Problem 1. (Problem 2.1 on page 47 of the text.) In Figure 2.1,

$$v(t) = \sqrt{2}\cdot 120\cdot\cos(\omega\cdot t + 30\cdot deg) \qquad i(t) = \sqrt{2}\cdot 10\cdot\cos(\omega\cdot t - 30\cdot deg)$$

a. Find p(t), S, P, and Q into the network.
b. Find a simple, two element, series circuit consistent with the prescribed terminal behavior as described in this problem.

$$p(t) = v(t)\cdot i(t)$$

$$p(t) = 2400\cdot\cos(\omega\cdot t + 30\cdot deg)\cdot\cos(\omega\cdot t - 30\cdot deg)$$

$$p(t) = 1200\cdot(\cos(60\cdot deg) + \cos(2\cdot\omega\cdot t))$$

$$p(t) = 600 + 1200\cdot\cos(2\cdot\omega\cdot t)$$

$$j := \sqrt{-1}$$

Use RMS Phasors

$$V := 120\cdot e^{j\cdot 30\cdot deg}\cdot volt \qquad I := 10\cdot e^{-j\cdot 30\cdot deg}\cdot A$$

Calculate the power quantities

$$VAr := volt\cdot amp$$

$$S := V\cdot\bar{I} \qquad S = 600 + 1039i\ volt\cdot amp$$

$$P := Re(S) \qquad P = 600\,W$$

$$Q := Im(S) \qquad Q = 1.039\times 10^{3}\,VAr$$

To find the series circuit, first find the equivalent impedance. The series circuit will be two elements, a resistor to model the real part of the impedance and a reactance (in this case, an inductive reactance, to model the reactive portion.

$$Z := \frac{V}{I} \qquad Z = 6 + 10.392i\,ohm$$

6 ohms **10.39 ohms**

Problem 2. (problem 2.4 on page 48). A 3phase load draws 200 kW at a PF of 0.707 lagging froma 440-V line. in parallel is a 3phase capacitor bank which suppoies 50 kVAr. Find the resultant power factor and current into the parallel combination.

$P_i := 200 \cdot kW \qquad PF_i := 0.707 \qquad V_{LLi} := 440 \cdot volt \qquad Q_{cap} := 50 \cdot kV \cdot A$

Find the initial values of apparant power S and reactive power Q

$S_i := \frac{P_i}{PF_i} \qquad Q_i := \sqrt{S_i \cdot S_i - P_i \cdot P_i} \qquad Q_i = 200.06 \circ kV \cdot A$

Real power P is unchanged by addition or deletion of reactive power Q.

$P_f := P_i \qquad P_f = 2 \cdot 10^5 \ W$

Add the two reactive power components together. Capacitive reactive power is negative under the convention assumed in the text.

$Q_f := Q_i + (-Q_{cap}) \qquad Q_f = 150.06 \circ kV \cdot A$

Calculate the new apparent power. The power factor is the ratio of real power to apparent power.

$S_f := \sqrt{P_f \cdot P_f + Q_f \cdot Q_f} \qquad S_f = 250.036 \circ kV \cdot A$

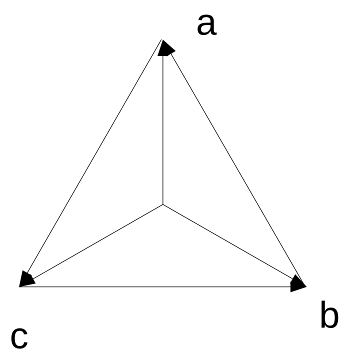
$PF_f := \frac{P_f}{S_f} \qquad PF_f = 0.8$ lagging

Use a three phase power formula to find the line current.

$I_a := \frac{S_f}{\sqrt{3} \cdot V_{LLi}} \qquad I_a = 328 \ A$

Problem 3. (Problem 2.6 on page 47) The system shown in the Figur below is balanced and positive sequence. Assume that Z=10 at 15 deg and Vca=208 at -120 deg. Find Vab, Vbc, Van, Vbn, Vcn, Ia, Ib, Ic, and S3phase.

