Chapter 2
Solar Radiation and Heat Transfer

2.1 Introduction

The Sun (with a diameter of 1.39×10^6 km) comprises intensely hot gaseous matter and is the most abundant source of solar energy in the universe. The Sun is a continuous fusion reactor, and it comprises 75% hydrogen, 24% helium, and 1% other elements. The surface temperature of the Sun is 5777 K. Various thermonuclear processes takes place at the interior of the Sun with the most important being hydrogen protons ($n = 4$) combining to form helium (helium nucleus), and the formed final mass is found to be less than the mass of the four protons (the initial mass). This conveys that the lost mass in the reaction has been converted to energy. This energy is produced at temperatures on the order of millions of degrees and is transferred outward at the outer surface of the Sun and further radiates into space [1]. On average, the Sun–Earth distance is 1.5×10^8 km. One half of the Earth is ignited by solar radiation at a time, and it reflects approximately one third of the solar radiation falling on it. This is known as the "albedo" of the Earth.

Heat-transfer mechanism plays an important role in renewable-energy technologies and practical applications. Heat transfer between any two systems takes place as a result of driving fore called "temperature difference." In general, heat is a form of energy that can be transferred from one system to another as a consequence of a difference of temperatures (ΔT) among the two systems. The energy transfer by way of mode of heat transfer occurs through electrical and mechanical processes. It is essential to understand and determine the rates of such heat transfer. In any particular direction, the rate of heat transfer depends on the temperature gradient in that direction, and it increases with an increase in the temperature gradient. Basically, heat is transferred through three modes: conduction, convection, and radiation. These modes of heat transfer are governed by different laws and differ profoundly in nature. Heat transfer through these modes always takes place from the higher surface temperature to the lower one [1, 2] in the modes of transfer of a

solar-distillation system (i.e., conduction, convection, radiation, and evaporation) are discussed in detail in this chapter.

2.2   Solar Radiation

Any object can emit electromagnetic radiation provided that its temperature is >0 K. Solar radiation is a part of electromagnetic radiation that is emitted by the Sun in large amounts through thermonuclear processes. The spectral radiance depends on the emitting properties of the surface and the temperature of an abject. The green portion of the visible spectrum gives the maximum solar intensity at a wavelength of 0.48 μm. The ultraviolet $(\lambda < 0.4\,\mu m)$ and visible region $(0.4\,\mu m < \lambda < 0.7\,\mu m)$ of the spectrum consists of approximately 8.73 and 38.15%, respectively. The remaining 52.12% of the total energy is present in the infrared region $(\lambda > 0.70\,\mu m)$. The mechanism of absorption and scattering of solar radiation takes place during the propagation of energy through the atmosphere. Nitrogen, oxygen, and other atmospheric gasses in the ionosphere highly absorb the X-ray and extreme ultraviolet rays of the Sun. The ultraviolet $(\lambda < 0.7\,\mu m)$ and infrared radiations $(\lambda > 2.3\,\mu m)$ are extensively absorbed by the ozone layer. There is almost comprehensive absorption of short-wavelength radiation $(\lambda < 0.29\,\mu m)$. Consequently, there is negligible solar energy available below and above the range of $0.29\,\mu m < \lambda < 2.3\,\mu m$ of the spectrum of solar radiation incident on the surface of the Earth [2].

In general, the atmosphere of the Earth has two unique properties:

 (i) It allows short-wavelength radiation having wavelength in the range of $0.29\,\mu m < \lambda < 2.3\,\mu m$, and it absorbs ultraviolet and far infrared radiations.
(ii) It does not allow radiation having wavelength, $\lambda \geq 2.3\,\mu m$ which is known as "long-wavelength radiation."

The Earth receives approximately 180×10^8 GW of solar radiation at any one point at a given geographical site. The amount differs between night and day due to the rotation of the Earth as well as between seasons due to the Earth's orbit. In addition, in space it varies due to the change in obliquity of solar rays with latitude and longitude at any given time. In general, the amount of power received by the Earth at a given location and time depends on the relative Sun–Earth position. Due to this reason, time and Sun–Earth geometry plays a vital role for solar-energy conversion. Therefore, it is essential to understand the Sun–Earth angles, which are discussed later in the text.

The determination of solar flux either on a horizontal surface or on a surface perpendicular to the solar radiation gives the energy flux of the beam radiation on an arbitrarily oriented surface. If the beam flux (I) on a plane surface is incident at an angle θ_i, then the flux incident on the plane surface becomes $I \cos \theta_i$.

Table 2.1 Different orientations of solar distillation system (Northern hemisphere)

Solar distillation system	Orientation	Inclination in different weather condition				Annual
		Solar still		FPC[a]		
		Summer	Winter	Summer	Winter	
Passive single slope	Due South	$\varphi - 15°$	$\varphi + 15°$	–		φ
Passive double slope	East–West	$\varphi - 15°$	$\varphi + 15°$	–		φ
Active single slope (FPC)	Due South	$\varphi - 15°$	$\varphi + 15°$	$\varphi - 15°$	$\varphi + 15°$	φ
Active double slope (FPC)	East–West	$\varphi - 15°$	$\varphi + 15°$	$\varphi - 15°$	$\varphi + 15°$	φ

Note [a]Orientated only due south

Latitude (ϕ): The latitude of a location is the angle made by the radial line joining the given location to the center of the Earth with its projection on the equatorial plane. For the Northern Hemisphere, the latitude is positive, and it is negative for the Southern Hemisphere. The different orientations of solar distillation systems for Northern Hemisphere are given in Table 2.1.

Declination (δ): Declination is defined as the angle between the line joining the centers of the Sun and the Earth and its projection on the equatorial plane, and it is due to the rotation of the Earth on its own axis. It makes an angle of 660.5° with the plane of its rotation around the Sun. The range of the declination angle varies from 230.45° on June 21 (maximum value) to –230.45° on December 21 (minimum value). It is expressed as follows:

$$\delta = (23.45)\sin\left[\left(\frac{360}{365}\right)(284 + \text{n})\right] \qquad (2.2.1)$$

Hour angle (ω): This is the angle through which the Earth must rotate to bring the meridian of the plane directly under the Sun. In other words, it is the angular displacement of the Sun east or west of the local meridian due to the rotation of the Earth on its axis at 15° per hour. The hour angle is zero at solar noon, negative in the morning, and positive in the afternoon. The expression for the hour angle is given by the following:

$$\omega = (\text{ST} - 12)15° \qquad (2.2.2)$$

where ST is local solar time.

Zenith (θ_z): This is defined as the angle between the Sun's rays and a perpendicular line to the horizontal plane.

Altitude or solar-altitude angle (β): This is defined as the angle between the Sun's rays and a horizontal plane. In addition, $\alpha = 90° - \theta_z$.

Slope (β): This is the angle between the plane surface under consideration and the horizontal. It is taken to be positive for surfaces sloping toward the south and negative for surfaces sloping toward the north.

Table 2.2 Surface azimuth angle (γ) for various orientations in Northern hemisphere [2]

Surface orientation	γ
Sloped towards South	0°
Sloped towards North	180°
Sloped towards East	−90°
Sloped towards West	+90°
Sloped towards South-East	−45°
Sloped towards South-West	+45°

Surface-azimuth angle (γ): This is the angle in the horizontal plane between the line due south and the projection of the normal to the surface (inclined plane) on the horizontal plane. By convention, the angle will be taken to be negative for the Northern Hemisphere (India) and positive for the Southern hemisphere (Australia) if the projection is east of south and positive if it is west of south. The values of γ for some orientations are listed in Table 2.2.

Solar-azimuth angle (γ_s): This is the angle in a horizontal plane between the line due south and the projection of beam radiation on the horizontal plane. By convention, the angle is taken to be positive and negative, respectively, if the projection is east of south and west of south for the Northern Hemisphere (India) and vice versa for the Southern Hemisphere.

Angle of incidence (θ_i): This is the angle between beam radiation on a surface and normal to that surface. In general, the angle of incidence (θ_i) can be expressed as follows:

$$\cos \theta_i = (\cos \varphi \cos \beta + \sin \varphi \sin \beta \cos \gamma) \cos \delta \cos \omega \\ + \cos \delta \sin \omega \sin \beta \sin \gamma + \sin \delta (\sin \varphi \cos \beta - \cos \varphi \sin \beta \cos \gamma) \tag{2.2.3}$$

For a horizontal plane facing due south, $\gamma = 0, \beta = 0, \theta = \theta_z$ (zenith angle)

$$\cos \theta_z = \cos \varphi \, \cos \delta \, \cos \omega + \sin \delta \, \sin \varphi. \tag{2.2.4}$$

There is only 1.7% variation between the Sun and Earth during the orbital motion of the Earth around the Sun. The solar radiation outside the Earth's atmosphere remains at nearly fixed intensities. The surface of unit area receives the radiant energy flux per second outside the atmosphere. Moreover, the unit surface area, which held normal to the direction of the solar radiations at the mean Sun–Earth distance, basically remains constant all year long. This is known as the "solar constant" (I_{sc}) and has a numerical value of 1367 W/m^2. Extraterrestrial radiation (I_{ext}) varies due to the elliptical path of the Earth around the Sun (at one of the foci). The solar intensity of extraterrestrial radiation, in terms of solar constant I_{sc} measured on the nth day of the year on a plane normal to the radiation, is given as follows

$$I_{ext} = I_{sc} \left[1 + 0.033 \cos \left(\frac{360n}{365} \right) \right] \tag{2.2.5}$$

2.3 Solar Radiation on an Inclined Surface

The total solar radiation incident on a surface is a combination of the following:

(a) Beam radiation (I_b) or direct radiation: The solar radiation propagating along the line joining receiving surface and the Sun;
(b) Diffuse radiation (I_d): The solar radiation scattered by dust, aerosols, and molecules etc.; it does not have any particular direction of propagation; and
(c) Solar radiation reflected from the ground and surroundings.

