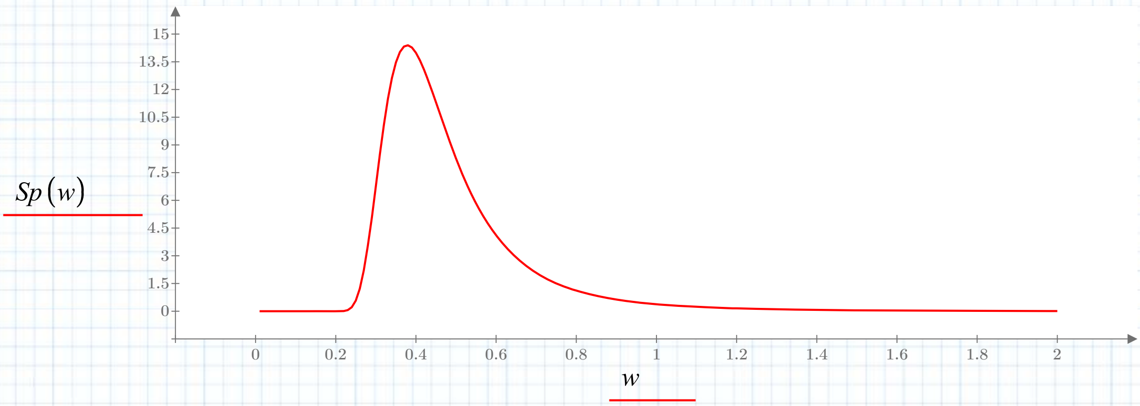
Geração de série temporal das elevações do mar: Espectro Pierson-Moskovitz

$Hs := 7.8$ Altura significativa de onda

$Tz := 11.8$ Período cruzamento zero

$$B := \frac{\left(\frac{2 \cdot \pi}{Tz}\right)^4}{\pi} \qquad A := B \cdot \frac{Hs^2}{4} \qquad Sp(w) := \frac{A}{w^5} \cdot \exp\left(\frac{-B}{w^4}\right)$$

$w := 0, 0.01 .. 2$



Intervalo de Frequências | Intervalo de Tempo | Tempo de Simulação

$\omega i := 0.2$ $\Delta t := 0.125$ $T := 10800$

$\omega f := 2.0$ $Ts := T$ $T = 1.08 \cdot 10^4$

Momentos Espectrais

$$mo := \int_{\omega i}^{\omega f} \omega^0 \cdot Sp(\omega)\, d\omega \qquad m2 := \int_{\omega i}^{\omega f} \omega^2 \cdot Sp(\omega)\, d\omega \qquad m4 := \int_{\omega i}^{\omega f} \omega^4 \cdot Sp(\omega)\, d\omega$$

$mo = 3.796$ $m2 = 1.029$ $m4 = 0.57$

$\sigma := \sqrt{mo}$ $\sigma = 1.948$ $\varepsilon := \sqrt{1 - \frac{m2^2}{mo \cdot m4}}$

$v_m := \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{m4}{m2}}$ $v_m = 0.118$ $v_o := \frac{1}{2 \cdot \pi} \cdot \sqrt{\frac{m2}{mo}}$ $v_o = 0.083$ $\varepsilon = 0.715$ Largura de banda

Relação entre Hs e m0

$$\frac{Hs}{\sqrt{mo}} = 4.003$$

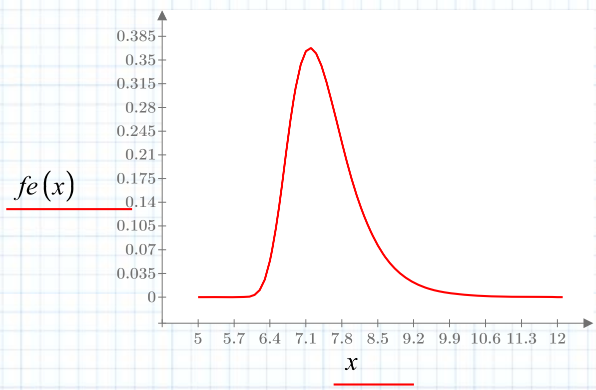
Estatística do Valor Extremo (amplitude da onda individual extrema)

$u := \sqrt{mo} \cdot \sqrt{2 \cdot \ln(v_o \cdot T)}$ $\quad u = 7.184$ $\quad v_o \cdot T = 895.092$

