Method for the measurement of the modulation transfer function of sampled imaging systems from bar-target patterns

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A rigorous and simple method for the determination of the modulation transfer function (MTF) of a sampled imaging system is presented. One calculates the MTF by imaging bar patterns and calculating the reduction in amplitude of the fundamental frequency components. The optimal set of bar-pattern frequencies that reduce errors from aliased frequency components is derived. Theoretical and experimental data are presented.

1. Introduction

The modulation transfer function (MTF) has become a standard measure for the determination of the imaging performance of precision optical-imaging systems. It has also been shown to be an effective measure of overall imaging-system performance because the MTF may be used to describe many sensors (e.g., film), as well as optical components. In recent years, however, the move toward digital imaging systems that use sampled detectors has introduced ambiguity into how the MTF should be defined and measured for sampled imaging systems. The MTF was originally defined with the assumption that the image could be measured over a continuous region. Considerable debate over the definition of the MTF for sampled systems has led to a definition that is consistent with the original formulation of the measure.[1] This paper presents a method for the accurate measurement of the MTF of sampled imaging systems from imaged bar patterns. The key aspect of the method is the proper choice of spatial frequencies, which avoids errors introduced by aliasing.

In most cases, the method described in this article will allow one to specify the MTF of a sampled imaging system rather than to have to use the commonly used contrast transfer function (CTF).[2] Traditionally, the MTF is determined by measurement of the reduction in modulation that occurs when a gray-scale sinusoid is imaged. The CTF, on the other hand, measures the overall reduction in contrast that occurs when a bar pattern is imaged. Unlike the MTF, the CTF includes the effects of higher harmonic terms, which are introduced by the bar pattern. The MTF is a superior measure for system analysis because it describes the system response to individual sinusoidal components. This property permits the MTF of an entire linear system to be given by the product of the MTF's of each linear component.

Section 2 of this paper provides a brief review of the definition of the MTF for continuous and sampled imaging systems. Section 3 presents the method for measurement of the MTF of sampled systems. Section 4 describes a bar target that was developed from the method and presents some experimental results.

2. Definition of the Modulation Transfer Function

The concept of the optical transfer function (OTF) developed from the application of Fourier analysis to optical-image formation. The OTF defines the spatial-filtering effect of the optical system on the object spectral distribution.[3] The OTF was originally defined for continuous-imaging applications that fulfill the isoplanatic condition (i.e., local-space invariance). The effect of the OTF on the object spectrum can be expressed by the following equation written in the spatial frequency domain:

$$I(u, v) = H(u, v)O(u, v) = |H(u, v)|\exp[j\theta_H(u, v)]O(u, v), \quad (1)$$

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where $I(u, \nu)$ represents the Fourier transform of the image, $O(u, \nu)$ represents the Fourier transform of the object, and $H(u, \nu)$ is the OTF. The last part of Eq. (1) shows that the OTF can be expressed as the product of an amplitude term, which represents the MTF, and a phase term, which defines the phase transfer function (PTF). Thus the MTF defines the reduction in amplitude of spatial frequency components caused by aberration and diffraction effects. The PTF describes any phase shift introduced by the imaging system.

For continuous systems the OTF may also be defined equivalently as the Fourier transform of the point-spread function (or impulse response) of the system, as given by

$$H(u, \nu) = \iint_{-\infty}^{+\infty} h(x, y)\exp[-j2\pi(ux + \nu y)]\mathrm{d}x\mathrm{d}y. \qquad (2)$$

Once the image is sampled, however, the property of space invariance of the system is lost.[4] Depending on the relative position on the sampling grid and the point-spread function, the sampled-image distribution will be different. The discrete-space Fourier transform of the sampled point-spread function is given by

$$\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} h(m\Delta, n\Delta)\exp[-j2\pi\Delta(um + \nu n)] = \frac{1}{\Delta^2}\sum_{m=-\infty}^{+\infty}\sum_{n=-\infty}^{+\infty} H\left(u - \frac{m}{\Delta}, \nu - \frac{n}{\Delta}\right), \qquad (3)$$

where Δ represents the sample spacing. As Eq. (3) describes, the sampling process causes the OTF of the continuous system to be replicated in the frequency domain. If the spatial-frequency content of $H(u, \nu)$ is high enough, the distributions will overlap. In this case, some of the high spatial frequencies of $H(u, \nu)$ appear as low spatial frequencies in the sampled distribution; this effect is referred to as aliasing.[5] Where frequency components overlap, the magnitude of the sum of the frequency components depends on the relative phases of the overlapped frequencies and therefore on the position of the point-spread function relative to the sampling grid. Thus, if the MTF were defined as the magnitude of the discrete-space Fourier transform of the point-spread function, the MTF would not be uniquely defined for frequencies for which overlapping were present.

