TwoPDETesta.xmcd. September 1, 2009. This worksheet modifies the PDE Solve Quicksheet to solve the linearized partial differential equations describing undrained, adiabatic shear of a velocity strengthening fault zone. The equations are written in terms of the gradients of pore pressure and of temperature.

In these equations, Φ is the gradient of pore pressure and Ψ is gradient of temperature in non-dimensional form and perturbations from the uniform solution. cth and chy are thermal and hydraulic diffusivities, z is a parameter that gives the magnitude of the velocity strengthening (shear rate strengthening) and Hc is a measure of magnitude of the shear heating (all in nondimensional form). Representative values are given below.

$cth := 10^{-2}$ $\quad Hc := 0.3$ $\quad chy := 1 \cdot 10^{-4}$ $\quad z := 40$

Linearized solution for temperature and pressure (and their gradients) is unstable (grows exponentially) for wavenumbers less than a critical value given by (Critical value of the velocity is slightly different because of the effect of the time dependence of the homogeneous solution.

$$Ncrit := \frac{1}{\pi} \cdot \sqrt{\frac{z \cdot Hc}{(cth + chy)}} \qquad Ncrit = 10.972$$

$N1 := \mathrm{floor}(Ncrit)$ $\quad \mathrm{floor}(Ncrit) = 10$

$N2 := \mathrm{ceil}(Ncrit)$ $\quad \mathrm{ceil}(Ncrit) = 11$

$\Phi0 := 1$ $\quad \Psi0 := 0$

Given

$$\Psi_t(y,t) = cth \cdot \Psi_{yy}(y,t) + Hc \cdot z \cdot \Phi(y,t)$$

$$\Phi_t(y,t) = \left(chy \cdot \Phi_{yy}(y,t) + cth \cdot \Psi_{yy}(y,t)\right) + Hc \cdot z \cdot \Phi(y,t)$$

Initial conditions (for single Fourier mode):

$$\Phi(y,0) = \Phi0 \cdot \sin(\pi \cdot Nnum \cdot y)$$

$$\Psi(y,0) = \Psi0 \cdot \sin(\pi \cdot Nnum \cdot y)$$

Boundary conditions (since solution is anti-symmetric about y = 0, bc's given for half the domain

$\Psi(L,t) = 0$ $\quad \Phi(L,t) = 0$

$\Psi(0,t) = 0$ $\quad \Phi(0,t) = 0$

Solve over the ranges 0 to L in y and 0 to T in t.

$$\begin{pmatrix} \Phi num \\ \Psi num \end{pmatrix} := \text{Pdesolve}\left[\begin{pmatrix} \Phi \\ \Psi \end{pmatrix}, y, \begin{pmatrix} 0 \\ L \end{pmatrix}, t, \begin{pmatrix} 0 \\ T \end{pmatrix}, spacepts, timepts\right]$$

Exact solution (verified and tested in LinearSolFulla.xmcd. Solutions are of the form exp(pt) sin(N*πY) (because of the boundary conditions). Then p must satisfy a quadratic equation with the following coefficients::

$$Bc(n) := \frac{1}{2}\cdot\left[(\pi\cdot n)^2(cth + chy) - z\cdot Hc\right] \qquad Cc(n) := (\pi\cdot n)^4\cdot(cth\cdot chy)$$

$$Discrim(n) := \sqrt{(Bc(n))^2 - Cc(n)}$$

Two solutions are:

$$pplus := -Bc(Nnum) + Discrim(Nnum) \qquad pminus := -Bc(Nnum) - Discrim(Nnum)$$

$$pplus = -0.497 \qquad pminus = -9.932$$

$$\Phi plus := \frac{pplus + cth\cdot(Nnum\cdot\pi)^2}{pplus - pminus}\cdot\left[\Phi 0 - \frac{\Psi 0}{Hc\cdot z}\cdot\left[pminus + cth\cdot(Nnum\cdot\pi)^2\right]\right]$$

$$\Phi minus := \Phi 0 - \Phi plus$$

$$\Psi plus := \frac{Hc\cdot z\cdot\Phi plus}{\left[pplus + cth\cdot(Nnum\cdot\pi)^2\right]} \qquad \Psi minus := \frac{Hc\cdot z\cdot\Phi minus}{\left[pminus + cth\cdot(Nnum\cdot\pi)^2\right]}$$

$$\Psi time(s) := \Psi plus\cdot\exp(pplus\cdot s) + \Psi minus\cdot\exp(pminus\cdot s)$$

$$\Phi time(s) := \Phi plus\cdot\exp(pplus\cdot s) + \Phi minus\cdot\exp(pminus\cdot s)$$

$$\Psi anal(y,s) := \Psi time(s)\cdot\sin(Nnum\cdot\pi\cdot y) \qquad \Phi anal(y,s) := \Phi time(s)\cdot\sin(Nnum\cdot\pi\cdot y)$$

Compare Exact and Numerical Solutions

$L \equiv 1$ $T \equiv 3$ $spacepts \equiv 100$ $timepts \equiv 1000$ $Nnum \equiv 15$

$y := 0, 0.01 .. L$ $t := 0, 0.01 .. T$ $Ncrit = 10.972$