After knowing beam and diffuse radiation on horizontal surface, the total radiation or global radiation on a surface of any arbitrary orientation (β) of solar thermal device (for east, south, north, and west) for $\gamma = \pm 90°, \pm 180°$ and $0°$ (Table 2.2) can be obtained from the formulae given by Liu and Jordan [3]:

$$I_T = I_b R_b + I_d R_d + \rho R_r (I_b + I_d) \tag{2.3.6}$$

where R_b, R_d and R_r are known as conversion factors for the beam, diffuse, and reflected components, respectively; and ρ is the reflection coefficient of the ground (0.6 and 0.2 for snow and ordinary covered ground, respectively). The expressions for R_b, R_d and R_r are given below:

(i) **R_b** : This is defined as the ratio of the flux of beam radiation incident on an inclined surface to that on a horizontal surface. Therefore, R_b for beam radiation can be obtained as follows:

$$R_b = \frac{I'_b}{I_b} = \frac{\cos \theta_i}{\cos \theta_z}$$

(ii) **R_d** : This is defined as the ratio of the flux of diffuse radiation falling on the tilted surface to that on the horizontal surface.

The distribution of diffuse radiation over the sky as well, as portion of the sky exposed to the surface, defines the value of R_d. No reasonable method has been developed for the evaluation of diffuse radiation distribution over the sky.

However, it is widely considered that the sky is an isotropic source of diffuse radiation. If the radiation shape factor for a tilted surface with respect to sky is $(1 + \cos \beta)/2$, then

$$R_d = \frac{1 + \cos \beta}{2}$$

(iii) **R_r** : The ground and other surrounding objects are responsible for the origin of the reflected component. Considering the reflected radiation as diffuse and isotropic (the situation opposite to the above case), then

$$R_r = \frac{1 - \cos \beta}{2}$$

It may be mentioned here that both R_b and R_d undergo reflection from the ground and surroundings. The effective ratio of incident solar energy on a surface to that on a horizontal surface is given as follows:

$$R' = \frac{I_T}{I_b + I_d} = \frac{I_b R_b + I_d R_d}{I_b + I_d} + \rho R_r$$

To obtain the total radiation from Eq. (2.3.6), the beam and diffused radiation can be calculated from the following methods.

2.3.1   First Method

According to Singh and Tiwari [4] (by knowing the experimental values of beam and diffused radiation), the rate of beam (direct) radiation reaching the terrestrial region can be expressed as

$$I_N = I_{ext} \exp[-(m\varepsilon T_R + \alpha')] \tag{2.3.7}$$

The expression of I_{ext} is given in Eq. (2.2.5).

Where m **is** air mass; T_R is the turbidity factor; ε is the atmosphere's optical thickness of an integrated Rayleigh scattering; and α' is the lumped atmospheric parameter for the beam radiation. The parameters m and ε are given as [5, 6] follows:

$$m = \frac{1}{\left[\cos \theta_z + 0.15(93.88 - \theta_z)^{-1.25}\right]} \tag{2.3.8}$$

and

$$\varepsilon = \left(4.53 \times 10^{-4}\right)m^2 - \left(9.67 \times 10^{-3}\right)m + 0.1080 \tag{2.3.9}$$

Therefore, the terrestrial beam radiation is expressed by classical equation as follows:

$$I_b = I_N \cos \theta_z = I_{ext} \exp[-(m\varepsilon T_R + \alpha')]\cos\theta_z \tag{2.3.10}$$

where α' determines an additional reduction of the direct normal radiation in the terrestrial region due to different climatic conditions such as cloudiness/haziness as well as transient and unpredictable changes.

On the horizontal surface, the diffused radiation can be rewritten as follows:

$$I_d = K_1(I_{ext} - I_N)\cos\theta_z + K_2 \tag{2.3.11}$$

where K_1 (dimensionless) and $K_2\left(\text{W/m}^2\right)$ are constants of lumped atmospheric parameters for diffuse radiation. Furthermore, the constant K_1 can be inferred as a *perturbation factor* for describing the scattering out of a beam traversing the lumped atmosphere, and K_2 can be expressed as background diffuse radiation.

2.3.2   Second Method

The following expression for the beam and diffuse radiation on the horizontal surface can be used (case of non-availability of data for the beam and diffuse radiation):

$$I_b = I_N\cos\theta_z \tag{2.3.12}$$

$$I_d = \frac{1}{3}[I_{ext} - I_N]\cos\theta_z \tag{2.3.13}$$

and

$$I_N = I_{ext}\exp\left[\frac{-T_R}{(0.9 + 9.4\sin\alpha)}\right] \tag{2.3.14}$$

The expression of I_{ext} is given in Eq. (2.2.5). T_R is the turbidity factor, and it depends on the sky conditions (clear, cloudy, and hazy) as well as different regions (mountain, flat land, city, desert, or forest etc.). The turbidity factors for different months and locations are listed in Table 2.4.

After knowing the beam and diffuse radiation on a horizontal surface, total radiation on a surface of arbitrary orientation (east, south, west and north $\gamma = -90°$, 0, $+90°$ and $\pm180°$ for a given latitude (ϕ)) can be evaluated from the Liu and Jordan formulae as given in Eq. (2.3.6).

Example 2.1 Calculate the total solar radiation from (a) the first method and (b) the second method for an inclined surface having an inclination of 10° facing east at 11 am for New Delhi on 23 September 2015 $(T_R = 4.75 \text{ and } \rho = 0.2)$. Use the parameters listed in Table 2.3 for the first method (Table 2.4).

Solution

$\varphi = 28.58°; \beta = 28.58°; \gamma = -90°(\text{East surface}); n = 266; ST = 11 \text{a.m.}$

$\delta = 23.45\sin\left[\left(\frac{360}{365}\right)(284 + n)\right] = -1.0088°; \omega = (11 - 12) \times 15° = -15°$

Table 2.3 Evaluated parameters (T_R, α', K_1, K_2) for average weather conditions for New Delhi climatic conditions New Delhi climatic conditions

Month factor	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Nov	Dec
T_R	2.65	2.94	3.39	3.49	4.58	7.33	4.49	3.77	4.75	4.33	3.60	4.28
α'	0.33	0.28	0.25	0.38	0.35	0.39	1.03	1.02	0.35	0.03	0.04	−0.04
K_1	0.40	0.41	0.43	0.44	0.46	0.49	0.40	0.37	0.39	0.28	0.16	0.34
K_2	−12.12	−14.51	−12.08	−08.62	−15.56	−49.59	−39.26	−24.69	−19.53	+29.68	+65.18	+4.66

Table 2.4 The turbidity factor (T_R) for different months and locations

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Mountain	1.8	1.9	2.1	2.2	2.4	2.7	2.7	2.7	2.5	2.1	1.9	1.8
Flat land	2.2	2.2	2.5	2.9	3.2	3.4	3.5	3.3	2.9	2.6	2.3	2.2
City	3.1	3.2	3.5	3.9	4.1	4.2	4.3	4.2	3.9	3.6	3.3	3.1

Note For cloudy condition $T_R = 10.0$

From Eqs. (2.2.3) and (2.2.4),

$$\cos(\theta_i) = 0.872 \quad \text{or} \quad \theta_i = \cos^{-1}(0.872) = 29.33°$$
$$\cos(\theta_z) = 0.8396 \quad \text{or} \quad \theta_z = \cos^{-1}(0.8396) = 32.9°$$

Conversion factors:

$$R_b = \frac{0.872}{0.8396} = 1.038; R_d = \frac{1 + \cos(10)}{2} = 0.9924; R_r = \frac{1 - \cos(10)}{2} = 0.0076$$

$$I_{ext} = 1367 \times \left[1 + 0.033 \cos\left(\frac{360 \times 266}{365}\right)\right] = 1361 \text{ W/m}^2$$

(a) **First method**

$$m = \frac{1}{\left[\cos\theta_z + 0.15(93.88 - \theta_z)^{-1.25}\right]} = 1.190$$
$$\varepsilon = (4.53 \times 10^{-4})m^2 - (9.67 \times 10^{-3})m + 0.1080 = 0.0965$$