Use a phasor diagram with a horizontal zero reference and counterclockwise positive direction:



Phase angles, by inspection:

given $\theta_{ca} := -120 \cdot deg$

$\theta_{ab} := 120 \cdot deg$ $\quad$ $\theta_{an} := 90 \cdot deg$

$\theta_{bc} := 0 \cdot deg$ $\quad$ $\theta_{bn} := -30 \cdot deg$

$\theta_{cn} := -150 \cdot deg$

A balanced system has the given 208V between any pair of lines; So, between line and neutral,

$$V_{an} := \left[\frac{208}{\sqrt{3}} \cdot \left(\cos\left(\theta_{an}\right) + j \cdot \sin\left(\theta_{an}\right)\right)\right] volt \qquad \left|V_{an}\right| = 120.1 volt \qquad j := \sqrt{-1}$$

The other two line-to-neutral voltages have the same magnitude. The phase angle of Van, from the phasor diagram, is 90 degrees. The other two line-to-neutral voltages have phase angles as given beside the diagram above. Therefore, the balanced system has 120.1V line to neutral in each phase.

Current is voltage divided by impedance.

$$Z := 10 \cdot e^{-j \cdot 15 \cdot deg} \cdot ohm \qquad I_a := \frac{V_{an}}{Z} \qquad \left|I_a\right| = 12 \text{ A}$$

$$\phi_a := angle\left(Re\left(I_a\right), Im\left(I_a\right)\right) \qquad \phi_a = 105 deg$$

Phase angles on the other two line currents are

$$\phi_b := \phi_a - 120 \cdot deg \qquad \phi_b = -15 deg \qquad \phi_c := \phi_a + 120 \cdot deg \qquad \phi_c = 225 deg$$

Three phase complex power is

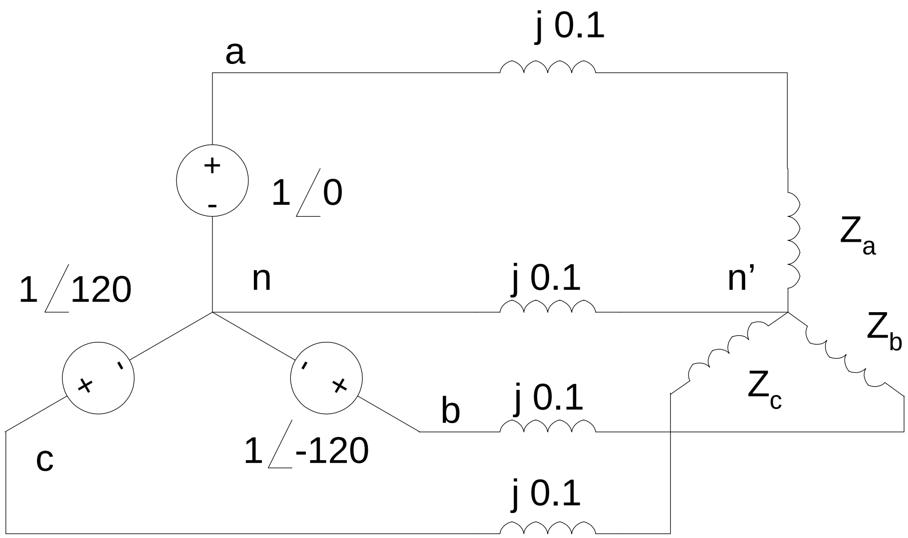
$$S_{3\phi} := 3 \cdot V_{an} \cdot \overline{I_a} \qquad S_{3\phi} = 4179 - 1120i \, volt \cdot amp$$

Summary of answers:

All line-to-neutral voltages are 120.1 Volts. All line-to-line voltages are 208 Volts. All line currents are 12.0 A.

Problem 4. (Problem 2.8 on page 47) In the system shown in the figure below, find Ia, Ib, and Ic for

a. Za=j1; Zb=j1; Zc=j0.9 (unbalanced)
b. Za=j1; Zb=j1; Zc=j1 (balanced)



$j := \sqrt{-1}$

Write down the given.

