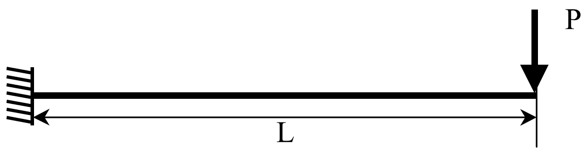
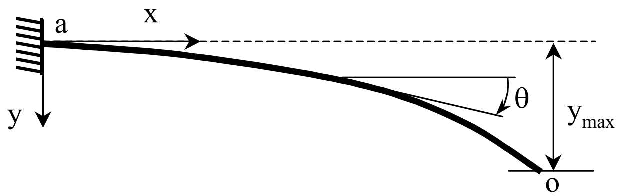
Simple Cantilever Beam Analysis

Ted Diehl
Bodie Technology, Inc.





This example demonstrates using Mathcad to analyze deflections of a cantilever beam. The analysis performs a simple small-deflection, linear beam theory evaluation for the problem depicted above. Mathcad features demonstrated in this document include:

1. Using units
2. Equations and functions
3. Importing PowerPoint images
4. Plotting
5. Tricks for pasting Mathcad graphs into PowerPoint

Notes:

- To enhance worksheet speed and efficiency, automatic calculation is turned off.
  - Hit the F9-key to update calculations as needed.
- Functions in Kornucopia® use the following SI definition of Hz, $\mathrm{Hz} = 1.000\frac{1}{\mathrm{sec}}$
- The origin of this worksheet is currently set to $\mathrm{ORIGIN} = 1$.

Inputs

Geometry of Beam

Beam Length (L), width (b), and thickness (h)

$L := 1 \cdot ft$ $\quad b := 1500 \cdot mil$ $\quad h := 1.5 \cdot mm$

$h = 0.059 \cdot in$

Bending moment of inertia $\quad I := \frac{1}{12} \cdot b \cdot h^3$ $\quad I = 2.574 \times 10^{-5} \cdot in^4$ $\quad I = 10.716 \cdot mm^4$

Material of Beam

Elastic Modulus $\quad E_{steel} := 30 \cdot 10^6 \cdot psi$ $\quad E_{steel} = 206.843 \cdot GPa$ $\quad E := E_{steel}$

Tip Load

$P := 1 \cdot lbf$

Calculate the Tip Deflection

The deflection curve for the beam is $\quad y(x) := \frac{-P}{6 \cdot E \cdot I} \cdot \left(x^3 - 3 \cdot L \cdot x^2\right)$

The tip deflection is $\quad y_{max} = y(x = L)$ $\quad$ => $\quad y_{max} := \frac{P \cdot L^3}{3 \cdot E \cdot I}$ $\quad y_{max} = 0.746 \cdot in$

check w/ general eqn $\quad y(L) = 0.746 \cdot in$

Graphing Deflected Shapes

Draw a graph showing how the deflection curve of the beam changes for different values of beam thickness

Write the bending inertia as a function of thickness, h $\quad I(h) := \frac{1}{12} \cdot b \cdot h^3$

The deflection curve of the beam is now $\quad y(x, h) := \frac{-P}{6 \cdot E \cdot I(h)} \cdot \left(x^3 - 3 \cdot L \cdot x^2\right)$

Consider evaluating three beam thicknesses $\quad h := \begin{pmatrix} 1.0 \\ 1.5 \\ 2.0 \end{pmatrix} \cdot mm$

A counter for h values $\quad j := 1 .. rows(h)$

Make a range variable for the x location down the beam $\quad num := 10$ $\quad i := 1 .. num$

Initialize x $\quad x := ""$ $\quad$ create a vector of x values $\quad x_i := \frac{i - 1}{num - 1} \cdot L$

$$x^T = (0.000 \quad 1.333 \quad 2.667 \quad 4.000 \quad 5.333 \quad 6.667 \quad 8.000 \quad 9.333 \quad 10.667 \quad 12.000) \cdot in$$

Calculation of the deflected shapes for each value of h

Approach 1 - use a separate variable for each h value.

$Y1 := y(x, h_1)$ $\quad$ $Y2 := y(x, h_2)$ $\quad$ $Y3 := y(x, h_3)$

Approach 2 - use one variable and range index looping.