From Table 2.3, $T_R = 4.75$, $\alpha' = 0.35$, $K_1 = 0.39$, $K_2 = -19.53$

$$I_N = I_{ext} \exp[-(m\varepsilon T_R + \alpha')] = 558.90 \text{ W/m}^2$$
$$I_b = I_N \cos\theta_z = I_{ext} \exp[-(m\varepsilon T_R + \alpha')]\cos\theta_z = 469.27 \text{ W/m}^2$$
$$I_d = K_1(I_{ext} - I_N)\cos\theta_z + K_2 = 243 \text{ W/m}^2$$

From Eq. (2.3.6)

$$I_T = I_b R_b + I_d R_d + \rho R_r(I_b + I_d) = 1082 \text{ W/m}^2$$

(b) **Second method**

$$I_N = 1361 \times \exp\left[\frac{-4.75}{(00.9 + 9.4\sin(90 - 32.9))}\right] = 793.12\,\text{W/m}^2$$
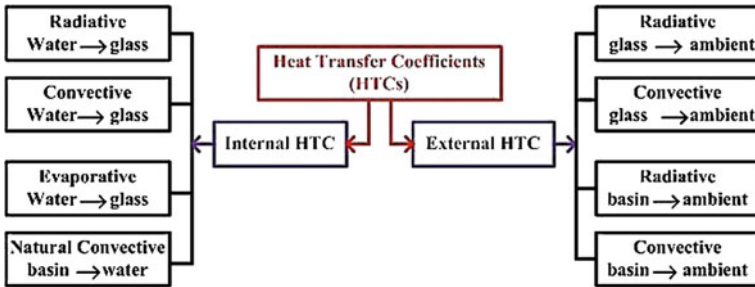
$$I_b = I_N\cos\theta_z = 665.92\,\text{W/m}^2$$

$$I_d = \frac{1}{3}[I_{\text{ext}} - I_N]\cos\theta_z = 158.93\,\text{W/m}^2$$

From Eq. (2.3.6),

$$I_T = I_bR_b + I_dR_d + \rho R_r(I_b + I_d) = 691.22 + 157.7 + 1.25 = 850.1\,\text{W/m}^2$$

2.4　Heat Transfer

The heat transfer in a solar-distillation system can be broadly categorized as (i) internal heat transfer and (ii) external heat transfer as described in the flow chart. These are the most desired heat transfers in a solar-distillation system.

The mechanism of internal heat transfer takes place through the modes of evaporation, convection, and radiation. In contrast, external heat transfer involves conduction, convection, and radiation processes. In the solar-distiller cavity, heat transfer is accomplished by transport of the vapor formed above the water surface through an air–vapor mixture. The internal heat-transfer mechanism is accountable for the conveyance of pure or potable water in vapor form leaving behind all of the impurities remains in the basin of the distillation system. The external heat transferring through the condensing cover is accountable for the condensation of pure vapor as distillate [2, 7, 8].

External heat transfer takes place from the glass cover, the sides of the insulation, and the bottom to the ambient air. The side and bottom losses (external heat transfers) of the distillation system are undesirable. Therefore, it is essential to

minimize heat transfer from the sides and bottom as much as possible for the better performance of the system. The different modes of heat transfer in a solar-distillation system are as discussed in detail later in the text.

2.4.1 Conduction

The process of conduction takes place in solids, liquids, or gases, in which the high energetic particles of the substance transfers energy to adjacent low energetic ones as a consequence of interaction between the particles.

The collision and diffusion of molecules occurs in gases and liquids due to their random motion, which is responsible for the conduction. In solids, it is due to the energy transfer by free electrons and the combination of vibrations of the molecules in a lattice.

Geometry of the medium plays an essential role for the heat-conduction rate, i.e., the temperature difference across the medium as well as thickness and material of the medium. For example, the rate of heat loss from a hot-water tank can be reduced by wrapping it with glass wool, which is an insulating material. Heat loss is less with greater thickness of the insulation. Moreover, the quick heat loss by a hot water tank with a lower surrounding temperature is a well-known fact.

Let us consider a steady-state heat condition through a large plane wall with thickness $\Delta x = L$, surface area A (always normal to the direction of heat transfer); and temperature difference across the wall ΔT. According to Fourier's law of heat conduction, the rate of heat transfer $(\dot{Q})$ through the wall is

$$\dot{Q} = -KA\frac{dT}{dx} \tag{2.4.15}$$

where constant K is the normal conductivity of the material, which determines the material's ability to conduct heat. The above-mentioned relation specifies that the rate of heat conduction in any arbitrary direction increases with an increase in temperature gradient in that direction. This implies that heat conduction occurs in the direction of low temperature such that the temperature gradient becomes negative when temperature decreases with increasing x. Therefore, a negative sign is added in Eq. (2.4.15) to indicate heat transfer in the positive direction.

2.4.2 Convection

The temperature gradient is the driving force for all of the modes of heat transfer. The temperature difference between the fluid and any hot surface in contact with the fluid causes the density gradient in between the layers of fluid above the surface, thus resulting in buoyancy. Consequently, the developed motion of fluid above the surface is called "free convection" or "natural convection." However, the

convection is called "forced convection" if forces arise from an externally imposed pressure difference. The natural- and forced-convection processes may sometimes occur simultaneously.

Mathematically, the rate of heat transfer by convection within the boundary layer of a cooled fluid that is in contact with a hot surface is described by a general equation:

$$\dot{Q} = h_{cw} A \Delta T \tag{2.4.16}$$

where h_{cw} convective heat-transfer coefficient (W/m^2 °C), A area responsible for heat transfer (m^2); and ΔT temperature difference between the fluid and surface of the body.

In Eq. (2.4.16), the crux of the problem is to evaluate the convective heat-transfer coefficient (h_{cw}). It is a convoluted function of the parameters (a) the geometry of the surface, (b) the flow characteristics of the fluid, (c) the physical properties of the fluid at operating temperature, and (d) the operating-temperature range.

Heat-transfer coefficients are evaluated from empirical relations in most of the practical cases. These empirical relations have been developed by correlating experimental outcomes with the help of the dimensional-analysis method. The thermo-physical properties and heat-transfer correlations of different nanofluids are also given in Appendix E.

Dimensionless heat-convection parameters
Nusselt number (Nu): The *Nusselt* number illustrates the process of heat transfer at the solid–fluid boundary, and it is the ratio of convective heat transfer to heat transfer by conduction in the fluid. It is usually unknown in problems of convection because it includes the heat-transfer coefficient (h_{cw}), which is an unknown parameter.

Let us consider a fluid layer of thickness d and temperature difference, (T). Heat transfer through the fluid layer could take place by way of the mode of conduction when the fluid layer is motionless, and it occurs by way of the mode of convection when the fluid involves some motion (natural or forced). The rate of heat transfer per unit time per unit surface area (heat flux) in either case will be as follows:

$$\dot{q}_{conv} = h_{cw} \Delta T \tag{2.4.17}$$

$$\dot{q}_{cond} = \left(\frac{k}{d}\right) \Delta T \tag{2.4.18}$$

From the above-mentioned two equations, according to the definition of the *Nusselt* number (*Nu*):

$$Nu = \frac{h_{cw}}{(k/d)} \tag{2.4.19}$$

This represents the enhancement of heat transfer through a fluid layer as a result of convection relative to conduction across the same fluid layer. For the larger value of *Nusselt* number, the convection is more effective. It is unity for a fluid layer, in which heat transfer takes place through the mode of pure conduction only.

Reynold's number (Re): This is the ratio of the fluid dynamic force $\left(\rho v_0^2\right)$ to the viscous drag force $(\mu v_0/X)$:

$$Re = \frac{\rho v_0 X}{\mu} \tag{2.4.20}$$

where **μ**. the dynamic viscosity; ρ is the density; and X is the characteristic length. *Reynold's* number describes the flow behavior in forced convection. Furthermore, it serves as a criterion for the stability of laminar flow.

Prandtl number (Pr): This is the ratio of momentum diffusivity (μ/ρ) to the thermal diffusivity $\left(K/\rho C_p\right)$:

$$Pr = \frac{\mu C_p}{K} \tag{2.4.21}$$

Through the diffusion mechanism, the specific heat (C_p) gives the relative effectiveness of momentum transport in the velocity boundary layer over the thermal-energy transport in the thermal boundary layer. It is unity for gasses.

Grashof number (Gr): This is the ratio of the buoyancy force to the viscous force.

$$Gr = \frac{g\beta d^3 \rho^2 \Delta T}{\mu^2} \tag{2.4.22}$$

where g is the gravitational acceleration; β is the coefficient of volumetric thermal expansion; and ΔT is the temperature gradient. It signifies the flow regime in natural convection. Low *Grashof* number corresponds to the laminar flow, whereas its large value indicates the turbulent flow in natural convection.