$V_{an} := 1.0 \qquad V_{bn} := 1.0 \cdot e^{-j \cdot \frac{2 \cdot \pi}{3}} \qquad V_{cn} := 1.0 \cdot e^{j \cdot \frac{2 \cdot \pi}{3}}$

$Z_{line} := j \cdot 0.1 \quad Z_a := j \cdot 1 \quad Z_b := j \cdot 1 \quad Z_c := j \cdot 0.9$

The mesh equations are as follows:

$$V_{an} = I_a \cdot Z_{line} + I_a \cdot Z_a + (I_a + I_b + I_c) \cdot Z_{line}$$

$$V_{bn} = I_b \cdot Z_{line} + I_b \cdot Z_b + (I_a + I_b + I_c) \cdot Z_{line}$$

$$V_{cn} = I_c \cdot Z_{line} + I_c \cdot Z_c + (I_a + I_b + I_c) \cdot Z_{line}$$

Forming into a matrix

$$\begin{bmatrix} Z_{line} + Z_a + Z_{line} & Z_{line} & Z_{line} \\ Z_{line} & Z_{line} + Z_b + Z_{line} & Z_{line} \\ Z_{line} & Z_{line} & Z_c + Z_{line} + Z_{line} \end{bmatrix} \cdot \begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} V_{an} \\ V_{bn} \\ v_{cn} \end{bmatrix}$$

Inverting the matrix to solve,

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} := \begin{bmatrix} Z_{line} + Z_a + Z_{line} & Z_{line} & Z_{line} \\ Z_{line} & Z_{line} + Z_b + Z_{line} & Z_{line} \\ Z_{line} & Z_{line} & Z_c + Z_{line} + Z_{line} \end{bmatrix}^{-1} \cdot \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} -5.584 \cdot 10^{-3} - 0.912i \\ -0.793 + 0.451i \\ 0.86 + 0.496i \end{bmatrix}$$

$$|I_a| = 0.912 \qquad angle(Re(I_a), Im(I_a)) - 360 \cdot deg = -90.351 \circ deg$$

$$|I_b| = 0.912 \qquad angle(Re(I_b), Im(I_b)) - 360 \cdot deg = -209.649 \circ deg$$

$$|I_c| = 0.993 \qquad angle(Re(I_c), Im(I_c)) = 30 \circ deg$$

Same process for the balanced system,

$$Z_{line} := j \cdot 0.1 \qquad Z_a := j \cdot 1 \quad Z_b := j \cdot 1 \qquad Z_c := j \cdot 1.0$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} := \begin{bmatrix} Z_{line} + Z_a + Z_{line} & Z_{line} & Z_{line} \\ Z_{line} & Z_{line} + Z_b + Z_{line} & Z_{line} \\ Z_{line} & Z_{line} & Z_c + Z_{line} + Z_{line} \end{bmatrix}^{-1} \cdot \begin{bmatrix} V_{an} \\ V_{bn} \\ V_{cn} \end{bmatrix}$$

$$\begin{bmatrix} I_a \\ I_b \\ I_c \end{bmatrix} = \begin{bmatrix} -0.909i \\ -0.787 + 0.455i \\ 0.787 + 0.455i \end{bmatrix}$$

$$|I_a| = 0.909 \qquad angle(Re(I_a), Im(I_a)) - 360 \cdot deg = -90 \circ deg$$

$$|I_b| = 0.909 \qquad angle(Re(I_b), Im(I_b)) - 360 \cdot deg = -210 \circ deg$$

$$|I_c| = 0.909 \qquad angle(Re(I_c), Im(I_c)) = 30 \circ deg$$

It is also possible (and easier) to solve the balanced system using a per phase equivalent.

$$I_a := \frac{V_{an}}{Z_{line} + Z_a} \qquad I_a = -0.909i \qquad |I_a| = 0.909 \qquad \text{angle}(\text{Re}(I_a), \text{Im}(I_a)) = 270 \cdot \text{deg}$$