$\alpha := \sqrt{2 \cdot \ln(v_o \cdot T)} \cdot \frac{1}{\sqrt{mo}}$ $\quad \alpha = 1.892$ $\quad \mu_e := u + \frac{0.5772}{\alpha}$ $\quad \mu_e = 7.489$

$\sigma_e := \frac{\pi}{\sqrt{6} \cdot \alpha}$ $\quad \sigma_e = 0.678$

$fe(x) := \exp(-\alpha \cdot (x - u) - \exp(-\alpha \cdot (x - u)))$ $\quad x := 5, 5.1 .. 14$



$Hmax := 2 \cdot u$ $\quad Hmax = 14.368$

$\frac{Hmax}{Hs} = 1.842$ $\quad N := v_o \cdot T = 895.092$

Se fosse 1000 a relação seria 1.86!

Geração de uma série temporal

No de componentes

$N\omega := 500$

Intervalo de frequência

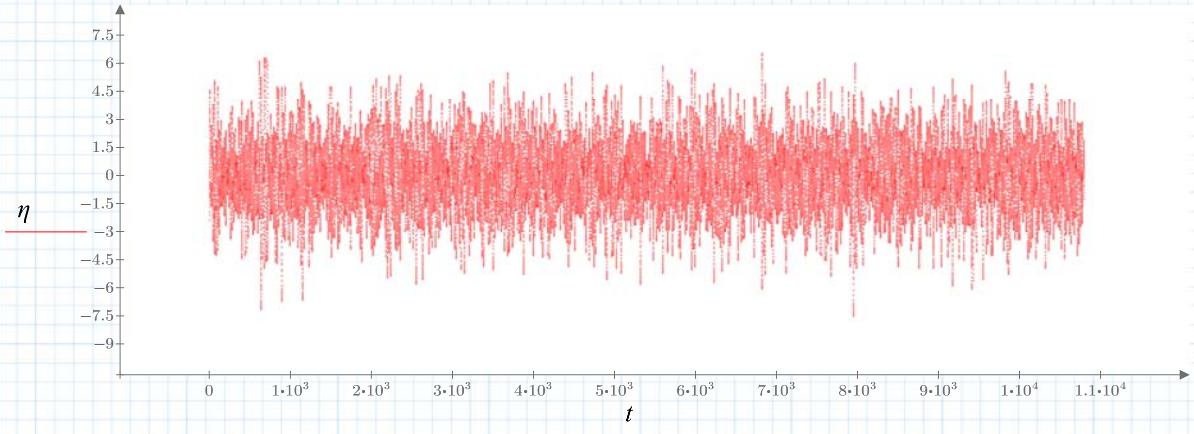
$\Delta\omega := \dfrac{\omega f - \omega i}{N\omega}$ $\quad \Delta\omega = 0.004$

Simulação no domínio do tempo

$NP := \dfrac{T}{\Delta t} + 1 \quad NP = 8.64 \cdot 10^4 \quad i := 1, 2 .. NP \quad t_i := (i-1) \cdot \Delta t$

$$Serie(N\omega, \Delta\omega, NP, \Delta t) := \begin{array}{l} \phi \leftarrow \text{runif}(N\omega, 0, 2 \cdot \pi) \\ \phi\omega \leftarrow \text{runif}(N\omega, 0, 1) \\ \text{for } i \in 1, 2 .. N\omega \\ \quad \omega m_i \leftarrow \dfrac{\Delta\omega \cdot (i-1) + \Delta\omega \cdot i + \omega i \cdot 2}{2} \\ \quad \omega_i \leftarrow \omega i + \Delta\omega \cdot (i-1) + \Delta\omega \cdot \phi\omega_i \\ \quad A_i \leftarrow \sqrt{2 \cdot Sp(\omega m_i) \cdot \Delta\omega} \\ \text{for } k \in 1, 2 .. NP \\ \quad t_k \leftarrow (k-1) \cdot \Delta t \\ \quad y_k \leftarrow \sum_{j=1}^{N\omega} \left(A_j \cdot \cos\left(\omega_j \cdot t_k + \phi_j\right)\right) \\ y \end{array}$$