Many sampled imaging systems can be modeled as a continuous linear space-invariant system followed by a sampling function, as illustrated in Fig. 1. The continuous linear space-invariant system may include spatial-filtering effects that are introduced by the detector (e.g., by finite detector apertures) as well as by the optical system. It follows that if one could remove the effects that aliasing introduces through

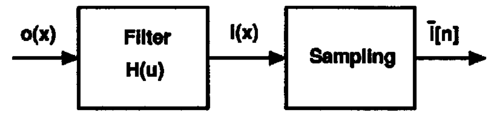

Fig. 1. Linear sampled system that is modeled as a continuous linear filtering operation and that is followed by a sampling operator.

the sampling function, the frequency response of the system could be accurately and reliably measured. In terms of imaging systems, we could measure the MTF of the effective continuous system. Section 3 presents a method to measure the MTF of a sampled system at specific frequencies in such a way that the aliasing effects are negligible.

3. Measurement of the MTF of Sampled Imaging Systems from Imaged Bar Targets

It is well known that for continuous imaging applications one may calculate the MTF by imaging bar-target patterns and determining the reduction in amplitude of the fundamental frequency component.[6] In this section, a similar method for sampled imaging systems is presented. The spatial frequencies of the bar patterns are chosen specifically to avoid aliasing and to provide an accurate measure of the system MTF. Although the MTF is a two-dimensional quantity, it is common to measure only one-dimensional slices of the MTF (usually in the horizontal and vertical planes of the detector). Throughout the rest of this paper, the analysis will be restricted to the measurement of a one-dimensional slice of the MTF in the direction of the bar-pattern periodicity.

The black and white bar pattern that is recorded on the target is represented schematically in Fig. 2 and is modeled by the function

$$o(x) = c + a\sum_{n=-\infty}^{+\infty}\mathrm{rect}\left(\frac{x - nd}{L}\right), \qquad (4)$$

where L represents the width of a bright or white bar as measured in the image plane, d represents the spatial period of the pattern as measured in the image plane, and the $\mathrm{rect}(x)$ function is defined by

$$\mathrm{rect}(x) = \begin{cases} 1 & \text{if } |x| < 0.5 \\ 0 & \text{otherwise} \end{cases}. \qquad (5)$$

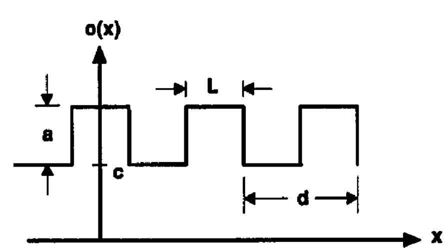

Fig. 2. Schematic diagram of the square-wave input with a period d and a duty cycle L/d that was used for the MTF test.

The constant c in Eq. (4) represents the measured irradiance in the image of a uniform black region recorded on the target. The value of $a + c$ represents the measured irradiance in the image of a constant white region recorded on the target. Thus, Eq. (4) actually describes the continuous image distribution that would be measured in the image plane if the imaging system were ideal and therefore able to image the object without a reduction in contrast.

The Fourier transform of the object distribution is given by

$$O(u) = c\delta(u) + \frac{aL}{d} \sum_{m=-\infty}^{+\infty} \operatorname{sinc}\left(\frac{mL}{d}\right)\delta\left(u - \frac{m}{d}\right), \tag{6}$$

where

$$\operatorname{sinc}(x) = \frac{\sin(\pi x)}{\pi x} \tag{7}$$

and $\delta(x)$ represents the Dirac delta function. The Fourier transform of the bar pattern is a series of discrete frequency components that correspond to the harmonic components of the periodic bar pattern.

