve the specific solution of the boundary value problem, we need to define the boundary and continuity conditions for the structure. The following figure shows the prescribed boundary conditions:



Thus the conditions will be as follows:

1) Boundary conditions at the left support of Beam 1:

$$\begin{aligned} (\%i28) \quad & bc1:w1(0)=0; \\ & bc2:M1(0)=0; \\ (\%o28) \quad & \%c7=0 \\ (\%o29) \quad & -\%c3 E1 I1=0 \end{aligned}$$

2) Boundary conditions at the right support of Beam 2:

$$\begin{aligned} (\%i30) \quad & bc3:w2(L2)=0; \\ & bc4:M2(L2)=0; \\ (\%o30) \quad & -\frac{k2 A2 G2 \left(\frac{\%c2 L2^3}{6} - \frac{f2 L2^4}{24 k2 A2 G2} \right)}{E2 I2} - \frac{f2 L2^2}{2 k2 A2 G2} + \frac{\%c5 L2^2}{2} + \%c6 L2 + \%c2 L2 + \%c8 = 0 \\ (\%o31) \quad & -E2 I2 \left(\%c5 - \frac{k2 A2 G2 \left(\%c2 L2 - \frac{f2 L2^2}{2 k2 A2 G2} \right)}{E2 I2} \right) = 0 \end{aligned}$$

3) Continuity conditions at the middle support, between beams 1 and 2:

```
(%i32) bc5:w1(L1)=0;
      bc6:w2(0)=0;
      bc7:M2(0)=M1(L1);
      bc8:psi1(L1)=psi2(0);

(%o32) -
$$\frac{k_1 A_1 G_1 \left(\frac{\%c1 L_1^3}{6} - \frac{f_1 L_1^4}{24 k_1 A_1 G_1}\right)}{E_1 I_1} - \frac{f_1 L_1^2}{2 k_1 A_1 G_1} + \frac{\%c3 L_1^2}{2} + \%c4 L_1 + \%c1 L_1 + \%c7 = 0$$


(%o33) \%c8=0

(%o34) -\%c5 E_2 I_2 = -E_1 I_1 \left(\%c3 - \frac{k_1 A_1 G_1 \left(\%c1 L_1 - \frac{f_1 L_1^2}{2 k_1 A_1 G_1}\right)}{E_1 I_1}\right)

(%o35) -
$$\frac{k_1 A_1 G_1 \left(\frac{\%c1 L_1^2}{2} - \frac{f_1 L_1^3}{6 k_1 A_1 G_1}\right)}{E_1 I_1} + \%c3 L_1 + \%c4 = \%c6$$

```

3.2 Calculation of the integration constants

Substituting the analysis parameters into the boundary conditions and solving for the integration constants we will get:

```
(%i36) eqs:subst(par,[bc1,bc2,bc3,bc4,bc5,bc6,bc7,bc8])$  

      sol:linsolve(eqs,[\%c1,\%c2,\%c3,\%c4,\%c5,\%c6,\%c7,\%c8])$  

      float(ratsimp(sol ));  

(%o38) [\%c1=3.5864 10^-5, \%c2=6.4146 10^-5, \%c3=0.0, \%c4=2.8188 10^-4, \%c5=0.00143, \%c6=2.27879 10^-4, \%c7=0.0, \%c8=0.0]
```

3.3 Evaluation of the specific governing equations

Having found the integration constants in the previous section, it is now possible to substitute all of the parameters and enumerate all of the required functions. Please note, that for consistency, the enumerated functions are denoted with postscript "s".

Therefore for Beam 1, we will get the following functions:

```
(%i39) define(w1s(x),subst(append(sol,par),w1(x)));
define(M1s(x),subst(append(sol,par),M1(x)));
define(V1s(x),subst(append(sol,par),V1(x)));
define(beta1s(x),subst(append(sol,par),beta1(x)));
define(psi1s(x),subst(append(sol,par),psi1(x)));
(%o39) w1s(x) := -42.06 
$$\left( \frac{357788211991 x^3}{59857128480417918} - 2.2014 \cdot 10^{-6} x^4 \right) - 2.64167 \cdot 10^{-5} x^2 + \frac{9043791533454323 x}{28462064592438720009}
(%o40) M1s(x) := 170300. 
$$\left( \frac{357788211991 x}{9976188080069653} - 2.64167 \cdot 10^{-5} x^2 \right)
(%o41) V1s(x) := 170300. 
$$\left( \frac{357788211991}{9976188080069653} - 5.2833 \cdot 10^{-5} x \right)
(%o42) beta1s(x) := \frac{357788211991}{9976188080069653} - 5.2833 \cdot 10^{-5} x
(%o43) psi1s(x) := \frac{8023021764644000}{28462064592438720009} - 42.06 
$$\left( \frac{357788211991 x^2}{19952376160139306} - 8.8056 \cdot 10^{-6} x^3 \right)$$$$$$$$

```

Similarly for Beam 2, we will get the following functions:

```

(%i44) define(w2s(x),subst	append(sol,par),w2(x)));
define(M2s(x),subst-append(sol,par),M2(x));
define(V2s(x),subst-append(sol,par),V2(x));
define(beta2s(x),subst-append(sol,par),beta2(x));
define(psi2s(x),subst-append(sol,par),psi2(x));

(%o44) w2s(x) := -42.06 
$$\left( \frac{1919799975973 x^3}{179571385441253754} - 1.46759 \cdot 10^{-6} x^4 \right) + 6.9613 \cdot 10^{-4} x^2 + \frac{8311625462514323 x}{28462064592438720009}
(%o45) M2s(x) := -4050. 
$$\left( \frac{13542965440000000}{9487354864146240003} - 42.06 \left( \frac{1919799975973 x}{29928564240208959} - 1.76111 \cdot 10^{-5} x^2 \right) \right)
(%o46) V2s(x) := 170300. 
$$\left( \frac{1919799975973}{29928564240208959} - 3.5222 \cdot 10^{-5} x \right)
(%o47) beta2s(x) := \frac{1919799975973}{29928564240208959} - 3.5222 \cdot 10^{-5} x
(%o48) psi2s(x) := -42.06 
$$\left( \frac{1919799975973 x^2}{59857128480417918} - 5.8704 \cdot 10^{-6} x^3 \right) + \frac{13542965440000000 x}{9487354864146240003} + \frac{6485895685364000}{28462064592438720009}$$$$$$$$

```

4 Analysis Results - Deformations and Internal Forces

Having evaluated the specific governing equations, it is now possible to evaluate the internal forces, rotations and displacements in both elements. This section presents the results of the analysis.