$\eta := Serie(N\omega, \Delta\omega, NP, \Delta t)$

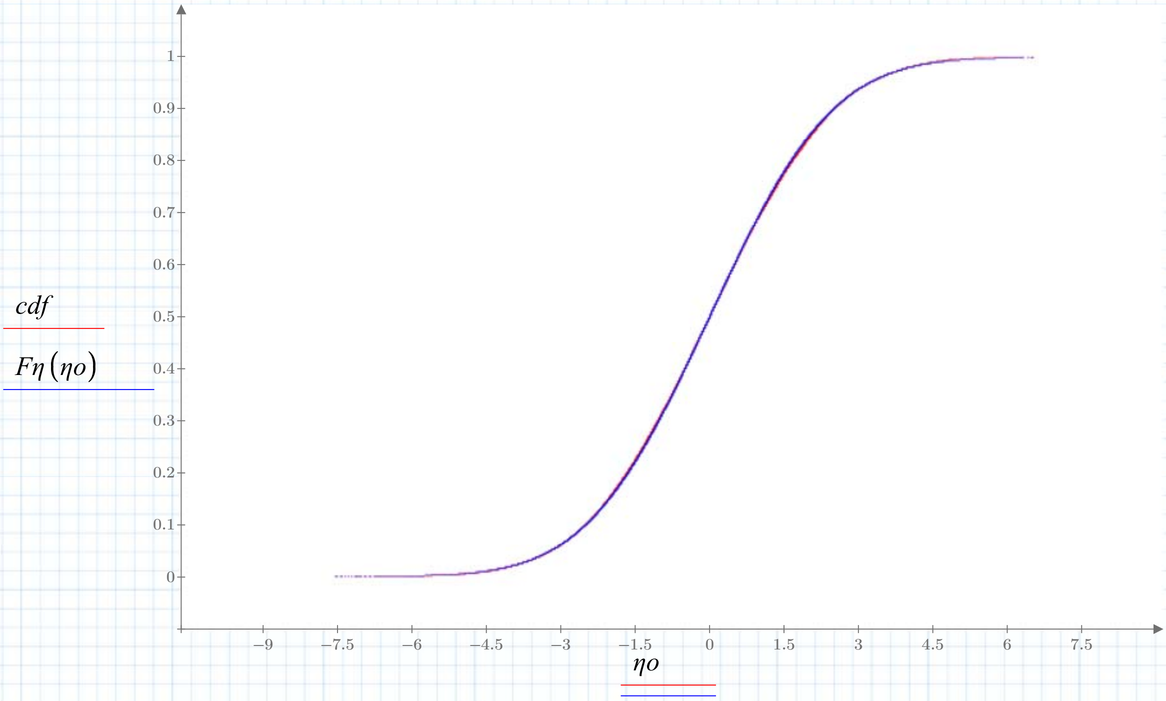


Distribuição de probabilidades do processo aleatório (comparação com a NORMAL)

$\eta o := \text{sort}(\eta)$ $\mu_\eta := \text{mean}(\eta)$ $\sigma_\eta := \text{stdev}(\eta)$

$cdf_i := \dfrac{i}{NP+1}$ $F\eta(\eta) := \text{cnorm}\left(\dfrac{\eta - \mu_\eta}{\sigma_\eta}\right)$

$\mu_\eta = 9.133 \cdot 10^{-4}$ $\sigma_\eta = 1.945$ $\sqrt{mo} = 1.948$



Distribuição dos máximos - Distribuição de Rice

$\nu_o = 0.083$ $\varepsilon = 0.715$

$$fmax(\eta) := \frac{\varepsilon}{\sqrt{mo}\cdot\sqrt{2\cdot\pi}}\cdot\exp\left(\frac{-1}{2}\cdot\frac{\eta^2}{mo}\right)+\frac{\eta}{mo}\cdot\sqrt{1-\varepsilon^2}\cdot\exp\left(\frac{-\eta^2}{2\cdot mo}\right)\cdot \text{cnorm}\left(\frac{\eta}{mo\cdot\varepsilon}\cdot\sqrt{1-\varepsilon^2}\right)$$

Distribuição a partir da amostra

$$Fmax(\eta) := \int_{-6}^{\eta} fmax(x)\,dx$$

Rotina para separar os máximos

```
pm(x) := | N ← rows(x)
         | v_1 ← 0
         | v_N ← 0
         | for j ∈ 2,3..N−1
         |   | i ← 0
         |   | if (x_{j−1} < x_j) ∧ (x_j > x_{j+1})
         |   |   | i ← 1
         |   | v_j ← i
         | m ← 0
         | for k ∈ 1,2..N
         |   | aux ← m
         |   | m ← v_k + aux
         |   | if v_k > 0
         |   |   | p_m ← x_k
         | p
```