Rayleigh number (Ra): This is the ratio of thermal buoyancy to viscous inertia. The *Rayleigh* number (Ra) for a fluid is related with buoyancy-driven flow, and it is a dimensionless number, also known as "free convection" or "natural convection." If the *Rayleigh* number is below a critical value for a particular fluid, then heat transfer primarily occurs in the form of conduction, whereas it takes place in the form of convection when it exceeds the critical value. In general, the transition in a boundary layer of the fluid in natural convection depends on the relative magnitude of the viscous and buoyancy forces. It is correlated in terms of the *Rayleigh's* number, which is the product of *Grashof* number (Gr) and *Prandtl* number (Pr).

$$Ra = Gr \, Pr \tag{2.4.23}$$

The *Reynold* (*Re*), *Prandtl* (*Pr*), and *Grashof* (*Gr*) numbers are evaluated by using the physical properties at the average temperatures (T_f) of two surfaces. The thermo-physical properties of water vapor; water vapor and air mixture; and water (basefluid) are given in Tables 2.5, 2.6, and 2.7 respectively. For the other shape, the characteristic dimension (X) is given by the following:

$$X = \frac{A}{P} \tag{2.4.24}$$

where A and P is area and perimeter of the surface, respectively, which is generally used for the irregular shape. For a rectangular horizontal surface $(L_0 \times B_0)$, it can be evaluated by (Table 2.8)

$$X = \frac{L_0 + B_0}{2} \tag{2.4.25}$$

Example 2.2 For a passive double-slope solar still, calculate the convective heat-transfer coefficient from the blackened surface (basin liner) to (a) the water (basefluid) and (b) the nanofluid (Al_2O_3 nanoparticles) for the following parameters:

Water mass $(M_w) = 40$ kg, solar still area $(A_b) = 2$ m $\times$ 1 m, ambient temperature $(T_a) = 35\,°C$, average glass temperature $(T_{gi}) = 50.7\,°C$, solar intensity $(I(t)) = 940$ W/m^2, wind velocity $(v) = 1.5$ m/s, $C = 0.54, n = 0.25$, inclination angle $(\theta) = 30°$, characteristic length $(d) = 1.5$ m, absorptivity of glass $(\alpha_g) = 0.85$, absorptivity of water $(\alpha_w) = 0.65$, absorptivity of basin $(\alpha_g) = 0.8$, $\varepsilon_{eff} = 0.741$, and thermal conductivity of glass $(K_g) = 0.004$ (W/m °C).

Properties of Al_2O_3 nanoparticles: $C_{pp} = 880$(J/kg K), $\beta_p = 8.1 \times 10^{-6}(K^{-1})$, $\varphi_p = 0.004\%$, $\rho_p = 3.89 \times 10^3$(kg/m^3), thermal conductivity of particle $k_p = 39.5$ (W/m °C), diameter of nanoparticle $(d_p) = 15$ nm.

Take temperature of water $(T_w) = 72.25\,°C$, Temperature of nanofluid $(T_{nf}) = 74.45\,°C$.

Solution

(a) From Table 2.7, all of the thermo-physical properties of water can be obtained as

$$\rho_w = 9.7716 \times 10^3 \,(\text{kg/m}^3), \mu_w = 0.3984 \times 10^{-3} \,(\text{kg/ms}), k_w = 0.661 \,(\text{W/m°C}), C_w$$
$$= 4.190 \times 10^3 \,(\text{J/kg K}), \beta_w = 0.0029 \,\text{K}^{-1}$$

$Nu = C(GrPr)^n = 2.289 \times 10^2$; where, $Gr = \frac{g\beta d^3 \rho^2 \Delta T}{\mu^2}$ and $Pr = \frac{\mu C_p}{k}$

Convective heat-transfer coefficient $(h_{cw}) = Nu \times \left(\frac{k_w}{d}\right) = 100.9 \,(\text{W/m}^2\,°C)$

Table 2.5 Thermo physical properties of vapor [2]

Quantity	Symbol	Expression
Specific heat	C_v	$999.2 + 0.1434 \times (T_v) + 1.101 \times 10^{-4} \times (T_v^2) - 6.7581 \times 10^{-8} \times (T_v^3)$
Density	ρ_v	$353.44/(T_v + 273.15)$
Thermal conductivity	k_v	$0.0244 + 0.7673 \times 10^{-4} \times (T_v)$
Viscosity	μ_v	$1.718 \times 10^{-5} + 4.620 \times 10^{-8} \times (T_v)$
Latent heat of vaporization of fluid	L	$3.1625 \times 10^6 + [1 - (7.616 \times 10^{-4} \times (T_v))]$; for $T_v > 70\,^\circ\mathrm{C}$ $2.4935 \times 10^6 [1 - (9.4779 \times 10^{-4} \times (T_v) + 1.3132 \times 10^{-7} \times (T_v^2) - 4.7974 \times 10^{-3} \times (T_v^3))]$; for $T_v < 70\,^\circ\mathrm{C}$
Partial vapor pressure at condensing cover and fluid temperature	P_{gi}	$P(gi) = \exp\left[25.317 - \left(\dfrac{5144}{T(gi)+273}\right)\right]$ $P(fluid) = \exp\left[25.317 - \left(\dfrac{5144}{T(fluid)+273}\right)\right]$
Thermal expansion coefficient	β_v	$1/(T_v + 273.15)$

Table 2.6 Thermo physical properties of water vapor and air mixture [28]

Quantity	Expression
Specific heat	$C_{pm} = \frac{C_{pa}(P_0 - P_v)M_a + C_{pv}P_vM_w}{M_a(P_0 - P_v) + P_vM_v}$ (kJ/kg K) Correlations of dry air and water vapor specific heat capacity; $C_{pa} = CA_0 + CA_1(T) + CA_2(T^2) + CA_3(T^3) + CA_4(T^4)$ $250\,\text{K} < T < 1050\,\text{K}$ $CA_0 = 1.034, CA_1 = -0.284 \times 10^{-3}, CA_2 = 0.781 \times 10^{-6}$, $CA_3 = -0.497 \times 10^{-9}, CA_4 = 0.1077 \times 10^{-12}$ $C_{pv} = CV_0 + CV_1(T) + CV_2(T^2)$ $0\,°C < T < 120\,°C$ $CV_0 = 1.869, CV_1 = -2.578 \times 10^{-4}, CV_2 = 1.941 \times 10^{-5}$ Specific heat capacity of the saturated mixture (kJ/kg K) is fitted by the following fourth order polynomial; $C_{pm} = CP_0 + CP_1(T) + CP_2(T^2) + CP_3(T^3) + CP_4(T^4)$ $CP_0 = 1.0880, CP_1 = -0.010577, CP_2 = 4.7691 \times 10^{-4}$, $CP_3 = -7.898 \times 10^{-6}, CP_4 = 5.122 \times 10^{-8}$
Density	$\rho_m = \left(\frac{P_0}{\bar{R}T}\right)\left\{M_a\left[1 - RH\left(\frac{P_{sv}}{P_0}\right)\right] + M_v RH\left(\frac{P_{sv}}{P_0}\right)\right\}$ (kg/m^3) $\bar{R} = 8.314\,(kJ/k\,mol°K); RH = \frac{P_v}{P_{sv}}$ $P_{sv} = A_0 + A_1(T) + A_2(T^2) + A_3(T^3) + A_4(T^4)$ $10\,°C < T < 110\,°C$ $A_0 = 1.132, A_1 = -3.75 \times 10^{-2}, A_2 = 5.591 \times 10^{-3}$, $A_3 = -6.6220 \times 10^{-5}, A_4 = 1.105 \times 10^{-6}$ The saturated humid air mixture density (kg/m^3) is fitted by the following third order polynomial function; $\rho_m = RO_0 + RO_1(T) + RO_2(T^2) + RO_3(T^3)$ $RO_0 = 1.299, RO_1 = -6.0436 \times 10^{-3}, RO_2 = 4.697 \times 10^{-5}$, $RO_3 = -5.760 \times 10^{-7}$

(continued)

Table 2.6 (continued)