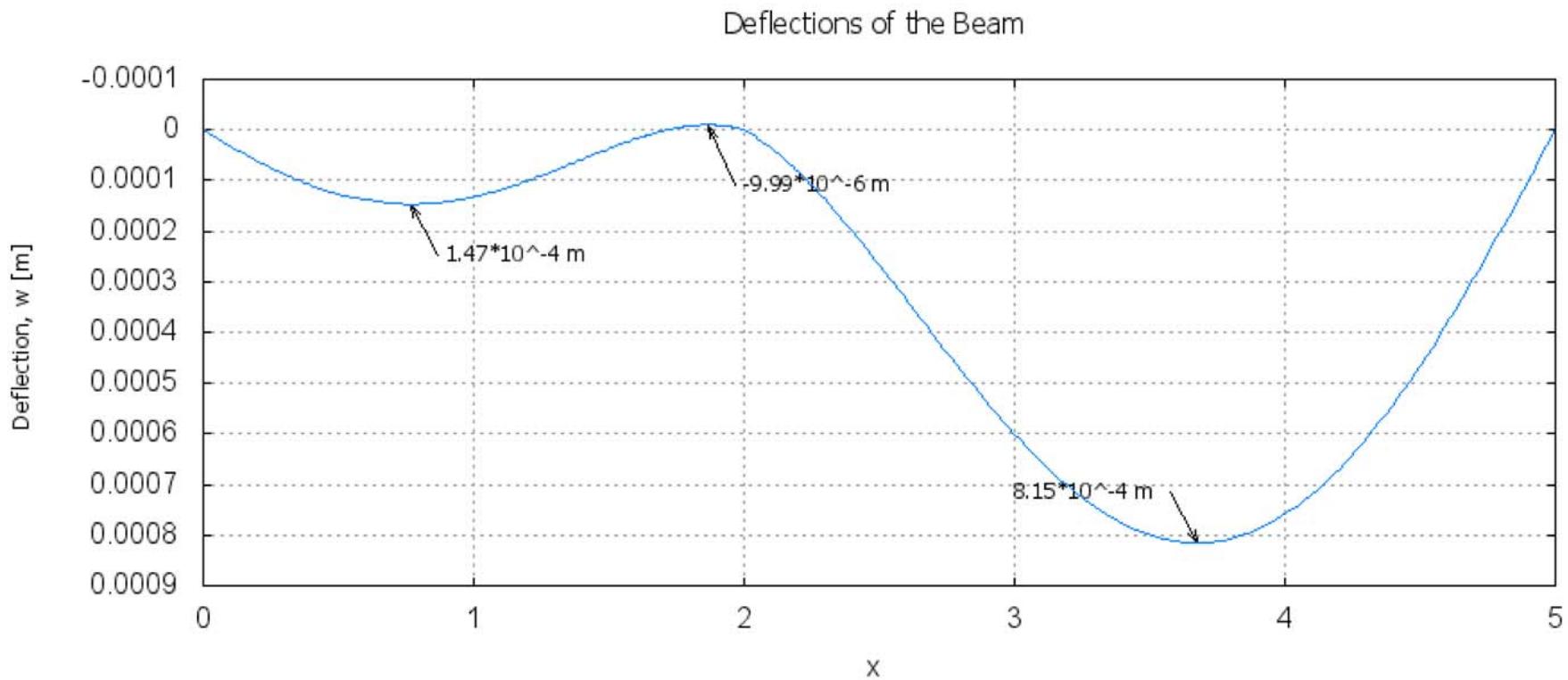
4.1 Deflections

The following plot shows the deflections along the length of the entire structure:

```
(%i49) wp(x):="(w1s(x)*between(x,0,subst(par,L1))+subst(x-subst(par,L1),x,w2s(x))*between(x,subst(par,L1),subst(par,L1+L2)))$
```

```
(%i50) wxplot2d(wp(x),[x,0,subst(par,L1+L2)],  
[gnuplot_preamble,"set yr range [:] reverse; set title 'Deflections of the Beam' font 'Tahoma, 12'; set grid;  
set ylabel 'Deflection, w [m]' font 'Tahoma, 10';  
set arrow 1 from 0.868,0.000247 to 0.768,0.000147; set label 1 '1.47*10^-4 m' at 0.9,0.000247 font 'Tahoma,9';  
set arrow 2 from 1.967,0.000109 to 1.867,-0.000009; set label 2 '-9.99*10^-6 m' at 2.0,0.000109 font 'Tahoma,9';  
set arrow 3 from 3.575,0.000715 to 3.675,0.000815; set label 3 '8.15*10^-4 m' at 3.0,0.000715 font 'Tahoma,9';"],  
wxplot_size=[900,400]$)
```

(%t50)



In order to find the locations with the extreme positive (downward) and negative (upward) displacements, the roots of the first derivative of the displacements is computed. Thus for Beam 1, the roots are:

```
(%i51) rw1:allroots(diff(w1s(x),x));  
(%o51) [x=0.768,x=-0.598,x=1.867]
```

Given the roots of the third order polynomial it is possible to see that only two of the roots apply. Thus the displacement at these two points would be:

```
(%i52) print("Extreme deflection of ", w1s(rhs(rw1[1])), "meters at ", rw1[1])$  
      print("Extreme deflection of ", w1s(rhs(rw1[3])), "meters at ", rw1[3])$
```

Extreme deflection of $1.46776 \cdot 10^{-4}$ meters at $x = 0.768$

Extreme deflection of $-9.9884 \cdot 10^{-6}$ meters at $x = 1.867$

Similarly for Beam 2, the location of the extreme displacements could be computed given the roots:

```
(%i54) rw2:allroots(diff(w2s(x),x));
```

```
(%o54) [x=-0.178,x=1.675,x=3.967]
```

In this case it can be observed that only one point will be of interest:

```
(%i55) print("Extreme deflection of ", w2s(rhs(rw2[2])), "meters at ", rw2[2], " (", rhs(rw2[2]+subst(par,L1)), " in global co-ordinate)")$
```

Extreme deflection of $8.149 \cdot 10^{-4}$ meters at $x = 1.675$ (3.675 in global co-ordinate)