$Y := ""$ $\quad$ $Y_j := y(x, h_j)$ $\quad$ $\mathrm{rows}(Y) = 3.000$ $\quad$ $\mathrm{cols}(Y) = 1.000$

Both approaches obtain same result

$Y1 = Y_1 = 1.000$ $\quad$ $Y2 = Y_2 = 1.000$ $\quad$ $Y3 = Y_3 = 1.000$

$Y_1 =$ (·in)

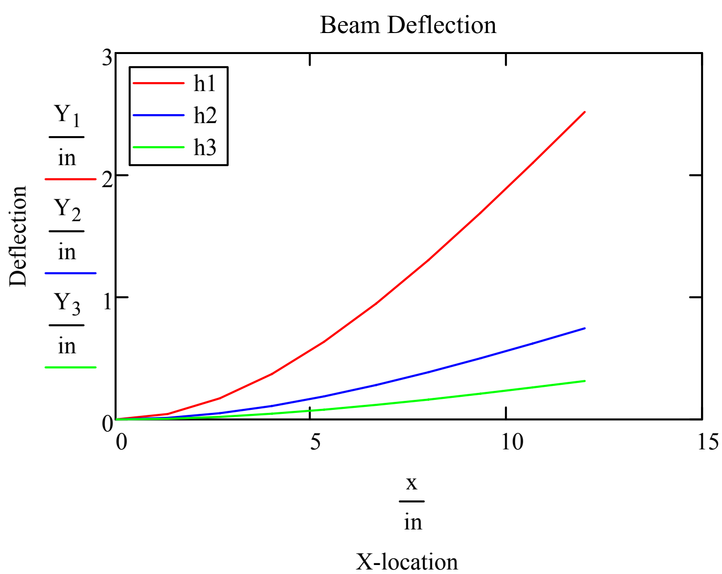
	1
1	0.000
2	0.045
3	0.173
4	0.373
5	0.635
6	0.950
7	1.305
8	1.692
9	2.099
10	2.517

$Y_2 =$ (·in)

	1
1	0.000
2	0.013
3	0.051
4	0.110
5	0.188
6	0.281
7	0.387
8	0.501
9	0.622
10	0.746

$Y_3 =$ (·in)

	1
1	0.000
2	$5.611 \cdot 10^{-3}$
3	0.022
4	0.047
5	0.079
6	0.119
7	0.163
8	0.211
9	0.262
10	0.315



$$h = \begin{pmatrix} 1.000 \\ 1.500 \\ 2.000 \end{pmatrix} \cdot \mathrm{mm}$$

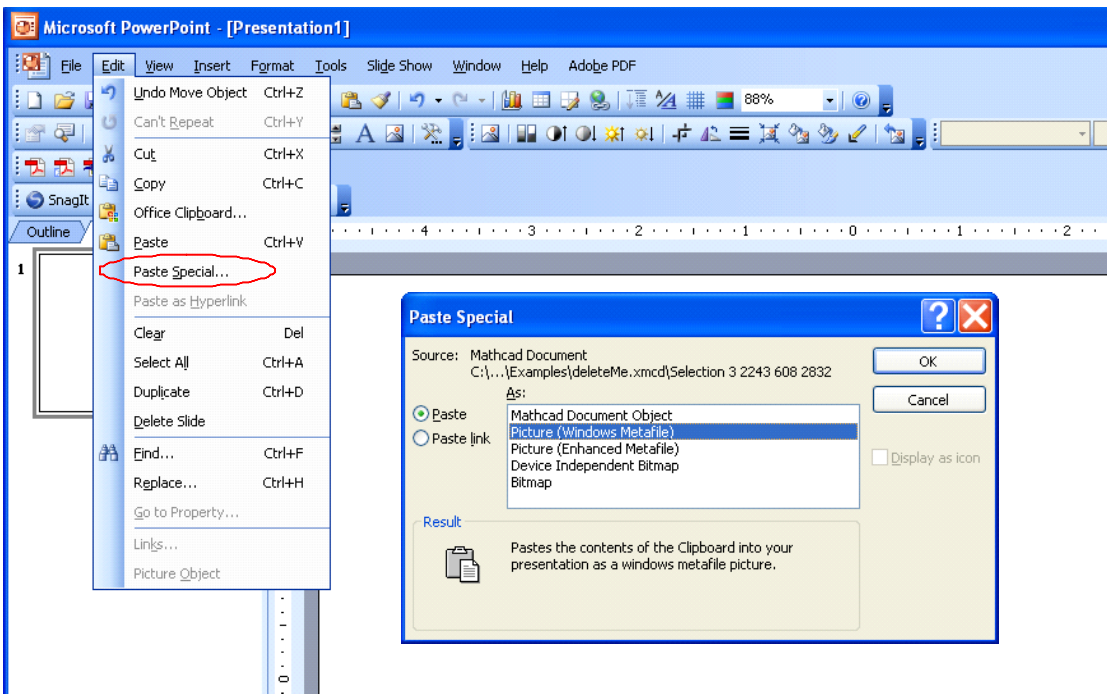
Tip:
For increased flexibility in PowerPoint, we set the traces in the graph to be solid lines. This allows use to do many more things in PowerPoint with the graph (as shown below)