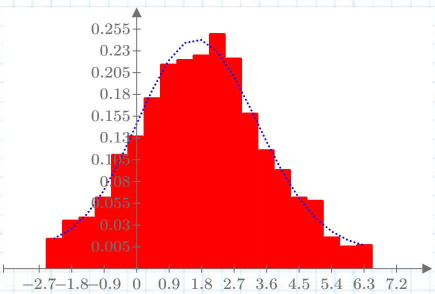
Histograma dos máximos

$xm := pm(\eta)$ $Nint := 20$ $v := \text{histogram}(Nint, xm)$ $xx := v^{\langle 1\rangle}$ $\varDelta := xx_2 - xx_1$

$Nm := \text{rows}(xm)$ $pdf := \frac{v^{\langle 2\rangle}}{Nm\cdot\varDelta}$ $\varDelta = 0.452$ $i := 1,2..Nint$

pdf_i

$fmax(xx_i)$



xx_i

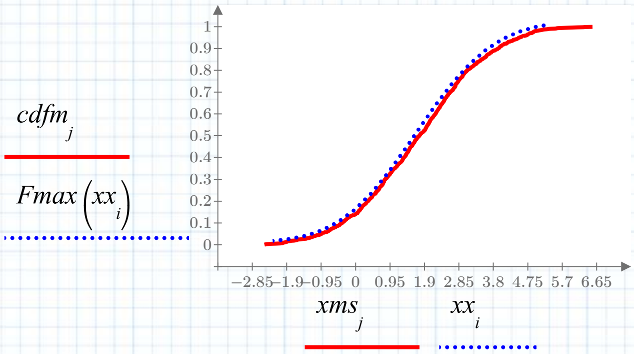
PDF

$\max(xm) = 6.534$ $xms := \text{sort}(xm)$

$\min(xm) = -2.515$ $Nm := \text{rows}(xm)$

$Nm = 1.259\cdot 10^3$ $j := 1,2..Nm$

$$cdfm_j := \frac{j}{Nm+1}$$



CDF

Geração de uma amostra de valores extremos (processo demorado)

$$Sample\_m(N) := \left\| \begin{array}{l} \text{for } j \in 1,2..N \\ \quad \| \; xm_j \leftarrow \max((Serie(N\omega, \Delta\omega, NP, \Delta t))) \\ xm \end{array} \right.$$

$Nm := 20$ $\qquad$ $xm := Sample\_m(Nm)$

$$xm = \begin{bmatrix} 7.61 \\ 6.715 \\ 7.793 \\ 6.947 \\ 8.139 \\ 7.105 \\ 6.528 \\ 7.455 \\ 7.113 \\ 7.053 \\ 7.518 \\ 7.623 \\ \vdots \end{bmatrix}$$

$mm := \text{mean}(xm)$ $\qquad$ $sm := \text{stdev}(xm)$

$mm = 7.296$ $\qquad$ $sm = 0.447$ $\qquad$ $\alpha m := \dfrac{\pi}{\sqrt{6} \cdot sm}$ $\qquad$ $\alpha m = 2.87$

$um := mm - \dfrac{0.5722}{\alpha m}$ $\qquad$ $um = 7.097$

Valores Teóricos

$\mu_e = 7.489$ $\qquad$ $\sigma_e = 0.678$ $\qquad$ $u = 7.184$ $\qquad$ $\alpha = 1.892$ $\qquad$ $i := 1,2..Nm$

$xms := \text{sort}(xm)$ $\qquad$ $Fms_i := \dfrac{i}{Nm + 1}$ $\qquad$ $Fm(x) := \exp(-\exp(-\alpha \cdot (x - u)))$