4.2 Bending moments

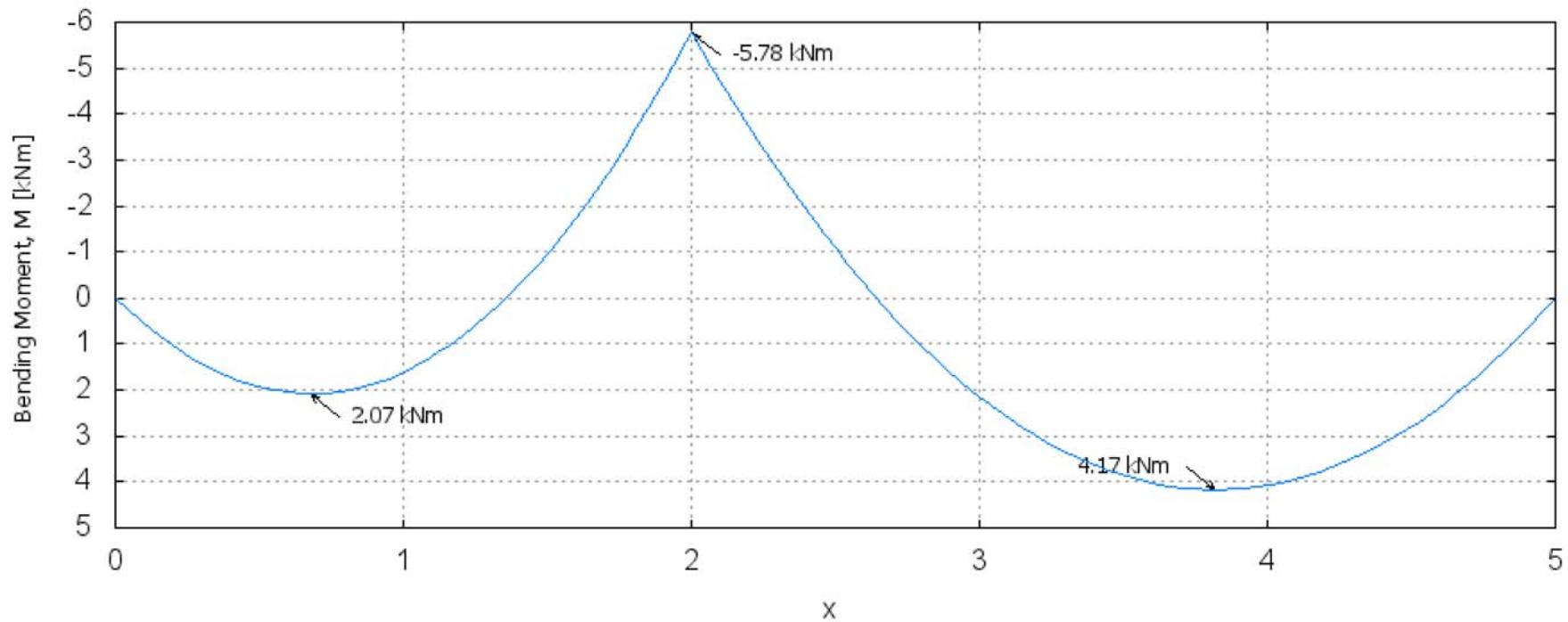
The following plot shows the bending moments along the length of the entire structure:

```
(%i56) Mp(x):="(M1s(x)*between(x,0,subst(par,L1))+subst(x-subst(par,L1),x,M2s(x))*between(x,subst(par,L1),subst(par,L1+L2)))$
```

```
(%i57) wxplot2d(Mp(x),[x,0,subst(par,L1+L2)],  
[gnuplot_preamble,"set yr range [:] reverse; set title 'Bending Moments in the Beam' font 'Tahoma, 12'; set grid;  
set ylabel 'Bending Moment, M [kNm]' font 'Tahoma, 10';  
set arrow 1 from 0.779,2.574 to 0.679,2.074; set label 1 '2.07 kNm' at 0.829,2.574 font 'Tahoma,9';  
set arrow 2 from 2.1,-5.281 to 2,-5.781; set label 2 '-5.78 kNm' at 2.15,-5.281 font 'Tahoma,9';  
set arrow 3 from 3.721,3.669 to 3.821,4.169; set label 3 '4.17 kNm' at 3.35,3.669 font 'Tahoma,9';"],  
wxplot_size=[900,400]$
```

(%)

Bending Moments in the Beam



By examination of the bending moment diagram, it is clear that the location with the highest bending is located over the intermediate support:

```
(%i58) print("Extreme bending moment of ", M1s(subst(par,L1)), "kNm at x = ", subst(par,L1))$
```

Extreme bending moment of -5.781 kNm at x = 2

In order to find the locations with the extreme bending moments within the spans, the roots of the first derivative of the bending moments is computed. Thus for Beam 1, the roots are:

```
(%i59) rM1:=allroots(diff(M1s(x),x));  
(%o59) [x = 0.679]
```

Given the roots of the first order polynomial it is possible to see that only one root exists. Thus the bending moment at this point would be:

```
(%i60) print("Extreme bending moment of ", M1s(rhs(rM1[1])), "kNm at ", rM1[1])$  
Extreme bending moment of 2.074 kNm at x = 0.679
```

Similarly for Beam 2, the location of the extreme bending moments could be computed given the roots:

```
(%i61) rM2:=allroots(diff(M2s(x),x));  
(%o61) [x = 1.821]
```

```
(%i62) print("Extreme bending moment of ", M2s(rhs(rM2[1])), "kNm at ", rM2[1], " (", rhs(rM2[1]+subst(par,L1)), " in global co-ordinate)")$  
Extreme bending moment of 4.169 kNm at x = 1.821 ( 3.821 in global co-ordinate)
```