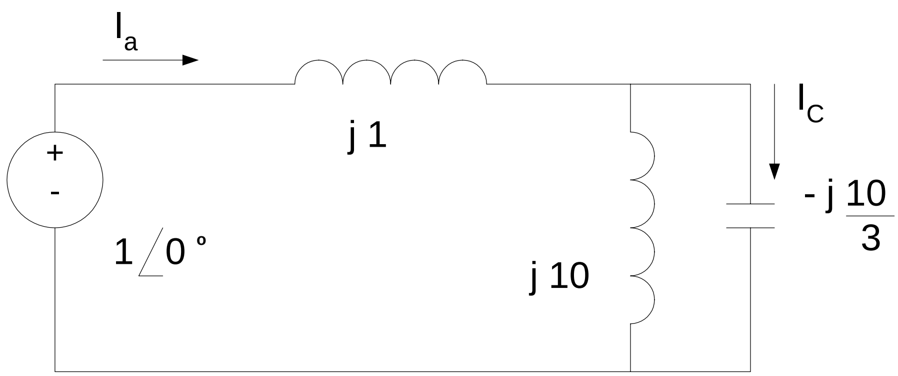
$$I_b := \frac{V_{bn}}{Z_{line} + Z_b} \qquad I_b = -0.787 + 0.455i \qquad |I_b| = 0.909 \qquad \text{angle}(\text{Re}(I_b), \text{Im}(I_b)) = 150 \cdot \text{deg}$$

$$I_c := \frac{V_{cn}}{Z_{line} + Z_c} \qquad I_c = 0.787 + 0.455i \qquad |I_c| = 0.909 \qquad \text{angle}(\text{Re}(I_c), \text{Im}(I_c)) = 30 \cdot \text{deg}$$

Problem 5. (Problem 2.12 on page 49 of the text) The system shown below is balanced. Assume that load inductors ZL=j10 and load capacitors ZC=-j10. Find Ia, Icap, and S3ϕload.

$Z_{C\Delta} := -j \cdot 10$ $\qquad Z_{LY} := j \cdot 10$ $\qquad j := \sqrt{-1}$

It is probably easiest to convert to a Y equivalent circuit. The per phase equivalent circuit becomes:



The delta to wye conversion reduces the balanced capacitance by a factor of 3. Finding the current Ia:

$$Z_{load} := \frac{(j \cdot 10) \cdot \left(-j \cdot \frac{10}{3}\right)}{j \cdot 10 - j \cdot \frac{10}{3}} \qquad Z_{load} = -5i \qquad I_a := \frac{1.0 + j \cdot 0.0}{j \cdot 1 + Z_{load}} \qquad I_a = 0.25i$$

By current division,

$$I_C := I_a \cdot \frac{(j \cdot 10)}{j \cdot 10 - j \cdot \frac{10}{3}} \qquad I_C = 0.375i$$

the current in a delta is a factor of sqrt of 3 less than the equivalent wye current and leads by 30 degrees.

$$I_{cap} := \frac{I_C}{\sqrt{3}} e^{j \cdot \frac{\pi}{6}} \qquad I_{cap} = -0.108 + 0.188i \qquad \left|I_{cap}\right| = 0.217$$

$$I_{cap\_angle} := \text{angle}\left(\text{Re}\left(I_{cap}\right), \text{Im}\left(I_{cap}\right)\right) \qquad I_{cap\_angle} = 120 \cdot \text{deg}$$

Finding the three phase output complex power (in this case all reactive),

$$S_{3\phi load} := 3 \cdot \left|I_a\right| \cdot \left|I_a\right| \cdot Z_{load} \qquad S_{3\phi load} = -0.9375i$$

Checking the complex power out,

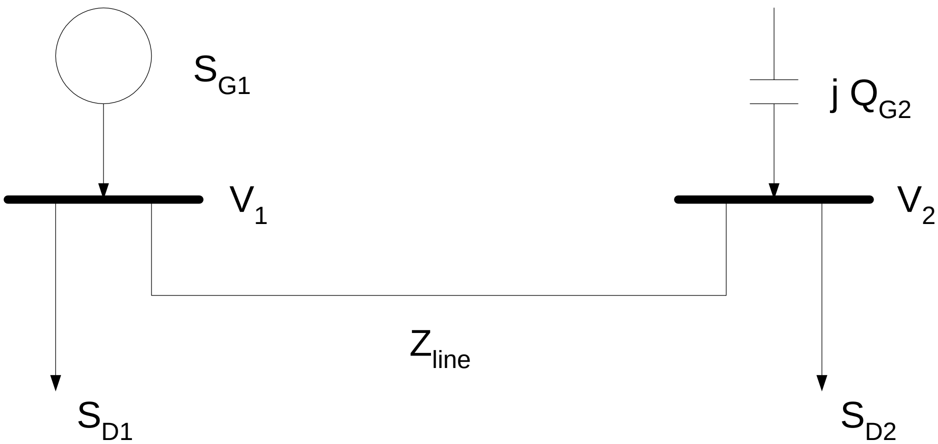
$V_{anx} := I_a \cdot Z_{load}$

$S_{3\phi load} := 3 \cdot V_{anx} \cdot \overline{I_a}$ $\qquad S_{3\phi load} = -0.937i$

