Simple Cantilever BeamAnalysis

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This example demonstrates using Mathcad to analyze deflections of a cantilever beam. The analysis performs a simple small-deflection, linear beam theory evaluation for the problem depicted above. Mathcad features demonstrated in this document include:

Using units

Equations and functions

Importing PowerPoint images

Plotting
Notes:

Tricks for pasting Mathcad graphs into PowerPoint

To enhance worksheet speed and efficiency, automatic calculation is turned off.

Hit the F9-key to update calculations as needed.

Functions in Kornucopia® use the following SI definition of Hz,Hz1sec1

The origin of this worksheet is currently set to ORIGIN1.

Inputs

Geometry of Beam

Beam Length (L), width (b), and thickness (h) $L := 1 \cdot ft$ $b := 1500 \cdot mil$ $h := 1.5 \cdot mm = 0.059\ in$

$h = 0.059\ in$

Bending moment of inertia $I := \frac{1}{12} \cdot b \cdot h^3$ $I = 0\ in^4$ $I = 10.716\ mm^4$

Material of Beam $E_{steel} := 30 \cdot 10^6 \cdot psi$ $E_{steel} = 206.843\ GPa$ $E := E_{steel}$

Elastic Modulus

Tip Load $P := 1 \cdot lbf$

Calculate the Tip Deflection

The deflection curve for the beam is $y(x) := \frac{-P}{6 \cdot E \cdot I} \cdot \left(x^3 - 3 \cdot L \cdot x^2\right)$

The tip deflection is y_{max} ■ $y(x$ ■ $L)$ => $y_{max} := \frac{P \cdot L^3}{3 \cdot E \cdot I}$ $y_{max} = 0.746\ in$

check w/ general eqn $y(L) = 0.746\ in$

Graphing Deflected Shapes

Draw a graph showing how the deflection curve of the beam changes for different values of beam thickness

Write the bending inertia as a function of thickness, h $I(h) := \frac{1}{12} \cdot b \cdot h^3$

The deflection curve of the beam is now $y(x, h) := \frac{-P}{6 \cdot E \cdot I(h)} \cdot \left(x^3 - 3 \cdot L \cdot x^2\right)$

Consider evaluating three beam thicknesses $h := \begin{bmatrix} 1.0 \\ 1.5 \\ 2.0 \end{bmatrix} \cdot mm$ A counter for h values $j := 1 .. rows(h)$

Make a range variable for the x location down the beam $num := 10$ $i := 1 .. num$

Initialize x $x := “”$ create a vector of x values $x_i := \frac{i-1}{num-1} \cdot L$

$b = 0.038\ m$

$x^T = [0.000\ \ 1.333\ \ 2.667\ \ 4.000\ \ 5.333\ \ 6.667\ \ 8.000\ \ 9.333\ \ 10.667\ \ 12.000]\ in$

$h_1 = 0.001\ m$

Calculation of the deflected shapes for each value ofh

At first it appears you cannot make functions in Prime, or at least you cannot use them. It does not complain above but does when you use them, giving you a bogus error messag

$x = \begin{bmatrix} 0 \\ 0.034 \\ 0.068 \\ 0.102 \\ 0.135 \\ 0.169 \\ 0.203 \\ 0.237 \\ 0.271 \\ 0.305 \end{bmatrix} m$

$I(h) = \ m^4$

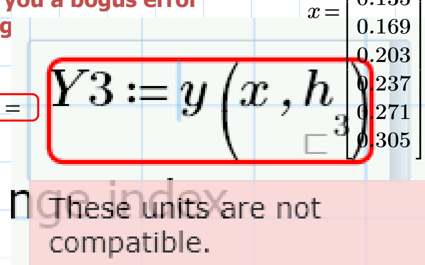
Approach 1 - use a separate variable for each h value.

$Y1 := y(x, h_1)$ $Y2 := y(x, h_2)$ $Y3 := y(x, h_3)$

Approach 2 - use one variable and range index looping.

$Y := “”$ $Y_j := y(x, h_j)$ $rows(Y) =$ $cols(Y) =$

$b = 0.038\ m$



Both approaches obtain same result

Approach 1 - use a separate variable for each h value.

Apparently Prime no longer supports arrays raised to a power. A BIG Backward Compatibility problem

$Y1 := \overrightarrow{y\left(x, h_1\right)}$ $Y2 := \overrightarrow{y\left(x, h_2\right)}$ $Y3 := \overrightarrow{y\left(x, h_3\right)}$

Swipe left to right does NOT allow you to get the Vectorize operator to apply. I had to swipe right to left

Approach 2 - use one variable and range index looping.