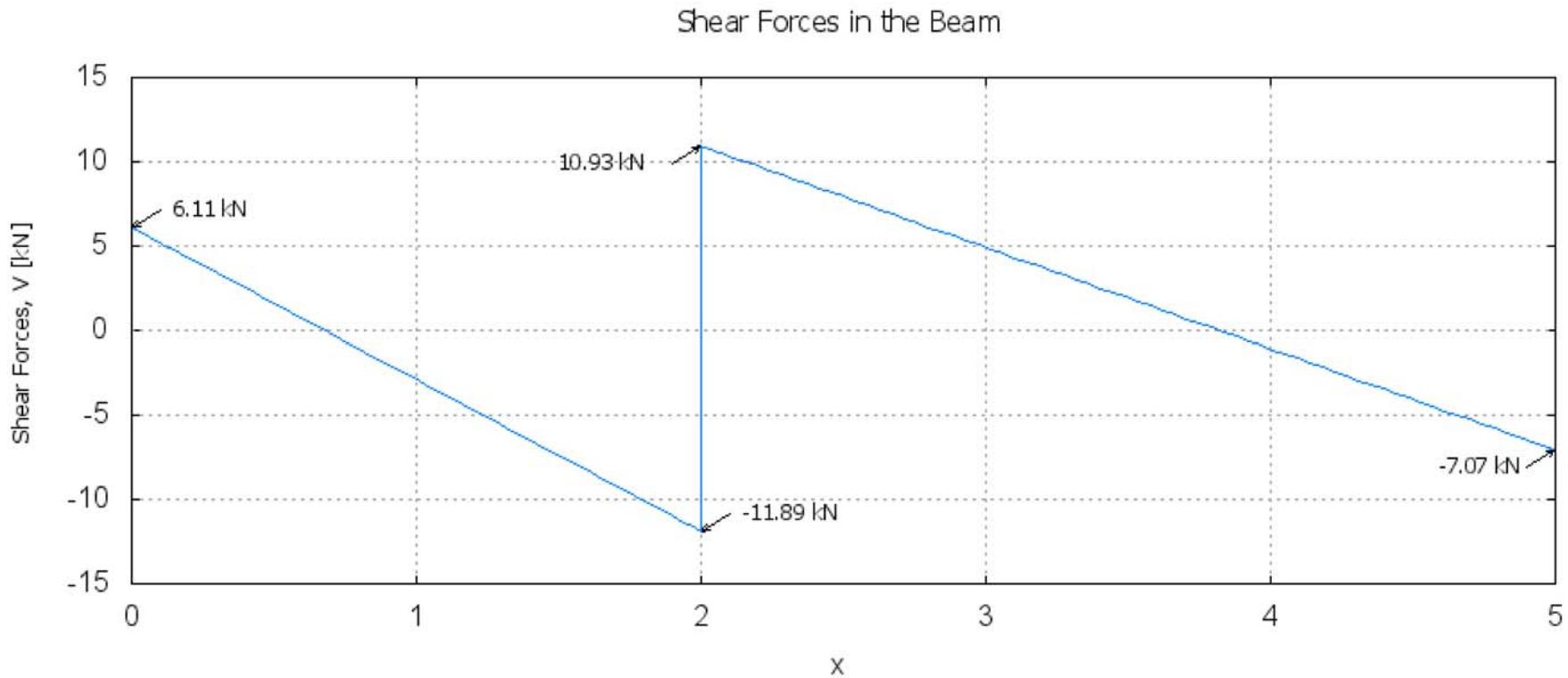
4.3 Shear forces

The following plot shows the shear forces along the length of the entire structure:

```
(%i63) Vp(x):=(V1s(x)*between(x,0,subst(par,L1))+subst(x-subst(par,L1),x,V2s(x))*between(x,subst(par,L1),subst(par,L1+L2)))$
```

```
(%i64) wxplot2d(Vp(x),[x,0,subst(par,L1+L2)],  
[gnuplot_preamble,"set title 'Shear Forces in the Beam' font 'Tahoma, 12'; set grid; set ylabel 'Shear Forces, V [kN]' font 'Tahoma, 10';  
set arrow 1 from 0.1,7.109 to 0.6,109; set label 1 '6.11 kN' at 0.15,7.109 font 'Tahoma,9';  
set arrow 2 from 2.1,-10.89 to 2,-11.89; set label 2 '-11.89 kN' at 2.15,-10.89 font 'Tahoma,9';  
set arrow 3 from 1.9,9.93 to 2,10.93; set label 3 '10.93 kN' at 1.5,9.93 font 'Tahoma,9';  
set arrow 4 from 4.9,-8.073 to 5,-7.073; set label 4 '-7.07 kN' at 4.6,-8.073 font 'Tahoma,9';"],  
wxplot_size=[900,400]$)
```

```
(%t64)
```



By examination of the shear force diagram, it is clear that the location with the highest shear forces is located over the intermediate support. It is also clear that the shear force is discontinuous over the support. Thus the value of the shear force will be given for both sides of the support:

```
✓ (%i65) print("Extreme shear force of ", V1s(subst(par,L1)), "kN at x = ", subst(par,L1), " (left side)")$  
      print("Extreme shear force of ", V2s(0), "kN at x = ", subst(par,L1), " (right side)")$  
Extreme shear force of -11.89 kN at x = 2 (left side)  
Extreme shear force of 10.93 kN at x = 2 (right side)
```

✓ The other two locations of high shear are at the end supports:

```
✓ (%i67) print("Extreme shear force of ", V1s(0), "kN at x = 0 (left support)")$  
      print("Extreme shear force of ", V2s(subst(par,L2)), "kN at x = ", subst(par,L1+L2), " (right support)")$  
Extreme shear force of 6.109 kN at x = 0 (left support)  
Extreme shear force of -7.073 kN at x = 5 (right support)
```

4.4 Slope due to shear and bending

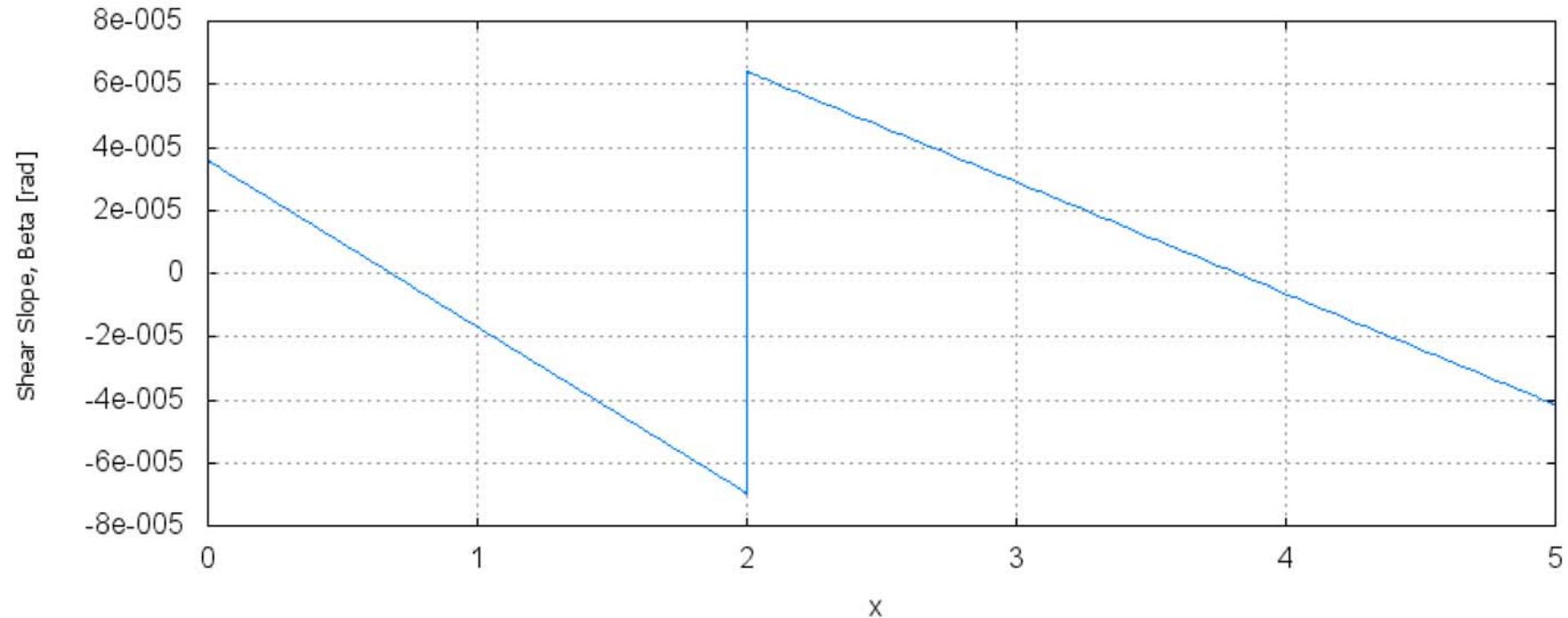
✓ The following plot shows the slope due to shear along the length of the entire structure:

```
✓ (%i69) betap(x):="(beta1s(x)*between(x,0,subst(par,L1))+subst(x-subst(par,L1),x,beta2s(x))*between(x,subst(par,L1),subst(par,L1+L2)))$
```

```
(%i70) wxplot2d(betap(x),[x,0,subst(par,L1+L2)],  
[gnuplot_preamble,"set title 'Slope Due to Shear in the Beam' font 'Tahoma, 12'; set grid; set ylabel 'Shear Slope, Beta [rad]' font 'Tahoma, 10';"],  
wxplot_size=[900,400]$
```