Another check of complex power out,

$S_{3\phi in} := 3 \cdot 1 \cdot \overline{I_a}$ $\qquad S_{3\phi out} := S_{3\phi in} - 3 \cdot \left| I_a \right| \cdot \left| I_a \right| \cdot (j \cdot 1)$ $\qquad S_{3\phi out} = -0.937i$

Problem 6. In the Figure shown below, assume that



$j := \sqrt{-1}$

$V_1 := 1 + j \cdot 0$

$Z_{line} := 0.01 + j \cdot 0.1$

$S_{D1} := 0.5 + j \cdot 0.5$

$S_{D2} := 0.5 + j \cdot 0.5$

Pick QG2 such that |V2|=1. In this case, what are QG2, SG1, and the angle of V2?

State the given voltage magnitude and find the angle on the line impedance.

$V_{2mag} := 1$ $\quad angle_Z := angle\left(Re\left(Z_{line}\right), Im\left(Z_{line}\right)\right)$ $\quad angle_Z = 84.289 \circ deg$

From the diagram, we see that the real power in the load SD2 must all be supplied from the transmission line.

$P_{D2} := Re\left(S_{D2}\right)$ $\quad P_{D2} = 0.5$

$P_{21} := -P_{D2}$ $\quad P_{21} = -0.5$

An expression for the real power from the line is as follows:

$$P_{21} = Re\left[\frac{\left(\left|V_2\right|\right)^2}{\left|Z_{line}\right|} \cdot e^{j \cdot angle_Z} - \frac{\left|V_1\right| \cdot V_{2mag}}{\left|Z_{line}\right|} \cdot e^{j \cdot angle_Z} \cdot e^{j \cdot \theta_{12}}\right]$$

Guess

$\theta_{12} := 0 \cdot deg$

Take the real part and solve for the voltage phase angle difference across the transmission line.

Given

$$P_{21} = \frac{\left(\left|V_{2mag}\right|\right)^2}{\left|Z_{line}\right|} \cdot \cos\left(angle_Z\right) - \frac{\left|V_1\right| \cdot V_{2mag}}{\left|Z_{line}\right|} \cdot \cos\left(angle_Z + \theta_{12}\right)$$

$\theta_{12} := \text{Find}(\theta_{12})$ $\theta_{12} = -2.902\cdot\text{deg}$

Therefore, **voltage V2 is 1.0 at an angle of - 2.902 degrees**.

A rectangular form to be used in calculations that follow is

$V_2 := V_{2mag}\cdot(\cos(\theta_{12}) + j\cdot\sin(\theta_{12}))$

An expression derived in class for S12 is

$$S_{12} := \frac{(|V_1|)^2}{\overline{Z_{line}}} - \frac{|V_1|\cdot|V_2|}{\overline{Z_{line}}}\cdot\overline{\left(e^{j\cdot\theta_{12}}\right)}$$

solving,

$S_{12} = 0.503 - 0.037i$

At bus 1, the complex power balance is

$S_{G1} := S_{D1} + S_{12}$ $S_{G1} = 1.003 + 0.463i$

Therefore, **SG1 is 1.003 per unit real power and 0.463 per unit reactive power.**

At bus 2, the complex power entering is

$$S_{21} := \left[\frac{(|V_2|)^2}{\overline{Z_{line}}} - \frac{|V_1|\cdot|V_2|}{\overline{Z_{line}}}\cdot e^{j\cdot\theta_{12}}\right]$$ $S_{21} = -0.5 + 0.063i$

At bus 2, the complex power balance is

$Q_{G2} := S_{21} + S_{D2}$ $Q_{G2} = -1.717\cdot10^{-7} + 0.563i$

Therefore, **QG2 is 0.563 per unit reactive power.**