Practical analysis of the imaged bar pattern requires the use of the discrete Fourier transform (DFT), which uses a finite number of N samples for analysis. The DFT of the sampled image is given by

$$\bar{I}[k] = \sum_{n=0}^{N-1} \bar{\imath}[n]\exp\left(-j2\pi \frac{kn}{N}\right), \tag{8}$$

where $\bar{\imath}[n]$ represents the sampled values of the image given by

$$\bar{\imath}[n] = i(n\Delta), \qquad n = 0, 1, \ldots, N - 1, \tag{9}$$

in which Δ represents the sample spacing in the image plane.

The DFT of the sampled image of the bar pattern is related to the Fourier transform of the bar pattern by

$$\bar{I}[k] = \left[\frac{1}{\Delta} \sum_{n=-\infty}^{+\infty} O\left(u - \frac{n}{\Delta}\right)H\left(u - \frac{n}{\Delta}\right)\right] * W(u)\bigg|_{u=\frac{k}{N\Delta}}, \tag{10}$$

where $W(u)$ represents the Fourier transform of the effective rectangular window function (inherent to the definition of the DFT), which is given by

$$W(u) = N\Delta \operatorname{sinc}(N\Delta u), \tag{11}$$

and where the asterisk denotes the convolution operation as defined by

$$f(x) * g(x) = \int_{-\infty}^{+\infty} f(\xi)g(x - \xi)\mathrm{d}\xi. \tag{12}$$

To simplify the analysis, it is assumed for the rest of the paper that frequency components greater than twice the Nyquist rate of the detector $(1/2\Delta)$ are not passed by the system. For most imaging systems, this assumption is well justified. This simplification allows one to consider only the aliased frequency components that originate from the neighboring replicas above and below the spectrum that is centered on zero frequency. In fact, because only the positive frequency components of the spectrum need to be analyzed to calculate the MTF, only the aliasing effects caused by the positively shifted replica need to be considered.

Through a similar approach for the calculation of the MTF for continuous systems, the MTF for sampled systems is determined through measurement of the reduction in amplitude of the fundamental frequency component of the bar pattern. This method is successful only if the fundamental frequency component does not overlap with an aliased harmonic component. Figure 3 illustrates the Fourier spectrum of the imaged bar targets. The fundamental frequency component at frequency $1/d$ and the mth harmonic (located at frequency $1/\Delta - m/d$) of the positively shifted replica overlap when the fundamental frequency of the bar pattern is given by

$$\frac{1}{d} = \frac{1}{(m + 1)\Delta}, \qquad m = 1, 2, 3, \ldots, . \tag{13}$$

Equation (13) indicates that overlapping between the fundamental frequency component and aliased harmonics occurs for a discrete set of frequencies. It follows that, to minimize the effects of aliasing, one should choose the test frequencies that are farthest from the overlapping condition. To find these frequencies, one may solve for the set of fundamental frequencies that are equally distant from the aliased mth harmonic and the aliased $(m + 1)$th harmonic, as given by

$$\frac{1}{d} - \left(\frac{1}{\Delta} - \frac{m + 1}{d}\right) = \left(\frac{1}{\Delta} - \frac{m}{d}\right) - \frac{1}{d}. \tag{14}$$

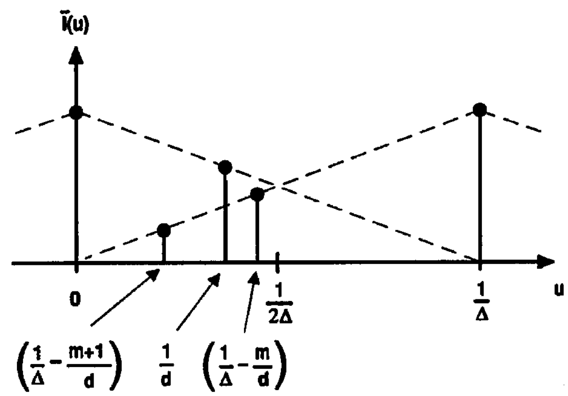

Fig. 3. Illustration of the fundamental frequency component and aliased harmonic components of the imaged-bar-pattern input in the spatial frequency domain. Only the fundamental frequency component and the aliased mth and $(m + 1)$th harmonic components are shown. The dashed curves illustrate the envelope of the MTF of the system.