(%t70)

Slope Due to Shear in the Beam

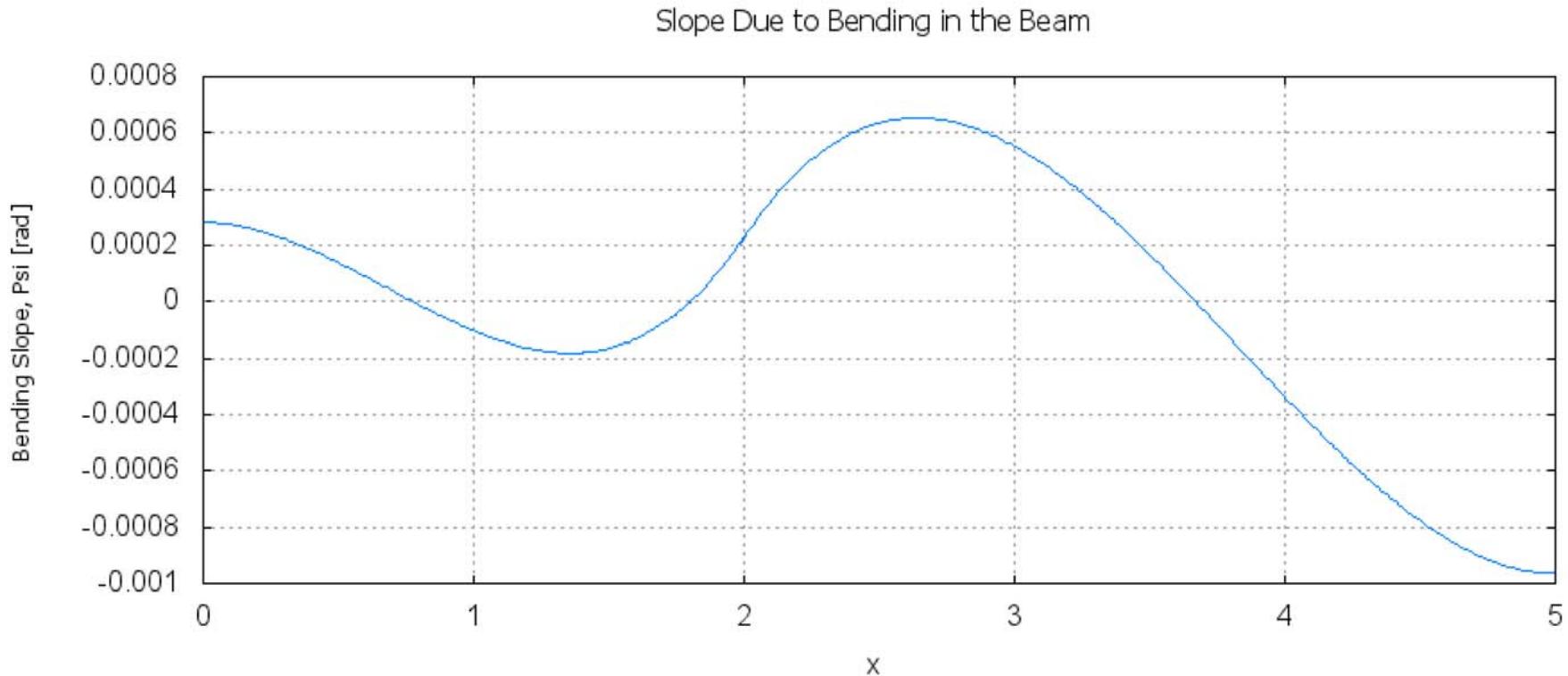


Similarly, the following plot shows the slope due to bending along the length of the entire structure:

```
(%i71) psip(x):="(psi1s(x)*between(x,0,subst(par,L1))+subst(x-subst(par,L1),x,psi2s(x))*between(x,subst(par,L1),subst(par,L1+L2)))$
```

```
(%i72) wxplot2d(psip(x),[x,0,subst(par,L1+L2)],  
[gnuplot_preamble,"set title 'Slope Due to Bending in the Beam' font 'Tahoma, 12'; set grid; set ylabel 'Bending Slope, Psi [rad]' font 'Tahoma, 10';"],  
wxplot_size=[900,400]$)
```

(%t72)