Cálculo da densidade espectral FFT

Cálculo da densidade espectral: FFT

$NP = 8.64 \cdot 10^4$

$NC := 2^{16}$ Maior número de pontos que pode ser escrito como potência de 2

$NC = 6.554 \cdot 10^4$

$i := 1, 2 .. NC$

$\eta a_i := \eta_i$

$u := \mathrm{FFT}(\eta a)$

$ncoef := \mathrm{length}(u) - 1$

$TC := (NC - 1) \cdot \Delta t$

$\Delta w := \frac{2 \cdot \pi}{TC}$

$k := 1, 2 .. ncoef$

$ncoef = 3.277 \cdot 10^4$

$w_k := (k - 1) \cdot \Delta w$

$an_k := 2 \cdot \mathrm{Re}(u_k)$

$bn_k := -2 \cdot \mathrm{Im}(u_k)$

$an_1 := 0.0$

$bn_1 := 0.0$

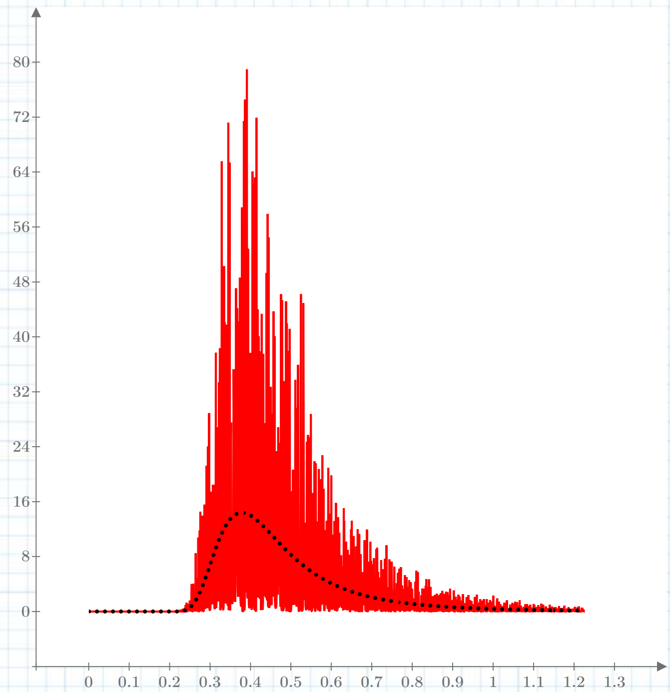
$$SS_k := \frac{(an_k)^2 + (bn_k)^2}{2 \cdot \Delta w}$$

$$\sum_{k=1}^{ncoef} \frac{(an_k)^2 + (bn_k)^2}{2} = 3.817$$

$mo = 3.796$

$j := 1, 2 .. 1600$

$S_j := SS_j$

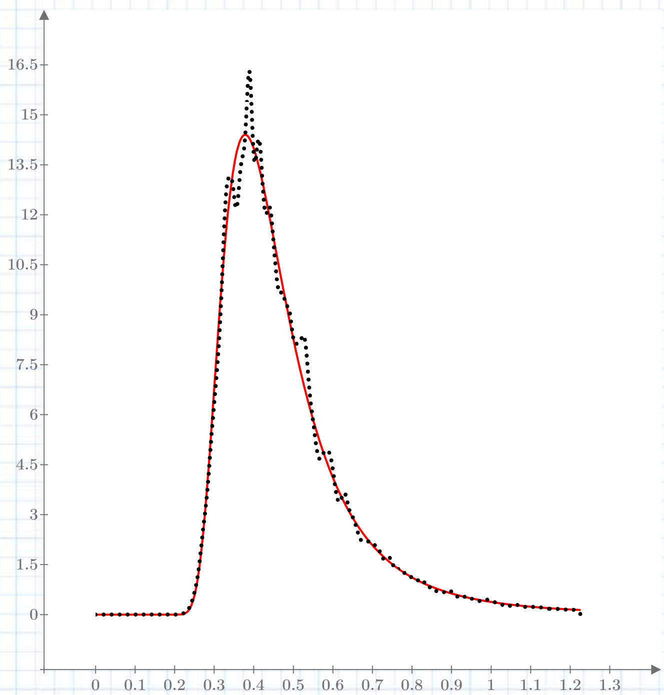


Suavização do Espectro

$$Hann(S, nvezes) := \left\| \begin{array}{l} Nw \leftarrow \text{rows}(S) \\ \text{for } i \in 1, 2 .. nvezes \\ \quad \left\| \begin{array}{l} SS \leftarrow S \\ \text{for } j \in 2, 3 .. Nw - 1 \\ \quad \left\| S_j \leftarrow 0.5 \cdot SS_j + 0.25 \cdot \left(SS_{j-1} + SS_{j+1}\right) \right. \end{array} \right. \\ S \end{array} \right.$$