Quantity	Expression
Thermal conductivity	$k_m = k_a(1 - x_v) + k_v x_v$ (W/m K) Correlations of dry air and water vapor thermal conductivity; $k_a = KA_0 + KA_1(T) + KA_2(T^2) + KA_3(T^3) + KA_4(T^4) + KA_5(T^5)$; (W/mK) 250 K < T < 1050 K $KA_0 = -2.276 \times 10^{-3}, KA_1 = 1.260 \times 10^{-4}, KA_2 = -1.481 \times 10^{-7},$ $KA_3 = 1.735 \times 10^{-10}, KA_4 = -1.0660 \times 10^{-13}, KA_5 = 2.476 \times 10^{-17},$ $k_v = KV_0 + KV_1(T) + KV_2(T^2)$; (W/m K) $\times 10^{-3}$ $0 < T < 120\,°C$ $KV_0 = 17.617; KV_1 = 0.0555, KV_2 = 0.000166$ Thermal conductivity of the saturated mixture (W/m K) is fitted by the following third order polynomial function; $k_m = K_0 + K_1(T) + K_2(T^2) + K_3(T^3)$ $K_0 = 0.0241; K_1 = 5.526 \times 10^{-5}; K_2 = 4.631 \times 10^{-7};$ $K_3 = -9.438 \times 10^{-9}$
Viscosity	$\mu_m = \frac{\mu_a(P_0 - P_v)\sqrt{M_a} + \mu_v P_v \sqrt{M_v}}{(P_0 - P_v)\sqrt{M_a} + P_v \sqrt{M_v}}$ (Ns/m²) Correlations of dry air and water vapor viscosity; $\mu_a = MA_0 + MA_1(T) + MA_2(T^2) + MA_3(T^3) + MA_4(T^4)$ (Ns/m²) $\times 10^{-6}$ $\quad -23\,°C \ll T \ll 327\,°C$ $MA_0 = -0.986, MA_1 = 9.08 \times 10^{-2}, MA_2 = -1.176 \times 10^{-4},$ $MA_3 = 1.123 \times 10^{-7}, MA_4 = -5.797 \times 10^{-11}$ $\mu_v = MV_0 + MV_1(T)$; (Ns/m²) $\times 10^{-6}$ $0\,°C \ll T \ll 120\,°C$ $MV_0 = 80.58, MV_1 = 0.400$ Viscosity of the saturated humid air (Ns/m²) is fitted by the fourth order polynomial function; $\mu_m = MU_0 + MU_1(T) + MU_2(T^2) + MU_3(T^3) + MU_4(T^4)$ $MU_0 = 1.685 \times 10^{-5}, MU_1 = 9.151 \times 10^{-8},$ $MU_2 = -2.16 \times 10^{-9}, MU_3 = 3.413 \times 10^{-11}, MU_3 = -2.645 \times 10^{-13}$

Table 2.6 (continued)

Quantity	Expression
Thermal diffusivity	$\alpha_m = \dfrac{k_m}{\rho_m c_m}$ (m²/s) Thermal diffusivity of saturated air mixture (m²/s) at the temperature range of interest, is fitted by the following fourth order polynomial expression; $\alpha_m = \mathrm{TD}_0 + \mathrm{TD}_1(T) + \mathrm{TD}_2(T^2) + \mathrm{TD}_3(T^3)$ $\mathrm{TD}_0 = 1.88 \times 10^{-5}$, $\mathrm{TD}_1 = 8.0276 \times 10^{-8}$, $\mathrm{TD}_2 = 1.496 \times 10^{-9}$, $\mathrm{TD}_3 = -2.1124 \times 10^{-11}$

a dry air, m mixture, v water vapor, M molar mass of humid air, T temperature, w water

(b) From Appendix E,

$$C_{nf} = \frac{\left[\left(\varphi_p \rho_p C_p + \left(1 - \varphi_p\right)\rho_w C_w\right)\right]}{\rho_w} = 4164.2 \text{ (J/kg K)}; \; \rho_{nf} = \varphi_p \rho_p + \left(1 - \varphi_p\right)\rho_w$$

$$= 1.0954 \times 10^3 \left(\text{kg/m}^3\right); \mu_{nf} = \mu_w\left[1 + 2.5\left(\varphi_p\right) + 6.2\varphi_p^2\right]$$

$$= 0.4327 \times 10^{-3} \text{ (kg/ms)}$$

$$k_{nf} = k_w\left[1 + (1.0112)\varphi_p + (2.4375)\varphi_p\left(\frac{47}{d_p(\text{nm})}\right) - (0.0248)\varphi_p\left(\frac{k_p}{0.613}\right)\right]$$

$$= 0.7544 (\text{W/m}\,^\circ\text{C}); \beta_{nf} = \left(1 - \varphi_p\right)\beta_w + \varphi_p \beta_p = 0.0028 \text{ K}^{-1}.$$

$Nu = C(GrPr)^n = 7.1848 \times 10^2$; where, $Gr = \frac{g\beta d^3 \rho^2 \Delta T}{\mu^2}$ and $Pr = \frac{\mu C_p}{k}$

Convectiveheat transfer coefficient $(h_{cw}) = Nu \times \left(\frac{k_{nf}}{d}\right) = 361.3 \text{ (W/m}^2\,^\circ\text{C)}$

Free convection

In free or natural flow, the homogeneity of mass forces acting on the considered volume of the fluid is responsible for the origin of motion of the volume. If this natural flow has not been distributed within a space by solid boundaries, then it is referred to as "free convection." Consequently, the temperature gradient among the fluid and the contacting surface originates the fluid motion for free convection. This occurs due to the terrestrial gravitational field acting on the fluid with a non-uniform density distribution. Particularly in rotating systems, free convective heat transfer is influenced through centrifugal and Coriolis forces.

Depending on type of the flow (laminar or turbulent, free or forced), the heat-transfer coefficient is usually incorporated with the *Nusselt* number.

For free convection:

$$Nu = C(GrPr)^n K' \tag{2.4.26}$$

where C and n are the constants. The above-mentioned relation is derived from the dimensionless-analysis method. The values of the constants C and n depend on the geometry of similar bodies and can be obtained by correlation of experimental data of the bodies. Free convective heat transfer of various geometries is listed in Table 2.8. The correlation factor K' in the relation is introduced to signify the entire physical behavior of the problem. Moreover, some empirical relations used for free convection are listed in Table 2.9.

Forced convection

In the process of forced convection, a pump or a fan is incorporated for forcing the fluid flow over the surface to create fluid motion artificially. For this purpose, an

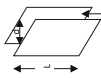**Table 2.7** Thermo physical properties of water (basefluid) [29]
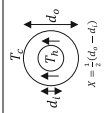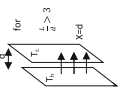
Quantity	Symbol	Expression
Density	ρ_{bf}	$999.79 + 0.0683 \times T_{\text{bf}} - 0.0107 \times T_{\text{bf}}^2 + 0.00082 \times T_{\text{bf}}^{2.5} - 2.303 \times 10^{-5} \times T_{\text{bf}}^3$
Specific heat	C_{bf}	$4.217 - 0.00561 \times T_{\text{bf}} + 0.00129 \times T_{bf}^{1.5} - 0.000115 \times T_{\text{bf}}^2 + 4.149 \times 10^{-6} \times T_{\text{bf}}^{2.5}$
Viscosity	μ_{bf}	$\dfrac{1}{\left(557.82 - 19.408 \times T_{\text{bf}} + 0.136 \times T_{\text{bf}}^2 - 3.116 \times 10^{-4} \times T_{\text{bf}}^3\right)}$
Thermal conductivity	k_{bf}	$0.565 + 0.00263 \times T_{\text{bf}} - 0.000125 \times T_{\text{bf}}^{1.5} - 1.515 \times 10^{-6} \times T_{\text{bf}}^2 - 0.000941 \times T_{\text{bf}}^{0.5}$

Table 2.8 Free convective heat transfer of various systems [2]

System	Schematic	C'	n	k'	Operating conditions
Horizontal cylinder		0.47	0.25	1	Laminar flow condition
		0.1	–	1	Turbulent flow condition
		0.0246	0.4	$[Pr'^{1/6}/(1+0.496Pr^{2/3})]^{2/3}$	Turbulent flow condition; to obtain local Nu use $C' = 0.0296$, $X = x$
Vertical cylinder with small diameter		0.686	0.25	$[Pr/(1+1.05Pr)]^{1/4}$	Laminar flow condition $\overline{Nu_{local}} = \overline{Nu} + 0.52(L/D)$
Heated horizontal plate facing upward		0.54	0.25	1	Laminar flow condition $(10^5 < GrPr < 2 \times 10^7)$ $X = (L_0 + B_0)/2$ Laminar flow condition $(10^7 < GrPr < 10^{11})$, $X = A/P$ for circular disc of diameter D, use $X = 0.9D$
		0.14	0.33	1	Turbulent flow condition $(2 \times 10^7 < GrPr < 3 \times 10^7)$, $X = (L_0 + B_0)/2$
		0.15	0.33	1	Turbulent flow condition $(10^7 < GrPr < 10^{11})$, $X = A/P$
Heated horizontal plate facing downward		0.27	0.25	1	Laminar flow condition only

(continued)

Table 2.8 (continued)

System	Schematic	C'	n	k'	Operating conditions
Moderately inclined plane		0.8	0.25	$\left[\dfrac{\cos\theta}{1+\left(1+\frac{1}{\sqrt{Pr}}\right)^2}\right]^{1/4}$	Laminar flow condition (multiply Gr by cos θ in the formula for vertical plate)
Two vertical parallel plates at the same temperature		0.04	1	$(d/L)^3$	Air layer
Hollow vertical cylinder with open ends		0.01	1	$(d/L)^3$	Air column
Two horizontal parallel plates hot plate uppermost		0.27	0.25	1	Pure conduction $\dot{q}=K(T_h-T_c)/d$ Laminar flow condition (air) $(3\times10^5 <GrPr<3\times10^{10})$
Two concentric cylinders		0.317	0.25	$\left[X^3\left(\dfrac{1}{d_i^{3/5}}+\dfrac{1}{d_o^{3/5}}\right)^5\right]^{-1/4}$	Laminar flow condition
Two vertical parallel plates of different temperatures (h for both surfaces)		0.18	0.25	$\left(\frac{L}{d}\right)^{-1/9}(Pr)^{-1/4}$	Laminar flow condition (air) $(2\times10^4<Gr<2\times10^5)$
		0.065	–	$\left(\frac{L}{d}\right)^{-1/9}(Pr)^{-1/3}$	Turbulent flow condition (air) $(2\times10^5<Gr<2\times10^7)$