$Y := \text{“”}$ $Y_j := \overrightarrow{y\left(x, h_j\right)}$ $rows(Y) = 3$ $cols(Y) = 1$

Why are these circled Red claiming Y is undefined if indeed it works.

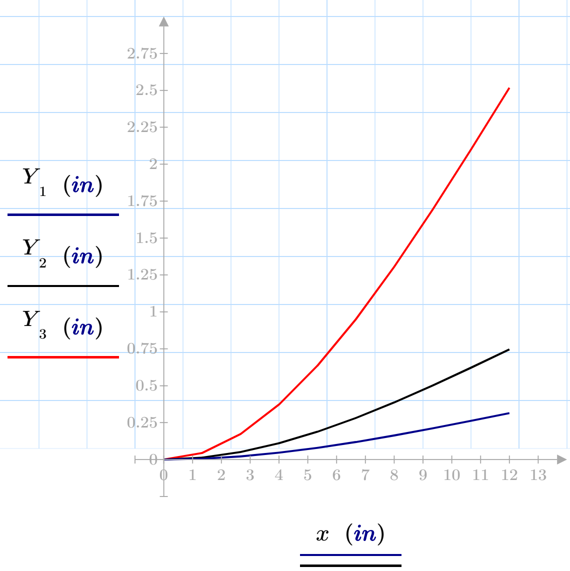
Both approaches obtain same result

$Y1 \blacksquare Y_1 = 1$ $Y2 \blacksquare Y_2 = 1$ $Y3 \blacksquare Y_3 = 1$

$Y_1 = \begin{bmatrix} 0 \\ 0.045 \\ 0.173 \\ 0.373 \\ 0.635 \\ 0.95 \\ 1.305 \\ 1.692 \\ 2.099 \\ 2.517 \end{bmatrix} in$ $Y_2 = \begin{bmatrix} 0 \\ 0.013 \\ 0.051 \\ 0.11 \\ 0.188 \\ 0.281 \\ 0.387 \\ 0.501 \\ 0.622 \\ 0.746 \end{bmatrix} in$ $Y_3 = \begin{bmatrix} 0 \\ 0.006 \\ 0.022 \\ 0.047 \\ 0.079 \\ 0.119 \\ 0.163 \\ 0.211 \\ 0.262 \\ 0.315 \end{bmatrix} in$

My points about PowerPoint are useless with Prime as graphs can no longer be copied into Powerpoint apparently

The 2-D graph never came over, so I must recreate it



$h = \begin{bmatrix} 1 \\ 1.5 \\ 2 \end{bmatrix} mm$

Tip:

For increased flexibility in PowerPoint, we set the traces in the graph to be solid lines.

This allows use to do many more things in PowerPoint with the graph (as shown below)

PowerPoint Tricks

This section explains several tricks that can be used to copy Mathcad entities into PowerPoint and further enhance/modify the entities for improved presentations.

Step 1:

The Mathcad graph is copied by using ctrl-click to select the graph (the graph will switch to show a dashed border around it) and then typing ctrl-c or selecting Edit Copy.

Step 2:

In PowerPoint select Edit, Paste Special, Picture (Windows Metafile).In some cases, using Picture (Enhanced Metafile) works better)

Step 3:

In PowerPoint, right click on the image and select "Edit Picture". Answer "Yes" to the pop-up question of converting the picture to a Microsoft Office drawing object.

Right-click again on the picture and select "Grouping, Ungroup" to begin ungrouping the image. repeat as needed.

You can now select various entities in the graph and do a variety of things

Delete the white background or change its color

Change the colors and thickness of the lines in the graph

Cut (remove), stretch, modify the curves in the graphs

Use PowerPoint animation to have graph curves appear, swipe, disappear in a presentation.

Add nicer formatted X/Y axes, titles, improved legends, etc.

A tip in trimming data

If you have zoomed the graph in Mathcad, or modified the X/Y graph limits such that some of the data curves are "clipped" by the graph axes, these curves will show up extending beyond the graph axes when they are converted to a Microsoft Office drawing object in PowerPoint. To avoid this problem, use the Kornucopia® function trim_k (or tweakXY_k) in Mathcad to first trim all your data prior to making the plot. This is demonstrated below.

Graph where max X value was manually set to 10

This is an image showing the manual setting of the axis

Below is a picture of the graph in PowerPoint, after it was converted to a Microsoft Office drawing object