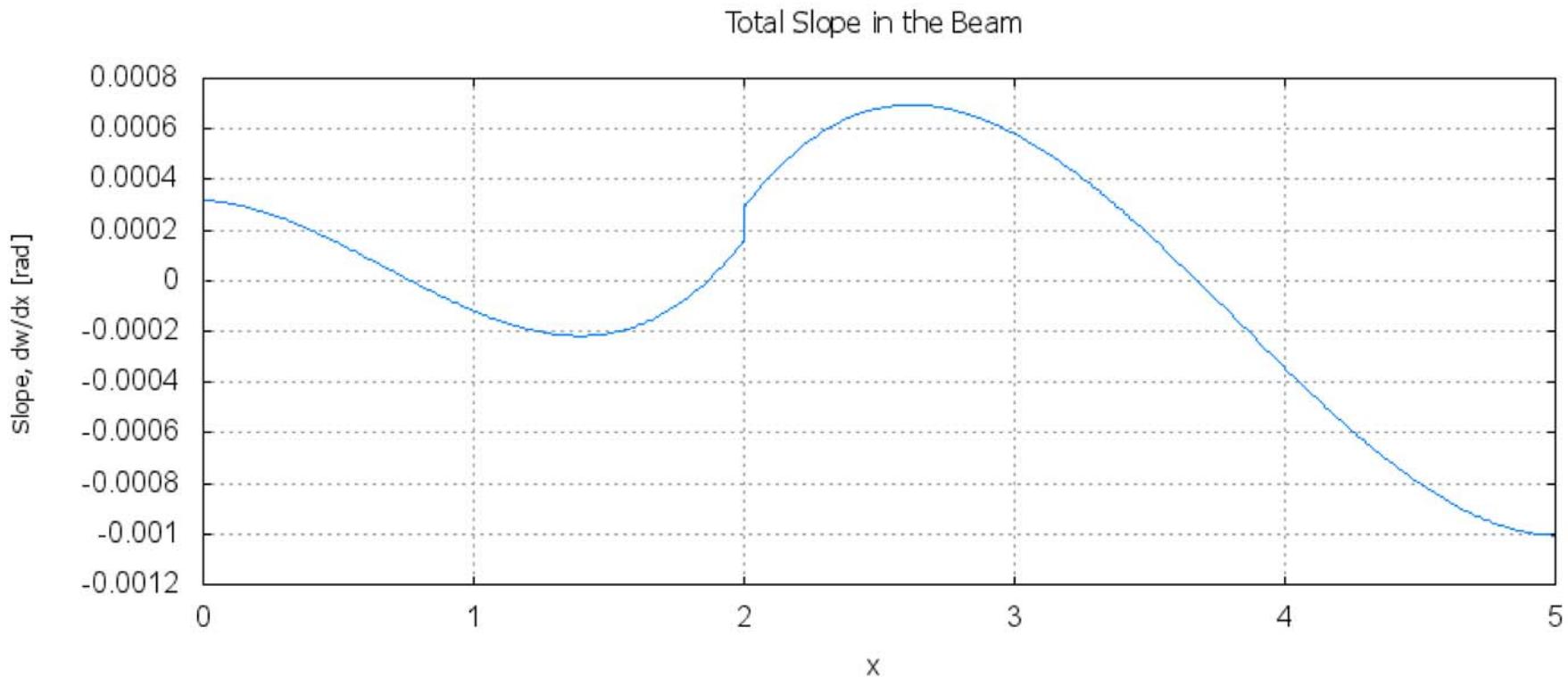


Finally, the combined slope of the beam (sum of shear and bending slopes) is presented in the following plot:

```
(%i73) dwp(x):=psip(x)+betap(x);  
(%o73) dwp(x) := psip(x) + betap(x)
```

```
(%i74) wxplot2d(dwp(x),[x,0,subst(par,L1+L2)],  
[gnuplot_preamble,"set title 'Total Slope in the Beam' font 'Tahoma, 12'; set grid; set ylabel 'Slope, dw/dx [rad]' font 'Tahoma, 10';"],  
wxplot_size=[900,400]$)
```

(%t74)



□ 5 Analysis Results - Normal and Shear Stresses

□ 5.1 Definition of normal and shear stresses

Given that there are no applied or internal axial forces, then the only remaining term for normal stresses would be the component due to bending:

```
✓ (%i75) sig1(x,z):=z*M1s(x)/l1;
    sig2(x,z):=z*M2s(x)/l2;
(%o75) sig1(x,z):=  $\frac{z \ M1s(x)}{l1}$ 
(%o76) sig2(x,z):=  $\frac{z \ M2s(x)}{l2}$ 
```

Furthermore, the corrected cross-section shear stresses will be:

```
✓ (%i77) tau1(x,z):=-V1s(x)/l1*(z^2/2-0.3^2/8);
    tau2(x,z):=-V2s(x)/l2*(z^2/2-0.3^2/8);
(%o77) tau1(x,z):=  $\frac{-V1s(x)}{l1} \left( \frac{z^2}{2} - \frac{0.3^2}{8} \right)$ 
(%o78) tau2(x,z):=  $\frac{-V2s(x)}{l2} \left( \frac{z^2}{2} - \frac{0.3^2}{8} \right)$ 
```

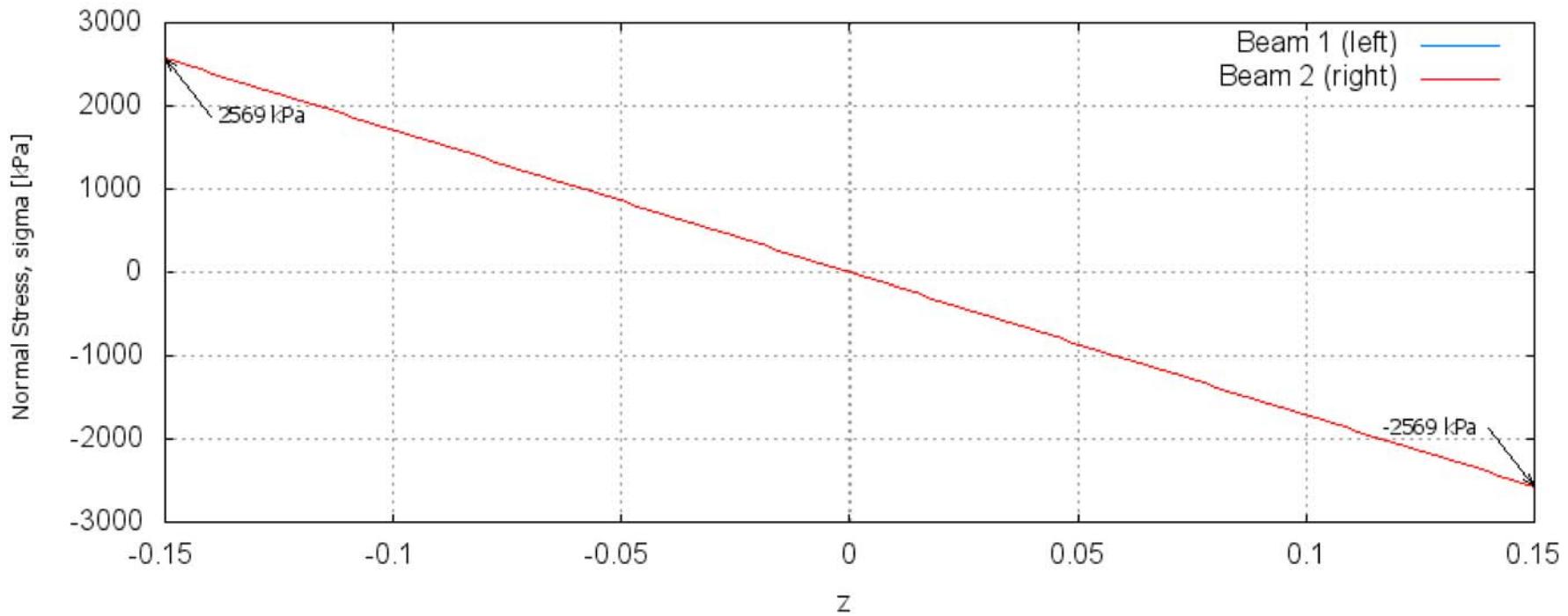
□ 5.2 Normal and shear stresses at the intermediate support

The following plot shows the normal stress distribution at the intermediate support. Notice that the stress line for both sides overlap. This is due to the fact that effects due to bending are continuous over the support.

```
(%i79) wxplot2d([subst(par,sig1(subst(par,L1),z)),subst(par,sig2(0,z))],[z,-.15,.15],[legend,"Beam 1 (left)","Beam 2 (right)"],  
[gnuplot_preamble,"set title 'Normal Stresses at Intermediate Support' font 'Tahoma, 12'; set grid; set ylabel 'Normal Stress, sigma [kPa]' font 'Tahoma, 10';  
set arrow 1 from -0.14,1869 to -0.15,2569; set label 1 '2569 kPa' at -0.138,1869 font 'Tahoma,9';  
set arrow 2 from 0.14,-1869 to 0.15,-2569; set label 2 '-2569 kPa' at 0.117,-1869 font 'Tahoma,9';"],  
wxplot_size=[900,400]$
```

(%t79)