(continued)

Table 2.8 (continued)

System	Schematic	C'	n	k'	Operating conditions
Two inclined parallel plates				$\overline{Nu} = \left[\dfrac{\overline{Nu_{vert}}\cos\theta + \overline{Nu_{horz}}\sin\theta}{2}\right]$	
Two horizontal parallel plates cold plate uppermost		0.195	0.25	$Pr^{1/4}$	Laminar flow condition (air) $\left(10^4 < Gr < 4 \times 10^5\right)$
		0.068	–	$Pr^{1/3}$	Turbulent flow condition (air) $Gr > 4 \times 10^5$

Table 2.9 Simplified equations for free convection from various surfaces to air at atmospheric pressure

Cases	Surface	Laminar $10^4 < Gr_f Pr_f < 10^9$	Turbulent $Gr_f Pr_f > 10^9$
1	Horizontal hot plate facing upward	$h = 1.32(\Delta T/L)^{1/4}$	$h = 1.52(\Delta T)^{1/3}$
2	Hot plate facing downward	$h = 0.59(\Delta T/L)^{1/4}$	
3	Vertical plane and cylinder	$h = 1.42(\Delta T/L)^{1/4}$	$h = 1.31(\Delta T)^{1/3}$
4	Horizontal cylinder	$h = 1.32(\Delta T/d)^{1/4}$	$h = 1.24(\Delta T)^{1/3}$

external energy is provided to balance the process, which includes two types of forces viz. (a) the frictional force produced by viscosity $\left(\mu \frac{dv}{dy}\right)$ and (b) the fluid pressure related to flow velocity $\left(\frac{1}{2}\rho\vartheta^2\right)$. The non-dimensional *Reynold* number signifies their relative importance in heat transfer. Furthermore, it directs the flow (laminar or turbulent) in the boundary layer, and the rate of heat transfer is closely connected to it.

Forced convection:

$$Nu = C(RePr)^n K' \tag{2.4.27}$$

Forced convective heat transfer by way of cylindrical tubes can be expressed by the following empirical relation:

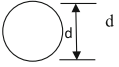$$\overline{Nu} = \frac{hD}{K_{th}} = CRe^m Pr^n K' \tag{2.4.28}$$

where $D = 4\left(\frac{A}{P}\right)$ is the hydraulic diameter (m); A is the area (m^2); P is the perimeter of the section (m); and K_{th} is the thermal conductivity (W/m K). For various conditions, the values of C, m, n and K are given in Table 2.10.

In the case of fully developed laminar flow in the tubes at constant wall temperature gives the following relation:

$$Nu_d = 0.36 + \left[0.0668\left(\frac{d}{L}\right)RePr\right]\left\{1 + 0.04\left[\left(\frac{d}{L}\right)RePr\right]^{\frac{2}{3}}\right\}^{-1} \tag{2.4.29}$$

The obtained heat-transfer coefficient from the above-mentioned relation is an average value over the entire length of the tube. The *Nusselt* number approaches a constant value of 3.66 for sufficient length of the tube. The *Nusselt* number for the heated plate over its entire length can be calculated by integrating the following equation over the plate length:

Table 2.10 The value of constants for forced convection

Cross-section	D	C	m	n	K	Operating conditions
(circle) d	d	3.66	0	0	1	Laminar flow case long tube $Re < 2000$, $Gz < 10$
	d	1.86	0.33	0.33	$(d/l)^{1/3}(\mu/\mu_w)^{0.14}$	Laminar flow case short tube for $Re < 2000$, $Gz > 10$
	d	0.027	0.8	0.33	$(\mu/\mu_w)^{0.14}$	Turbulent flow case of highly viscous liquids for $0.6 < Pr < 100$
	d	0.023	0.8	0.4	1	Turbulent flow case of gases $Re > 2000$

$$Nu_x = 0.332(Pr)^{1/3}(Re_x)^{1/2} \tag{2.4.30}$$

$$\bar{h} = \frac{1}{L}\int_0^L \frac{K}{x}(0.332)(Pr)^{1/3}(Re_x)^{1/2}\mathrm{d}x = \frac{K}{L}(0.332)(Pr)^{1/3}\int_0^L \frac{1}{x}\left(\frac{v_0 x}{\vartheta}\right)\mathrm{d}x$$

$$\frac{K}{L}(0.332)(Pr)^{1/3}(2)\left(\frac{v_0 L}{\vartheta}\right)^{1/2} = \frac{K}{L}(0.664)(Pr)^{1/3}(Re_L)^{1/2}$$

Therefore,

$$\overline{Nu}_L = 0.664(Pr)^{1/3}(Re_L)^{1/2} \tag{2.4.31}$$

In a solar-distillation system, the internal heat-transfer coefficient of convection from the blackened (basin) horizontal surface to the fluid mass, and vice versa, is evaluated from Eqs. (2.4.26) and (2.4.27) for free convection and forced convection, respectively. The numerical values of C and n for different cases are given as follows:

Direction of heat flow	C	n
Upward (hot surface facing upward)	0.54	1/4
Downward (hot surface facing downward)	0.27	1/4

In contrast, the internal heat-transfer coefficient of convection from a fluid surface to the inner surface of a condensing cover can be evaluated from the famous Dunkle's relation:

$$h_{cw} = 0.844 \left[(T_w - T_{gi}) + \frac{(P_w - P_{gi})(T_w + 273)}{(268.9 \times 10^3 - P_w)} \right]^{\frac{1}{3}} \qquad (2.4.32)$$

where

$$P(T) = \exp \left[25.317 - \left(\frac{5144}{273 + T} \right) \right] \qquad (2.4.33)$$

The Dunkle's relation was discussed in detail in Sect. 2.6. The variation of saturation vapor pressure (Eq. 2.4.33) with temperature is shown in Fig. 2.1.

Example 2.3 Calculate (a) the *Grashof* number (Gr), the *Prandlt* number (Pr), and the *Rayleigh* number (Ra) for temperature difference 20 °C and fluid-layer thickness 0.30 m and (b) the *Nusselt* number for free convection for a hot surface facing upward $(C = 0.54, n = 0.25)$. Use $X_0 = 1.5$.

Solution

(a) From the thermo-physical properties of vapor listed in Table 2.5
       Specific heat,

$$C_p = 999.2 + 0.1434 \times (40) + 1.101 \times (40)^2 - 6.7581 \times 10^{-8} \times (40)^3$$
$$= 1005.11 \, J/kg \, °C$$

Density, $\rho = \frac{353.44}{(40 + 273.15)} = 1.128 \, kg/m^3$

Thermal conductivity, $k = 0.0244 + 0.7673 \times 10^{-4} \times (40) = 0.0275 \, W/m \, K$

Viscosity, $\mu = 1.718 \times 10^{-5} + 4.620 \times 10^{-8} \times (40) = 1.9028 \times 10^{-5} kg/m \, s$

Thermal-expansion coefficient, $\beta = \frac{1}{(40 + 273.15)} = 0.00319 \, K^{-1}$

(i) $Gr = \dfrac{g\beta d^3 \rho^2 \Delta T}{\mu^2} = \dfrac{9.8 \times 0.00319 \times (0.30)^3 \times (1.128)^2 \times 20}{(1.9028 \times 10^{-5})^2}$
 $= 0.29 \times 10^7 \cong 3 \times 10^6$

(ii) $Pr = \dfrac{(1.9028 \times 10^{-5}) \times (1005.11)}{0.0275} = 0.695$

(iii) $Ra = (3 \times 10^6) \times 0.695 = 2.095 \times 10^6$

(b) *Nusselt* number: $Nu = C(GrPr)^n = 0.54 \times (2.085 \times 10^6)^{0.25} = 37.9$

Example 2.4 Estimate the rate of heat loss from a horizontal rectangular surface (1.0 m × 0.8 m) maintained at 134 °C and exposed in the upward direction to an environment at 22 °C. Use $\frac{\mu}{\rho} = 2.08 \times 10^{-5} m^2/s, Pr = 0.697$.