The left-hand side of Eq. (14) corresponds to the frequency separation between the fundamental frequency and the aliased $(m + 1)$th harmonic; the right-hand side corresponds to the frequency separation between the fundamental frequency and the aliased mth harmonic. Solving for the fundamental frequency in Eq. (14) yields

$$\frac{1}{d} = \frac{2}{(2m + 3)\Delta}, \qquad m = 1, 2, 3, \ldots . \tag{15}$$

Figure 4 illustrates the set of MTF test frequencies that incur the minimum aliasing effects. Restricting the fundamental frequency to the discrete set of sample frequencies provided by the DFT $(1/d = k/N\Delta)$ yields

$$\bar{k} = \frac{2N}{2m + 3}, \qquad m = 1, 2, 3, \ldots, \tag{16}$$

where the values of $\bar{k}$ and N must be integers. By choosing k to be an even integer, one may solve Eq. (16) for the corresponding number of data samples, N. The value of k defines the number of cycles of the bar pattern for analysis. In addition, if k is chosen to be the same for all the MTF test frequencies, then, as is shown in Appendix A the sensitivity to magnification errors is identical for each test frequency.

It should be noted that, because the spectra in the Fourier domain are convolved with the Fourier transform of the window function [as described by Eq. (10)], some leakage may occur from the sidelobes of each spectral component. By substituting Eq. (15) into either side of Eq. (14), one can show that the distance in terms of the number of frequency samples between the fundamental frequency peak and the nearest aliasing component is given by $N/(2m + 3)$. Thus, increasing N generally decreases noise effects and error contributions from sidelobes, but at the expense of one's having more data to analyze. In addition, the imaged target area should be as small as possible to maintain local space invariance. A more practical solution for reducing cross talk arising from sidelobes may be to apply a window function with low sidelobes to the data before taking the DFT.[7]

Because aliasing effects are minimal at the fundamental frequencies defined above, one may solve Eq. (10) using Eqs. (6) and (11) directly, to give the system MTF as

$$\left| H\left(\frac{1}{d}\right) \right| = \left| \frac{d\,\bar{I}[\bar{k}]}{NaL\ \mathrm{sinc}(L/d)} \right| . \tag{17}$$

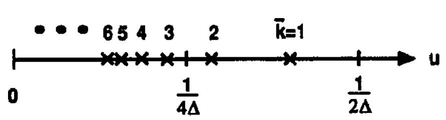

Fig. 4. Set of spatial frequencies ($\bar{k} = 1, 2, 3, 4, 5, 6$) that introduce minimal aliasing effects during the measurement of the MTF. The filled circles indicate the set of points $\bar{k} = 7, 8, 9, \ldots$, which cannot be represented adequately in the figure.

To apply Eq. (17), one must know the values of a and L/d. The value of a can be determined if a large uniform white area and a large uniform black area are included on the bar target. The mean of the uniform black area in the image provides an estimate for c, and the mean of the uniform white area in the image provides an estimate for $a + c$. The difference of the white mean and the black mean gives an estimate for a. In addition, if the duty cycle of each bar pattern (L/d) is not known, it can be estimated from the zero spatial frequency value of the DFT of the bar pattern. Because, by definition, the MTF for the zero spatial frequency is equal to unity, one may solve Eq. (10) at the zero frequency to give

$$\frac{L}{d} = \frac{\bar{I}[0]/N - c}{a}, \tag{18}$$

where a and c are determined as described above. If, on the other hand, the duty cycle of the bar pattern is accurately known, Eq. (18) can be solved for the value of a to compensate for slowly varying nonuniform illumination effects across the target.