Normal Stresses at Intermediate Support



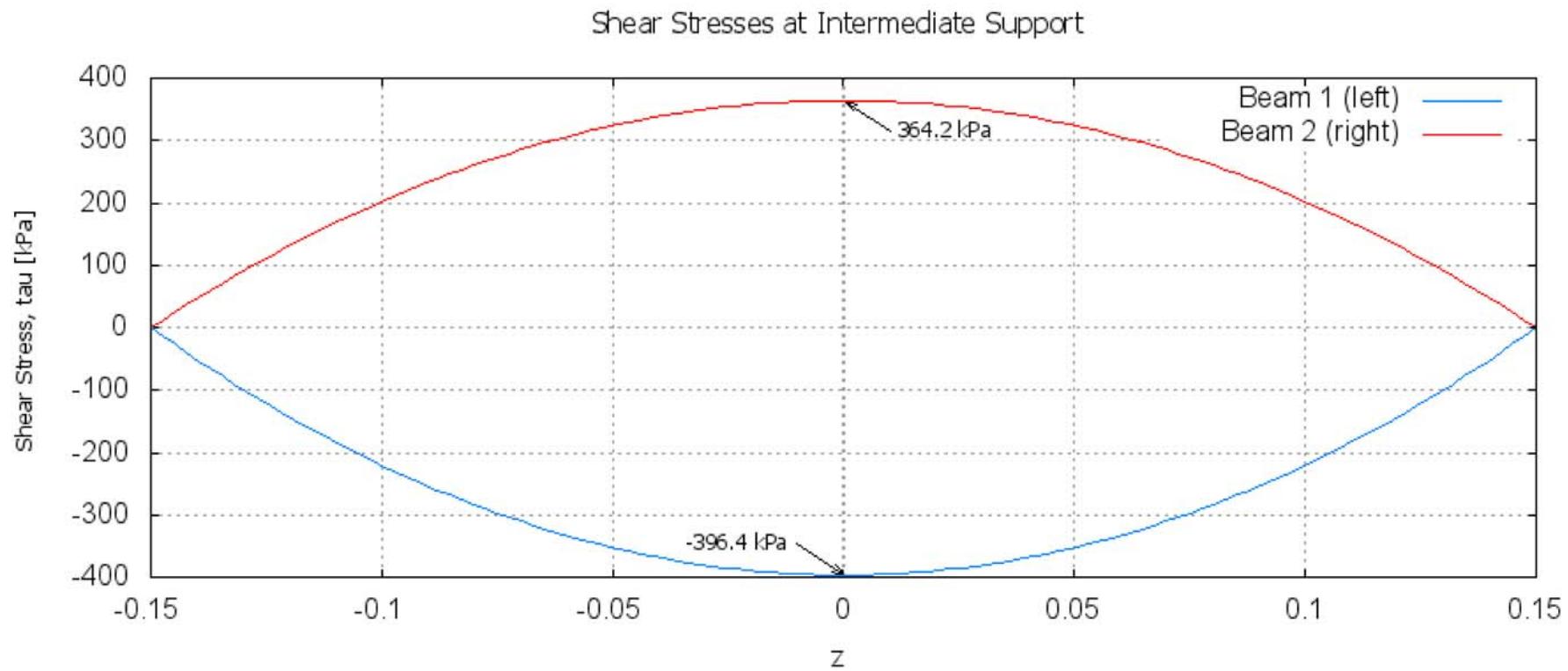
Evaluating the stresses at the outer fibers for both beam elements, we will get:

```
(%i80) print("Normal stress at the extreme bottom fiber (z = -0.15) is ", subst(par,sig1(L1,-0.15)), " kPa (left side)")$  
print("Normal stress at the extreme top fiber (z = 0.15) is ", subst(par,sig1(L1,0.15)), " kPa (left side)")$  
print("")$  
print("Normal stress at the extreme bottom fiber (z = -0.15) is ", subst(par,sig2(0,-0.15)), " kPa (right side)")$  
print("Normal stress at the extreme top fiber (z = 0.15) is ", subst(par,sig2(0,0.15)), " kPa (right side)")$  
Normal stress at the extreme bottom fiber (z = -0.15) is 2569. kPa (left side)  
Normal stress at the extreme top fiber (z = 0.15) is -2569. kPa (left side)  
  
Normal stress at the extreme bottom fiber (z = -0.15) is 2569. kPa (right side)  
Normal stress at the extreme top fiber (z = 0.15) is -2569. kPa (right side)
```

Using a similar approach the shear stresses at the cross-section at the intermediate support could be evaluated.

```
(%i85) wxplot2d([subst(par,tau1(L1,z)),subst(par,tau2(0,z))],[z,-.15,.15],[legend,"Beam 1 (left)","Beam 2 (right)"],
[gnuplot_preamble,"set title 'Shear Stresses at Intermediate Support' font 'Tahoma, 12'; set grid; set ylabel 'Shear Stress, tau [kPa]' font 'Tahoma, 10';
set arrow 1 from 0.01,314.2 to 0,364.2; set label 1 '364.2 kPa' at 0.012,314.2 font 'Tahoma,9';
set arrow 2 from -0.01,-346.4 to 0,-396.4; set label 2 '-396.4 kPa' at -0.034,-346.4 font 'Tahoma,9';"],wxplot_size=[900,400]$
```

(%t85)



Evaluating the shear stresses at the centre of the cross-section for both beam elements, we will get:

```
(%i86) print("Shear stress at the centre of cross-section (z = 0) is ", subst(par,tau1(L1,0)), " kPa (left side)")$  
print("Shear stress at the centre of cross-section (z = 0) is ", subst(par,tau2(0,0)), " kPa (right side)")$
```

Shear stress at the centre of cross-section (z = 0) is -396.4 kPa (left side)

