

Simple program that returns three values:

$$\text{PRGM}(x) := \begin{cases} a \leftarrow x \\ b \leftarrow x^2 \\ c \leftarrow \sqrt{x} \\ [a \ b \ c] \end{cases} \quad \text{ORIGIN} \equiv 1$$

Sample data to feed PRGM:

$$\text{Data1} := \begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \\ 11 \\ 13 \\ 17 \\ 19 \end{bmatrix} \quad \text{Data2} := \begin{bmatrix} 23 \\ 29 \\ 31 \\ 37 \\ 39 \end{bmatrix}$$

$i1 := \text{ORIGIN} \dots \text{last}(\text{Data1})$ $i2 := \text{ORIGIN} \dots \text{last}(\text{Data2})$

Results from using data arrays as input objects:

$$P1a := \text{PRGM}(\text{Data1}) = \begin{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \\ 11 \\ 13 \\ 17 \\ 19 \end{bmatrix} & \begin{bmatrix} 4 \\ 9 \\ 25 \\ 49 \\ 121 \\ 169 \\ 289 \\ 361 \end{bmatrix} & \begin{bmatrix} 1.414 \\ 1.732 \\ 2.236 \\ 2.646 \\ 3.317 \\ 3.606 \\ 4.123 \\ 4.359 \end{bmatrix} \end{bmatrix}$$

$$P2a := \text{PRGM}(\text{Data2}) = \begin{bmatrix} \begin{bmatrix} 23 \\ 29 \\ 31 \\ 37 \\ 39 \end{bmatrix} & \begin{bmatrix} 529 \\ 841 \\ 961 \\ 1369 \\ 1521 \end{bmatrix} & \begin{bmatrix} 4.796 \\ 5.385 \\ 5.568 \\ 6.083 \\ 6.245 \end{bmatrix} \end{bmatrix}$$

Results from using elements of arrays as input objects:

$$P1e_{i1} := \text{PRGM}(\text{Data1}_{i1}) = \begin{bmatrix} \begin{bmatrix} 2 & 4 & 1.414 \end{bmatrix} \\ \begin{bmatrix} 3 & 9 & 1.732 \end{bmatrix} \\ \begin{bmatrix} 5 & 25 & 2.236 \end{bmatrix} \\ \begin{bmatrix} 7 & 49 & 2.646 \end{bmatrix} \\ \begin{bmatrix} 11 & 121 & 3.317 \end{bmatrix} \\ \begin{bmatrix} 13 & 169 & 3.606 \end{bmatrix} \\ \begin{bmatrix} 17 & 289 & 4.123 \end{bmatrix} \\ \begin{bmatrix} 19 & 361 & 4.359 \end{bmatrix} \end{bmatrix}$$

$$P2e_{i2} := \text{PRGM}(\text{Data2}_{i2}) = \begin{bmatrix} \begin{bmatrix} 23 & 529 & 4.796 \end{bmatrix} \\ \begin{bmatrix} 29 & 841 & 5.385 \end{bmatrix} \\ \begin{bmatrix} 31 & 961 & 5.568 \end{bmatrix} \\ \begin{bmatrix} 37 & 1369 & 6.083 \end{bmatrix} \\ \begin{bmatrix} 39 & 1521 & 6.245 \end{bmatrix} \end{bmatrix}$$

$$\overrightarrow{\text{PRGM}(\text{Data2})} = \begin{bmatrix} \begin{bmatrix} 23 & 529 & 4.796 \end{bmatrix} \\ \begin{bmatrix} 29 & 841 & 5.385 \end{bmatrix} \\ \begin{bmatrix} 31 & 961 & 5.568 \end{bmatrix} \\ \begin{bmatrix} 37 & 1369 & 6.083 \end{bmatrix} \\ \begin{bmatrix} 39 & 1521 & 6.245 \end{bmatrix} \end{bmatrix}$$