PowerPoint Tricks

This section explains several tricks that can be used to copy Mathcad entities into PowerPoint and further enhance/modify the entities for improved presentations.

Step 1:
The Mathcad graph is copied by using ctrl-click to select the graph (the graph will switch to show a dashed border around it) and then typing ctrl-c or selecting Edit Copy.

Step 2:
In PowerPoint select Edit, Paste Special, Picture (Windows Metafile). In some cases, using Picture (Enhanced Metafile) works better)



Step 3:
In PowerPoint, right click on the image and select "Edit Picture". Answer "Yes" to the pop-up question of converting the picture to a Microsoft Office drawing object.

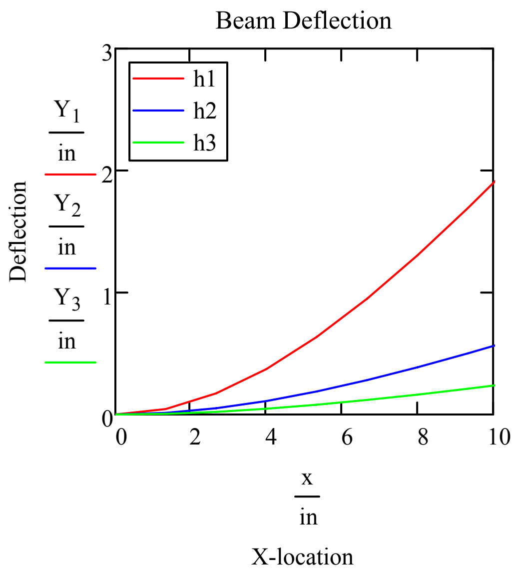
Right-click again on the picture and select "Grouping, Ungroup" to begin ungrouping the image. repeat as needed.

You can now select various entities in the graph and do a variety of things
- Delete the white background or change its color
- Change the colors and thickness of the lines in the graph
- Cut (remove), stretch, modify the curves in the graphs
- Use PowerPoint animation to have graph curves appear, swipe, disappear in a presentation.
- Add nicer formatted X/Y axes, titles, improved legends, etc.

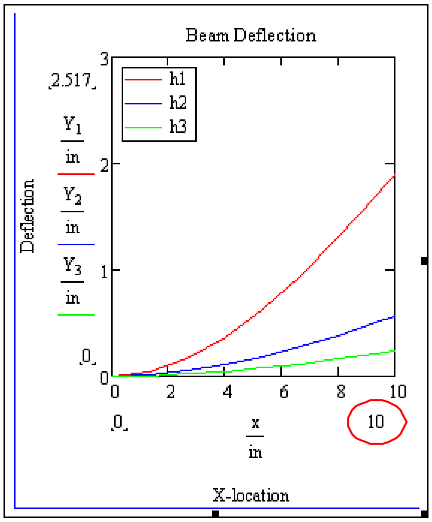
A tip in trimming data

- If you have zoomed the graph in Mathcad, or modified the X/Y graph limits such that some of the data curves are "clipped" by the graph axes, these curves will show up extending beyond the graph axes when they are converted to a Microsoft Office drawing object in PowerPoint. To avoid this problem, use the Kornucopia® function trim_k (or tweakXY_k) in Mathcad to first trim all your data prior to making the plot. This is demonstrated below

Graph where max X value was manually set to 10



This is an image showing the manual setting of the axis



Below is a picture of the graph in PowerPoint, after it was converted to a Microsoft Office drawing object