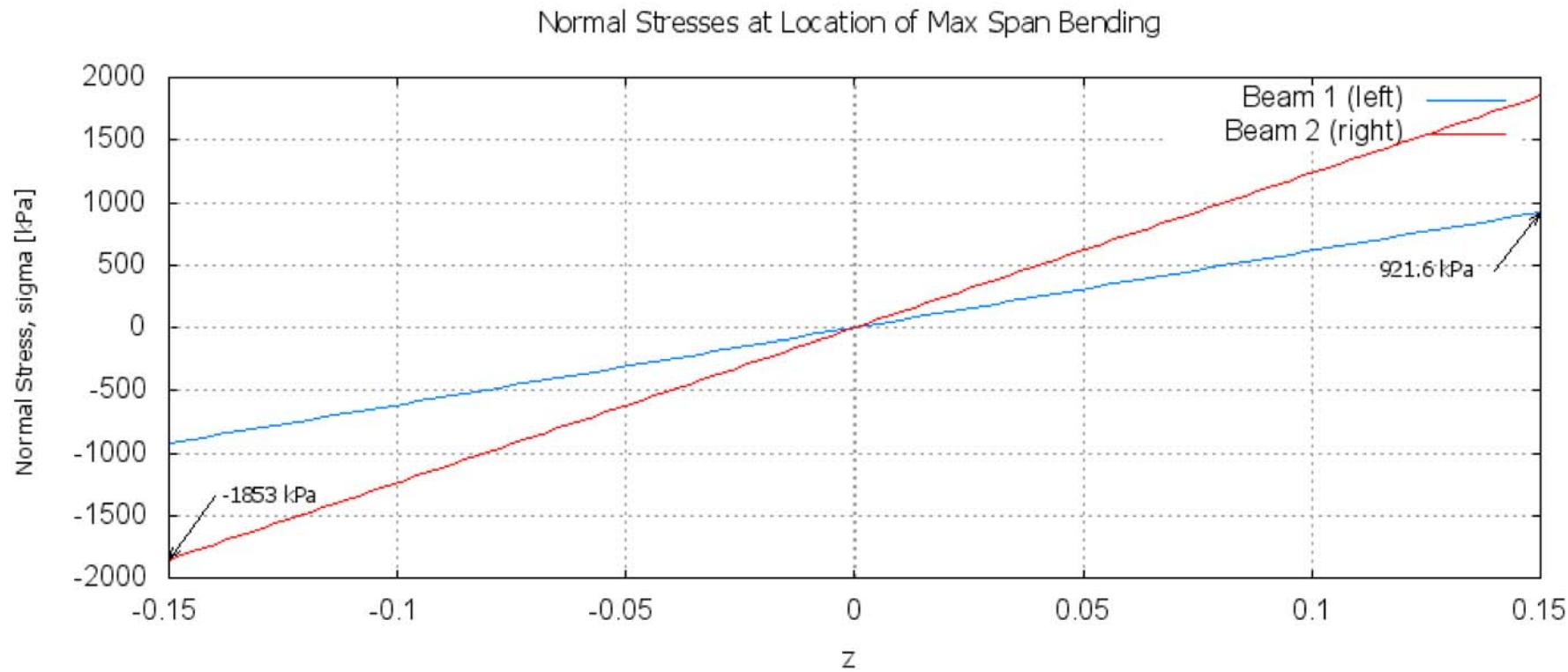
Shear stress at the centre of cross-section (z = 0) is 364.2 kPa (right side)

5.3 Normal stresses at location of extreme bending for each span

The following plot shows the normal stress distribution at the location of maximum bending moment along beams 1 and 2. Please refer to Section 4.2 to find out how this location was computed.

```
(%i88) wxplot2d([subst(par,sig1(rhs(rM1[1]),z)),subst(par,sig2(rhs(rM2[1]),z))],[z,-.15,.15],[legend,"Beam 1 (left)","Beam 2 (right)"],  
[gnuplot_preamble,"set title 'Normal Stresses at Location of Max Span Bending' font 'Tahoma, 12'; set grid; set ylabel 'Normal Stress, sigma [kPa]' font 'Tahoma,  
set arrow 1 from -0.14,-1353 to -0.15,-1853; set label 1 '-1853 kPa' at -0.138,-1353 font 'Tahoma,9';  
set arrow 2 from 0.14,451.6 to 0.15,921.6; set label 2 '921.6 kPa' at 0.115,451.6 font 'Tahoma,9';"],  
wxplot_size=[900,400]$
```

```
(%t88)
```



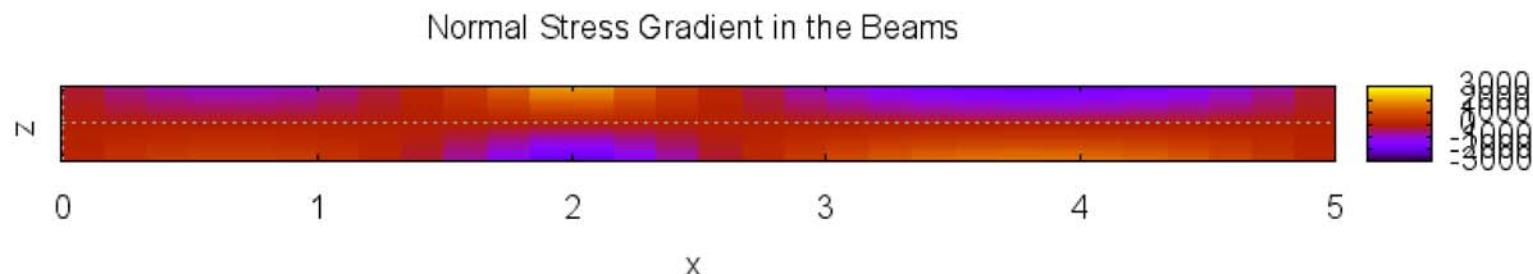
Evaluating the stresses at the outer fibers for both beam elements, we will get:

```
(%i89) print("Normal stress at the extreme bottom fiber (z = -0.15) at ",rM1[1] , " (Beam 1) is ", subst(par,sig1(rhs(rM1[1]),-0.15)), " kPa")$  
print("Normal stress at the extreme top fiber (z = 0.15) at ",rM1[1] , " (Beam 1) is ", subst(par,sig1(rhs(rM1[1]),0.15)), " kPa")$  
print("")$  
print("Normal stress at the extreme bottom fiber (z = -0.15) at ",rM2[1] , " (Beam 2) is ", subst(par,sig2(rhs(rM2[1]),-0.15)), " kPa")$  
print("Normal stress at the extreme top fiber (z = 0.15) at ",rM2[1] , " (Beam 2) is ", subst(par,sig2(rhs(rM2[1]),0.15)), " kPa")$  
Normal stress at the extreme bottom fiber (z = -0.15) at x = 0.679 (Beam 1) is -921.6 kPa  
Normal stress at the extreme top fiber (z = 0.15) at x = 0.679 (Beam 1) is 921.6 kPa  
  
Normal stress at the extreme bottom fiber (z = -0.15) at x = 1.821 (Beam 2) is -1853. kPa  
Normal stress at the extreme top fiber (z = 0.15) at x = 1.821 (Beam 2) is 1853. kPa
```

5.4 Normal and shear stress contour band plots

Since we have expressed the normal and shear forces as functions in two dimensions, it is possible to plot the stress distribution over the longitudinal cross section of the beams. Therefore the normal stress gradient would look like:

```
(%i94) sigp(x,z):="(sig1(x,z)*between(x,0,subst(par,L1))+subst(x-subst(par,L1),x,sig2(x,z))*between(x,subst(par,L1),subst(par,L1+L2)))$  
  
(%i95) wxplot3d(subst(par,sigp(x,-z)),[x,0,subst(par,L1+L2)], [z,-0.15,0.15],  
[gnuplot_preamble,"set pm3d map; set size ratio .3/5; set title 'Normal Stress Gradient in the Beams'; unset ytics;"],  
wxplot_size=[900,200]$  
(%t95)
```



And the shear stress gradient would be:

```
(%i96) taup(x,z):="(tau1(x,z)*between(x,0,subst(par,L1))+subst(x-subst(par,L1),x,tau2(x,z))*between(x,subst(par,L1),subst(par,L1+L2)))$
```

```
(%i97) wxplot3d(subst(par,taup(x,-z)),[x,0,subst(par,L1+L2)],[z,-0.15,0.15],  
[gnuplot_preamble,"set view map;set size ratio .3/5; set title 'Shear Stress Gradient in the Beams'; unset ytics;"],  
wxplot_size=[900,200]$
```

(%t97)