$nvezes := 200$

$Snew := Hann(S, nvezes)$



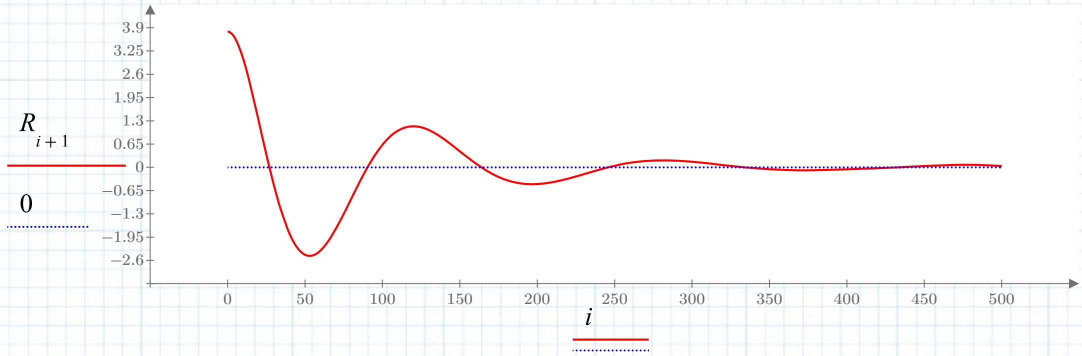
Função de Auto-correlação

$NPP := 500$

$i := 0, 1 .. NPP$

$tt_{i+1} := i \cdot \Delta t$

$$R_{i+1} := \sum_{j=1}^{NP-NPP} \frac{\eta_j \cdot \eta_{j+i}}{NP-NPP}$$



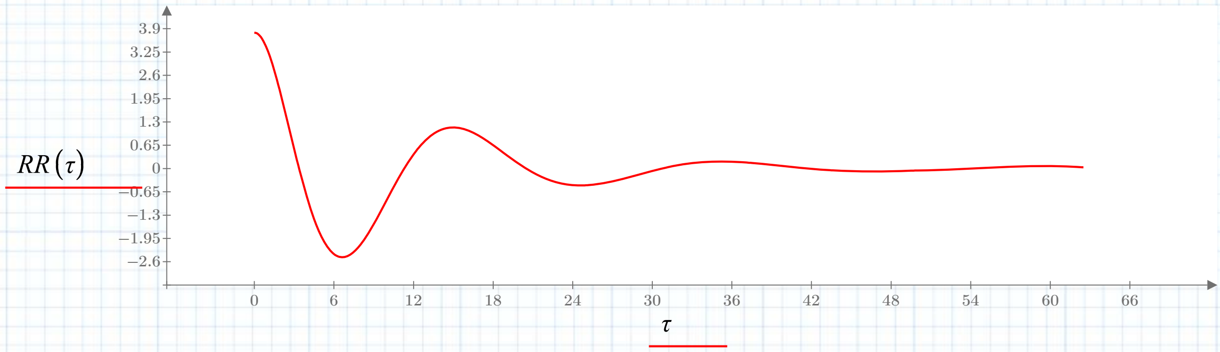
$vs := \text{lspline}(tt, R)$

$Tm := tt_{\text{rows}(tt)}$

$Tm = 62.5$

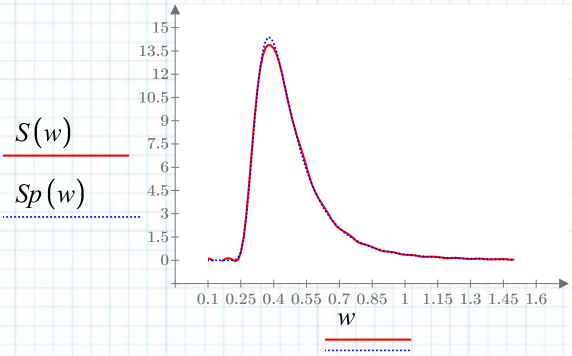
$RR(\tau) := \text{interp}(vs, tt, R, \tau)$

$\tau := 0, 0.1 .. Tm$



$$S(w) := 4 \cdot \frac{\int_0^{Tm} RR(\tau) \cdot \cos(w \cdot \tau)\, d\tau}{2 \cdot \pi}$$

$w := 0.1, 0.1125 .. 1.5$



S(w)
Sp(w)
15
13.5
12
10.5
9
7.5
6
4.5
3
1.5
0
0.1 0.25 0.4 0.55 0.7 0.85 1 1.15 1.3 1.45 1.6
w