Fig. 2.1 Variation of
saturation vapor pressure with
temperature
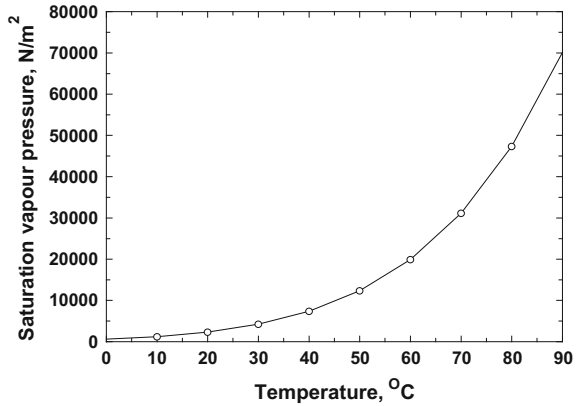
Solution

The average film temperature, $T_f = (134 + 20)/2 = 77\,°C$

$$\beta = \frac{1}{(77 + 273.15)} = \frac{1}{(293.15)}\,K^{-1}$$

For the characteristic dimension of

(a)
$$L = \frac{A}{P} = \frac{0.8}{3.6} = 0.222\,m$$

$$GrPr = \left[\frac{9.8 \times 0.00319 \times (0.222)^3 \times 114}{(293.15) \times (2.08 \times 10^{-5})^2}\right](0.679) = 6.72 \times 10^7$$

For the condition of the hot surface facing upward and the turbulent flow, the heat-transfer coefficient can be evaluated as follows:

$$h = \left(\frac{K}{L}\right)(0.15)(GrLPr)^{0.333} = \left(\frac{0.03}{0.222}\right)(0.15)(6.72 \times 10^7)^{0.333}$$
$$= 8.23\,W/m^2\,°C$$

Hence, the rate of heat loss from the hot plate to the surrounding ambient air is:

$$\dot{Q} = hA(T_1 - T_2) = 8.23(1.0 \times 0.8)(134 - 20) = 751\,W$$

(b) $$X = \frac{(L_0 + B_0)}{2} = 0.9$$

$GrPr = 4.47 \times 10^7$ and $h = \left(\frac{0.03}{0.9}\right)(0.14)(4.47 \times 10^7)^{1/3} = 7.74\,\text{W/m}^2\,{}^\circ\text{C}$

Hence, the rate of heat loss from the hot plate to the surrounding ambient air is:

$$\dot{Q} = hA(T_1 - T_2) = 7.742(1.0 \times 0.8)(134 - 20) = 706\text{W}$$

Example 2.5 Calculate the convective heat-transfer coefficient from the Dunkle's model for a passive single-slope solar-distillation system having a temperature of water surface and inner surface of the condensing cover of 50 and 30 °C, respectively.

Solution

From Eq. (2.4.33),

$$P_w = \exp\left[25.317 - \left(\frac{5144}{273 + 50}\right)\right] = 11983.71$$

$$P_{gi} = \exp\left[25.317 - \left(\frac{5144}{273 + 30}\right)\right] = 4188.5$$

Hence,   from   Eq. (2.4.32),

$$h_{cw} = 0.844\left[(20) + \frac{(11983.71 - 4188.5)(50 + 273)}{(0.269 - 11983.71)}\right]^{\frac{1}{3}} = 2.74\,\text{W/m}^2\,\text{K}$$

2.4.3   Radiation

Thermal radiation comes from a source and propagates through some material or medium. It is the transfer of heat by electromagnetic waves ($0.1\,\text{mum} \le \lambda \le 100\,\mu\text{m}$) from a body at greater temperature to another one at lower temperature. Thermal radiation follows the same rules as that of light, and it lies in the infrared range. The collision of thermal radiation with any object is absorbed by that object and is converted into the heat form.

Radiation involving real surfaces

When a body is exposed to radiant energy, a part of it is reflected; another part of it is absorbed; and the rest is transmitted through that body. According to the law of conservation, the total sum must be equal to the incident radiation as follows:

$$I_T = I_r + I_a + I_t \quad \text{and} \quad \rho' + \alpha' + \tau' = 1$$

where ρ', α', and τ are the reflectivity, absorptivity, and transmissivity of the intercepting body, respectively. For an opaque surface, $\tau = 0$; therefore, $\rho' + \beta' =$

1. However, when $\rho' = \tau' = 0$, $\alpha' = 1$, that is, the substance absorbs the entire energy incident on it, and such a substance is called a "black body." Similarly, $\alpha' = \tau' = 0$ and $\rho' = 1$ for a white body, which reflects the total radiation falling on it.

The absorbed energy is transformed into heat, and this heated body emits radiation by virtue of its temperature. Therefore, the radiant energy emitted per unit area of a surface in unit time is referred to as the "emissive power."

Sky radiation

A certain equivalent black-body sky temperature is established in order to determine the radiation exchange between the sky and a body. It accounts for the fact of non-uniformity of the atmospheric temperature and radiates only in particular wavelength sections. Accordingly, the net radiation to a surface with emittance ε and temperature T is as follows:

$$\dot{Q} = A\epsilon\sigma\left(T_{\text{sky}}^4 - T^4\right) \tag{2.4.34}$$

Various expressions have been given by many researchers and engineers to express the equivalent sky temperature (T_{sky}). These relations are only approximations, although they are simple to use. The relation between the sky temperature and the local air temperature can be given by the following relation:

$$T_{\text{sky}} = 0.0552T_{\text{a}}^{1.5} \tag{2.4.35}$$

where T_{sky} and T_{a} are both in degrees Kelvin. Another commonly used relation is given as [8]

$$T_{\text{sky}} = T_{\text{a}} - 6 \tag{2.4.36}$$

or

$$T_{\text{sky}} = T_{\text{a}} - 12 \tag{2.4.37}$$

Radiative heat-transfer coefficient

The water surface and the surface of the condensing cover of a solar-distillation system are treated as the case of infinite parallel planes for understanding radiation. Justification of the validity of this consideration is due to the larger width and smaller inclination angle of the transparent top glass cover of the system. The rate of radiative heat transfer $(\dot{q}_{\text{rw}})$ among the evaporative and the condensing cover, which are considered infinite parallel planes, is given as follows:

$$\dot{q}_{\text{rw}} = \sigma\epsilon_{\text{eff}}F_{12}\left[(T_{\text{w}} + 273)^4 - (T_{\text{gi}} + 273)^4\right] \tag{2.4.38}$$

or

$$\dot{q}_{rw} = h_{rw} F_{12} \left(T_w + T_{gi} \right) \tag{2.4.39}$$

where F_{12} is view factor, which is considered unity only for a small inclination of the condensing cover. Therefore, the radiative heat-transfer coefficient (h_{rw}) from the evaporative surface to the inner surface of the condensing cover can be expressed as follows:

$$h_{rw} = \sigma \epsilon_{eff} \left[(T_w + 273)^2 + (T_{gi} + 273)^2 \right] (T_w + T_{gis} + 546) \tag{2.4.40}$$

where $\frac{1}{\epsilon_{eff}} = \frac{1}{\epsilon_w} + \frac{1}{\epsilon_g} - 1$

The value of the radiative heat-transfer coefficient (h_{rw}) varies between $7-10 \, \text{W/m} \, {}^\circ\text{C}$, and it is almost independent of temperature for a normal operating range of the temperature $<80 \, {}^\circ\text{C}$.

Example 2.6 Calculate the radiative heat-transfer coefficient between the water surface $(T_w = 50 \, {}^\circ\text{C})$ and the inner surface of a condensing cover $\left(T_{gi} = 30 \, {}^\circ\text{C} \right)$ of a passive solar-distillation system. Use $\varepsilon_w = \varepsilon_g = 0.9$ and $F_{12} = 1$.

Solution

$$\varepsilon_{eff} = \left[\frac{1}{0.9} + \frac{1}{0.9} - 1 \right]^{-1} = 0.82$$

$$h_{rw} = 0.82 \times 5.64 \times 10^{-8} \left[(50 + 273)^2 + (30 + 273)^2 \right] (80 + 546) = 5.66 \, \text{W/m}^2 \, \text{K}$$

Example 2.7 Calculate the rate of long wavelength–radiation exchange (R) between the ambient air and the sky temperature for $T_a = 15 \, {}^\circ\text{C}$.

Solution
By using $\Delta R = \left[(T_a + 273)^4 - (T_{sky} + 273)^4 \right]$

$$T_{sky} = 0.0552(15)^{1.5} = 3.2 \, {}^\circ\text{C}; T_{sky} = 15 - 6 = 9 \, {}^\circ\text{C}; T_{sky} = 15 - 12 = 3 \, {}^\circ\text{C}$$

Now,

$$\Delta R = 5.64 \times 10^{-8} [(15 + 273)^4 - (3.2 + 273)^4] = 59.58 \, \text{W/m}^2$$
$$= 5.64 \times 10^{-8} [(15 + 273)^4 - (9 + 273)^4] = 31.22 \, \text{W/m}^2$$
$$= 5.64 \times 10^{-8} [(15 + 273)^4 - (3 + 273)^4] = 60.52 \, \text{W/m}^2$$

It is essential to mention here that the value of T_{sky} is nearly same for both the cases; thus, the numerical value of ΔR should be considered as $60 \, \text{W/m}^2$.