4. Experimental Results

A bar target was designed at the Oak Ridge National Laboratory with the principles given in Section 3. The chart was drawn in AUTOCAD (a trademark of Autodesk, Inc.), which permitted it to be easily scaled to test systems that work at different imaging magnifications. The chart was photographically reduced to the proper size. Figure 5 shows the chart, which includes six sets of bar patterns to test the MTF in the vertical and horizontal directions and fiduciary marks for automatic location and verification of the proper orientation of the chart in the digital image. It

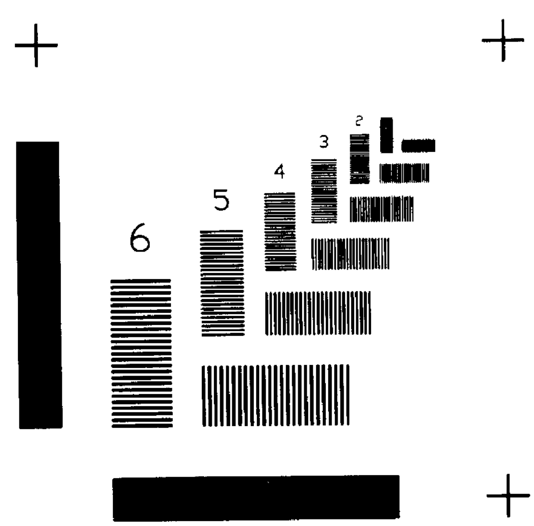

Fig. 5. Bar-target chart for measurement of the MTF of a sampled imaging system. The chart includes six sets of bar patterns for measuring the MTF in the vertical and horizontal directions. The long solid black bars are used to verify the proper magnification of the system. Fiduciary markings (pluses) border the chart to permit location and verification of the proper orientation of the target in the digital image.

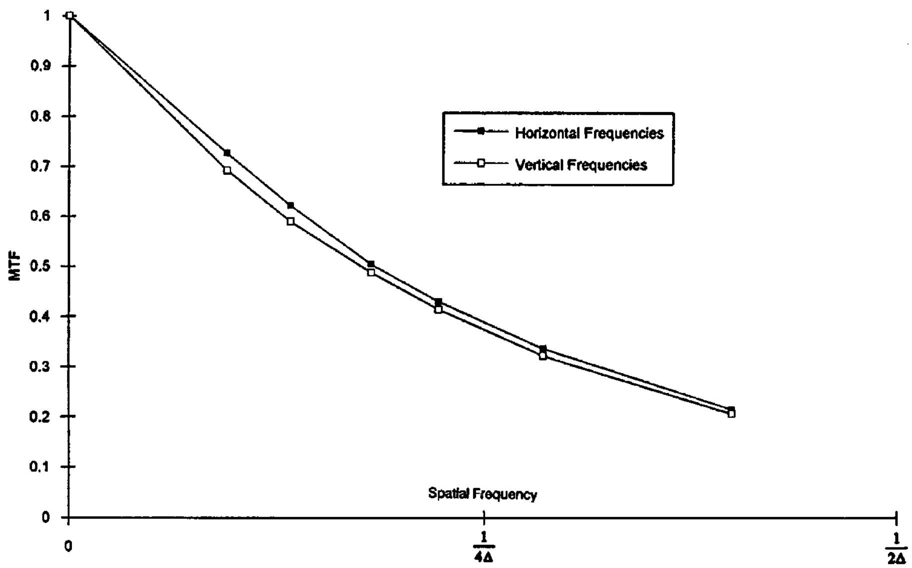

Fig. 6. Plot of the computed MTF in the vertical and horizontal directions of a commercial scanner. The bar-target chart shown in Fig. 5 was used along with the analysis described in this paper.

should be noted that the target was developed for a sensor with equal sample spacing in the vertical and horizontal directions. For detectors with unequal sample spacing in the vertical and horizontal directions, the bar patterns in the vertical and horizontal directions would have to be scaled appropriately. The chart includes large vertical and horizontal black bars whose lengths correspond to 500 pixels in the sampled image. The lengths of the bars can be measured in the digital image to verify the proper magnification of the optical system. In addition, the large black region within the bars provides a uniform area in the image for the estimation of c, as described above. There is also sufficient white area on the chart to obtain a measurement of $a + c$. The set of bar periods, $\{d_m\}$, was chosen by substitution of $m = 1$, 2, 3, 4, 6, 9 into Eq. (15). These frequencies defined a sufficiently dense sampling of the system's frequency range for our application. The value of k was chosen to be 20. Thus, 20 cycles of the bar patterns were analyzed for each test frequency. (Twenty-five cycles are printed on the target to alleviate positioning errors and edge effects.)