What I really want...single arrays without subarrays:

$$P1 = \text{?????} = \begin{bmatrix} 2 & 4 & 1.414 \\ 3 & 9 & 1.732 \\ 5 & 25 & 2.236 \\ 7 & 49 & 2.646 \\ 11 & 121 & 3.317 \\ 13 & 169 & 3.606 \\ 17 & 289 & 4.123 \\ 19 & 361 & 4.359 \end{bmatrix}$$

$$P2 = \text{?????} = \begin{bmatrix} 23 & 529 & 4.796 \\ 29 & 841 & 5.385 \\ 31 & 961 & 5.568 \\ 37 & 1369 & 6.083 \\ 39 & 1521 & 6.245 \end{bmatrix}$$

$$P1_{\widehat{i1}} := \overrightarrow{\text{PRGM}}(Data1_{i1})$$

$$P1 = \begin{bmatrix} 2 & 4 & 1.414 \\ 3 & 9 & 1.732 \\ 5 & 25 & 2.236 \\ 7 & 49 & 2.646 \\ 11 & 121 & 3.317 \\ 13 & 169 & 3.606 \\ 17 & 289 & 4.123 \\ 19 & 361 & 4.359 \end{bmatrix}$$

$$P2_{\widehat{i2}} := \overrightarrow{\text{PRGM}}(Data2_{i2})$$

$$P2 = \begin{bmatrix} 23 & 529 & 4.796 \\ 29 & 841 & 5.385 \\ 31 & 961 & 5.568 \\ 37 & 1369 & 6.083 \\ 39 & 1521 & 6.245 \end{bmatrix}$$

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 $unnest(V) :=$ 
 $O \leftarrow \text{ORIGIN}$ 
if  $\min(\text{rows}(V), \text{cols}(V)) \neq 1$ 
   $\parallel$  return  $V$ 
if  $\min(\text{rows}(V_{o,o}), \text{cols}(V_{o,o})) \neq 1$ 
   $\parallel$  return  $V$ 
if  $(\min(\overrightarrow{\text{cols}}(V)) \neq \max(\overrightarrow{\text{cols}}(V))) \vee (\min(\overrightarrow{\text{rows}}(V)) \neq \max(\overrightarrow{\text{rows}}(V)))$ 
   $\parallel$  return  $V$ 
if  $\text{cols}(V) > 1$ 
   $\parallel V \leftarrow V^T$ 
if  $\text{cols}(V_o) = 1$ 
   $\parallel$  for  $i \in O .. \text{last}(V)$ 
   $\parallel \parallel M^{(i)} \leftarrow V_i$ 
else
   $\parallel$  for  $i \in O .. \text{last}(V)$ 
   $\parallel \parallel M_{\widehat{i}} \leftarrow V_i$ 
 $M$ 

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$$P1a = \begin{bmatrix} \begin{bmatrix} 2 \\ 3 \\ 5 \\ 7 \\ 11 \\ 13 \\ 17 \\ 19 \end{bmatrix} & \begin{bmatrix} 4 \\ 9 \\ 25 \\ 49 \\ 121 \\ 169 \\ 289 \\ 361 \end{bmatrix} & \begin{bmatrix} 1.414 \\ 1.732 \\ 2.236 \\ 2.646 \\ 3.317 \\ 3.606 \\ 4.123 \\ 4.359 \end{bmatrix} \end{bmatrix} \quad \text{unnest}(P1a) = \begin{bmatrix} 2 & 4 & 1.414 \\ 3 & 9 & 1.732 \\ 5 & 25 & 2.236 \\ 7 & 49 & 2.646 \\ 11 & 121 & 3.317 \\ 13 & 169 & 3.606 \\ 17 & 289 & 4.123 \\ 19 & 361 & 4.359 \end{bmatrix}$$

$$P2e = \begin{bmatrix} \begin{bmatrix} 23 & 529 & 4.796 \end{bmatrix} \\ \begin{bmatrix} 29 & 841 & 5.385 \end{bmatrix} \\ \begin{bmatrix} 31 & 961 & 5.568 \end{bmatrix} \\ \begin{bmatrix} 37 & 1369 & 6.083 \end{bmatrix} \\ \begin{bmatrix} 39 & 1521 & 6.245 \end{bmatrix} \end{bmatrix} \quad \text{unnest}(P2e) = \begin{bmatrix} 23 & 529 & 4.796 \\ 29 & 841 & 5.385 \\ 31 & 961 & 5.568 \\ 37 & 1369 & 6.083 \\ 39 & 1521 & 6.245 \end{bmatrix}$$