The chart was scanned with a commercial scanner that used filtered blue light illumination. The captured digital image was analyzed in the following way: We estimated the value for c by averaging pixel values in a 20 pixel $\times$ 20 pixel array that was located in the middle of the large black bars. We determined the value for a by averaging pixel values in a 20 pixel $\times$ 20 pixel array that was located in the extended white area below the top fiduciary marks and by subtracting the value of c. Then, from the center of each bar pattern, a one-dimensional vector of data was taken in the direction of the periodicity. Because the exact duty cycle of the bar patterns in the final target was not known, the duty cycle for each bar pattern was determined by application of Eq. (18). The MTF for each bar region was computed with Eq. (17). Figure 6 shows a plot of the MTF of the sampled imaging system.

5. Discussion

A new method for the measurement of the MTF of sampled imaging systems has been presented. The optimal set of measurement frequencies that minimize aliasing effects has been derived. Initial experimental tests have shown the measurement technique to be repeatable and robust. In addition to sampled imaging and detector systems, the general analysis method described in this paper may be applied to other sampled systems. An AUTOCAD drawing file of the chart developed at the Oak Ridge National Laboratory can be obtained upon request from the authors.

Appendix A

The method presented in this paper determines the MTF of a sampled system through the measurement of the fundamental frequency components of imaged bar-target patterns. The frequency components are calculated through determination of the DFT of the imaged bar patterns. However, if the magnification between the object and the image plane is slightly off, the peak of the fundamental frequency component does not coincide with the correct frequency sample. Thus, a fraction of the true value is measured because the peak lies somewhere between frequency samples. Because the peaks in the Fourier transform are sharp, magnification errors can contribute significant errors to the estimation of the MTF. In this appendix it is shown that, if k is chosen to be the same for each bar-pattern group, then the sensitivity to magnification errors is the same for each MTF measurement.

If the fundamental frequency of a bar pattern is chosen such that it coincides with a frequency sample of the DFT, then the fundamental frequency is given by

$$u = \frac{1}{d} = \frac{\bar{k}}{N\Delta}, \quad (A1)$$

where d is the spatial period of the bar pattern and $\bar{k}$ represents the corresponding sample index in the N-point DFT. If a magnification error exists such that the bar pattern is scaled by $1 + \varepsilon$, where ε represents an error in the magnification, then the fundamental peak is shifted to

$$u' = \frac{(1 + \varepsilon)}{d} = \frac{\bar{k}(1 + \varepsilon)}{N\Delta}. \quad (A2)$$

Thus the shift in the frequency domain that is introduced by the error in magnification is given by

$$\frac{\bar{k}\varepsilon}{N\Delta}. \quad (A3)$$

By substitution of expression (A3) into Eq. (11), one can show that the error in the magnification reduces the measured value of the main lobe by a factor of

$$\text{sinc}(\varepsilon\bar{k}). \quad (A4)$$

From expression (A4), it follows that if $\bar{k}$ is chosen to be the same for each bar pattern, then the sensitivity to magnification errors is the same for all test frequencies.

References

1. W. Wittenstein, J. C. Fontanella, A. R. Newbery, and J. Baars, "The definition of the OTF and the measurement of aliasing for sampled imaging systems," Opt. Act. **29,** 41–50 (1982).
2. L. J. Pinson, *Electro-Optics* (Wiley, New York, 1985), pp. 118–127.
3. J. W. Goodman, *Introduction to Fourier Optics* (McGraw-Hill, New York, 1968), pp. 113–133.
4. S. K. Park, R. Schowengerdt, and M. Kaczynski, "Modulation-transfer function analysis for sampled image systems," Appl. Opt. **23,** 2572–2582 (1984).
5. A. V. Oppenheim and R. W. Schafer, *Digital Signal Processing* (Prentice-Hall, Englewood Cliffs, N.J., 1975), pp. 26–30.
6. W. J. Smith, *Modern Optical Engineering*, 2nd ed. (McGraw-Hill, New York, 1990), pp. 345–363.
7. Ref. 5, Chap. 11.