$$P1e^T = \begin{bmatrix} \begin{bmatrix} 2 & 4 & 1.414 \end{bmatrix} & \begin{bmatrix} 3 & 9 & 1.732 \end{bmatrix} & \begin{bmatrix} 5 & 25 & 2.236 \end{bmatrix} & \begin{bmatrix} 7 & 49 & 2.646 \end{bmatrix} & \begin{bmatrix} 11 & 121 & 3.317 \end{bmatrix} & \begin{bmatrix} 13 & 169 & 3.606 \end{bmatrix} & \begin{bmatrix} 17 & 289 & 4.123 \end{bmatrix} & \begin{bmatrix} 19 & 361 & 4.359 \end{bmatrix} \end{bmatrix}$$

$$P2a^T = \begin{bmatrix} \begin{bmatrix} 23 \\ 29 \\ 31 \\ 37 \\ 39 \end{bmatrix} & \begin{bmatrix} 529 \\ 841 \\ 961 \\ 1369 \\ 1521 \end{bmatrix} & \begin{bmatrix} 4.796 \\ 5.385 \\ 5.568 \\ 6.083 \\ 6.245 \end{bmatrix} \end{bmatrix} \quad \text{unnest}(P1e^T) = \begin{bmatrix} 2 & 4 & 1.414 \\ 3 & 9 & 1.732 \\ 5 & 25 & 2.236 \\ 7 & 49 & 2.646 \\ 11 & 121 & 3.317 \\ 13 & 169 & 3.606 \\ 17 & 289 & 4.123 \\ 19 & 361 & 4.359 \end{bmatrix}$$

The routine is written on purpose so that it returns a matrix in those cases and not a row- or column vector

$$\text{unnest}(P2a^T) = \begin{bmatrix} 23 & 529 & 4.796 \\ 29 & 841 & 5.385 \\ 31 & 961 & 5.568 \\ 37 & 1369 & 6.083 \\ 39 & 1521 & 6.245 \end{bmatrix}$$

$$\text{PRGM1}(x) := \begin{cases} a \leftarrow |x| \\ b \leftarrow x \cdot x \\ c \leftarrow \sqrt{x} \\ [a \ b \ c] \end{cases}$$

$$\text{PRGM2}(x) := \begin{cases} a \leftarrow |x| \\ b \leftarrow x \cdot x \\ c \leftarrow \sqrt{x} \\ [a \ b \ c]^T \end{cases}$$

$$\text{Data} := \begin{bmatrix} -4 \\ -1 \\ 0 \\ 9 \end{bmatrix} \xrightarrow{\text{PRGM1}(\text{Data})} \begin{bmatrix} [4 \ 16 \ 2i] \\ [1 \ 1 \ 1i] \\ [0 \ 0 \ 0] \\ [9 \ 81 \ 3] \end{bmatrix} \xrightarrow{\text{PRGM2}(\text{Data})} \begin{bmatrix} [4] \\ [16] \\ [2i] \\ [1] \\ [1] \\ [1i] \\ [0] \\ [0] \\ [0] \\ [9] \\ [81] \\ [3] \end{bmatrix}$$

$$P1 := \text{unnest}(\overrightarrow{\text{PRGM1}(\text{Data})}) = \begin{bmatrix} 4 & 16 & 2i \\ 1 & 1 & 1i \\ 0 & 0 & 0 \\ 9 & 81 & 3 \end{bmatrix} \quad P2 := \text{unnest}(\overrightarrow{\text{PRGM2}(\text{Data})}) = \begin{bmatrix} 4 & 1 & 0 & 9 \\ 16 & 1 & 0 & 81 \\ 2i & 1i & 0 & 3 \end{bmatrix}$$

For obvious reasons the functions should not be called without vectorization!

$$\text{PRGM1}(\text{Data}) = \begin{bmatrix} & & [2i] \\ & & [1i] \\ 9.899 & 98 & [0] \\ & & [3] \end{bmatrix}$$

$$\text{PRGM2}(\text{Data}) = \begin{bmatrix} 9.899 \\ 98 \\ [2i] \\ [1i] \\ [0] \\ [3] \end{bmatrix}$$