2.4.4 Evaporation

Internal heat transfer is responsible for transporting the pure water in the form of vapor leaving behind impurities in the basin of the distillation system. Evaporative and convective heat transfer are the most desired types of internal heat transfer in a solar-distillation system. Heat transfer by way of the mode of evaporation takes place from the water surface to the inner surface of the condensing cover, and convective heat transfer occurs simultaneously with the evaporative heat transfer. These two mechanisms of heat transfer are independent of radiative heat transfer. The temperature difference among the evaporative surface and the inner surface of the condensing cover acts as a driving force for a solar-distillation system. The rate of evaporative heat transfer, or productivity (yield) of the system, increases with increasing temperature difference between these two surfaces in the distiller cavity. The transfer of heat per unit area per unit time by the mode of evaporation in a solar-distillation system and is given by the *Cooper* and *Dunkle* model, which is discussed in detail later in the text.

Cooper and Dunkle model The mass of air per unit area per unit time by free convection is given by the following:

$$M_a = \frac{\dot{q}_{cw}}{C_{pa}(T_w - T_{gi})} = \frac{h_{cw}}{C_{pa}} \qquad (2.4.41)$$

It is assumed that the air next to the evaporative surface is saturated at the water temperature. Consequently, the specific humidity or water mass per mass of the dry air can be expressed as follows:

$$\left(\frac{M_w}{M_a}\right)\left(\frac{P_w}{P_T - P_w}\right) \qquad (2.4.42)$$

Thus, the mass of water vapor transferred per unit area per unit time from the water surface is:

$$M_{v,ws} = \left(\frac{M_w}{M_a}\right)\left(\frac{P_w}{P_T - P_w}\right)\left(\frac{h_{cw}}{C_{pa}}\right) \qquad (2.4.43)$$

Similarly, the mass of water vapor transferred per unit area per unit time from the glass surface is given by the following:

$$M_{v,gs} = \left(\frac{M_w}{M_a}\right)\left(\frac{P_{gi}}{P_T - P_{gi}}\right)\left(\frac{h_{cw}}{C_{pa}}\right). \qquad (2.4.44)$$

Baum and Bairamov [9] found that the bulk of the air has an insignificant rate of exchange of water vapor with the interface layers between the water and the glass

surface. The net mass of water vapor transferred per unit area per unit time is given by the difference of expression of Eqs. (2.4.43) and (2.4.44) as follows:

$$\Delta M = \left(\frac{M_w}{M_a}\right)\left(\frac{h_{cw}}{C_{pa}}\right)\left[\frac{P_w}{P_T - P_w} - \frac{P_{gi}}{P_T - P_{gi}}\right] \qquad (2.4.45)$$

The rate at which heat is transferred from the water surface to the glass cover on account of the mass transfer of the water vapor is:

$$\dot{q}_{ew} = L\left(\frac{M_w}{M_a}\right)\left(\frac{h_{cw}}{C_{pa}}\right)\left[\frac{P_T}{(P_T - P_w)(P_T - P_{gi})}\right](P_w - P_{gi}) \qquad (2.4.46)$$

or

$$\dot{q}_{ew} = h_{ew}(P_w - P_{gi}) \qquad (2.4.47)$$

The values of P_w and P_g are significantly low than the values of P_T in a practical situation. Consequently, it is justified to assume that $(P_T - P_w)(P_T - P_{gi})$ is approximately equal to P_T^2. Hence, from Eq. (2.4.46), the evaporative or equivalent mass transfer coefficient (h_{ew}) may then be written in terms of the convective heat-transfer coefficient (h_{cw}) as follows:

$$\frac{h_{ew}}{h_{cw}} = \left(\frac{L}{C_{pa}}\right)\left(\frac{M_w}{M_a}\right)\left(\frac{1}{P_T}\right) \qquad (2.4.47)$$

The above-mentioned equation can also be obtained by using the Lewis' relation (1922, 1933):

$$\frac{h_{cw}}{h_D \rho_a C_{pa}} = 1 \qquad (2.4.48)$$

where h_D is the mass-transfer coefficient (m/h); and ρ is the partial mass density of water vapor (kg/m^3). At a constant pressure of the mixture, the specific heat per unit volume is equal to the ratio of the heat-transfer coefficient to the mass-transfer coefficient.

h_D is defined as the mass-transfer rate $(\dot{m})$ per unit area from the water surface as follows:

$$\frac{\dot{m}}{A} = h_D(P_w - P_a) \qquad (2.4.49)$$

On substituting h_D from Eq. (2.4.48) in the perfect gas equation ($PV = RT'$, for water vapor), one can obtain:

$$\frac{\dot{m}}{A} = \left(\frac{h_{cw}}{\rho_a C_{pa}}\right)\left(\frac{M_w}{RT'}\right)(P_w - P_a) \qquad (2.4.50)$$

The reasonable approximation $T' \approx T'_w \approx T'_a$ is assumed. Therefore, the quantity of evaporative heat transferred on account of the mass transfer of the water vapor per unit area per unit time is as follows:

$$\dot{q}_{ew} = L\left(\frac{\dot{m}}{A}\right) = L\left(\frac{h_{cw}}{\rho_a C_{pa}}\right)\left(\frac{M_w}{RT'}\right)(P_w - P_a) = h_{ew}(P_w - P_a) \qquad (2.4.51)$$

where

$$h_{ew} = L\left(\frac{h_{cw}}{\rho_a C_{pa}}\right)\left(\frac{M_w}{RT'}\right) \quad \text{Or} \quad \frac{h_{ew}}{h_{cw}} = \left(\frac{L}{\rho_a C_{pa}}\right)\left(\frac{M_w}{RT'}\right) \qquad (2.4.52)$$

Using the perfect gas equation for air, one obtains:

$$RT' = \frac{M_a P_a}{\rho_a} \quad (P_a = \text{atmospheric pressure})$$

With above consideration, Eq. (2.4.52) can be expressed as follows:

$$\frac{h_{ew}}{h_{cw}} = \left(\frac{L}{C_{pa}}\right)\left(\frac{M_w}{M_a}\right)\left(\frac{1}{P_T}\right) \qquad (2.4.53)$$

Since $P_T = P_a$ for small P_w as discussed previously, Eq. (2.4.53) is similar to Eq. (2.4.47). On substituting the suitable values of different parameter, the evaporative heat-transfer coefficient can be obtained as follows:

$$h_{ew} = 0.013(h_{cw}) \qquad (2.4.54)$$

The value of $\left(\frac{h_{ew}}{h_{cw}}\right)$, obtained in the above-mentioned equation, is smaller than the value obtained by Bowen [10] and Dunkle [11]. This is due to the approximation made previously to neglect the terms P_w and P_{gi} on expanding the factor $(P_T - P_w)(P_T - P_{gi})$. It has been studied that the most significant depiction of the evaporative or mass heat-transfer phenomena is obtained for the value 16.273×10^{-3} of $\left(\frac{h_{ew}}{h_{cw}}\right)$. Thus; the evaporative heat transferred per unit area per unit time can be given by:

$$\dot{q}_{ew} = 16.273 \times 10^{-3}(h_{cw})(P_w - P_{gi}) \qquad (2.4.55)$$

Or the evaporative heat-transfer coefficient can be expressed as follows:

$$h_{ew} = 16.273 \times 10^{-3}(h_{cw})\left(\frac{P_w - P_{gi}}{T_w - T_{gi}}\right) \qquad (2.4.56)$$

The generalized expression of pressure, which is a function of temperature, is given as follows:

$$P(T) = \exp\left[25.317 - \left(\frac{5144}{273 + T}\right)\right] \qquad (2.4.57)$$

Equation (2.4.55) can also be re-expressed as follows:

$$\dot{q}_{ew} = h_{ew}(T_w - T_{gi}) \qquad (2.4.58)$$

The rate of mass transfer $(\dot{m}_{ew})$ is given by:

$$\dot{m}_{ew} = \frac{\dot{q}_{ew}}{L} \qquad (2.4.59)$$

Example 2.8 Evaluate the evaporative heat-transfer coefficient for Example 2.5.

Solution

From example 2.5, $h_{cw} = 2.74$ W/m^2 K

   Thus, $h_{ew} = 16.273 \times 10^{-3}(2.74)\left(\frac{7795.21}{20}\right) = 17.378$ W/m^2 K

Total internal heat transfer

As discussed in Chap. 1, internal heat transfer from the evaporative surface to the inner surface of the condensing cover takes place by three modes viz. evaporation, convection, and radiation. The rate of total internal heat transfer $(\dot{q}_{1w})$ is the combination of these three modes of heat transfer. The rate of total internal heat transfer can be expressed as follows:

$$\dot{q}_{1w} = \dot{q}_{cw} + \dot{q}_{rw} + \dot{q}_{ew} \quad (W/m^2) \qquad (2.4.60)$$

and the total internal heat-transfer coefficient is:

$$h_{1w} = h_{cw} + h_{rw} + h_{ew} \quad (W/m^2\,^{\circ}C) \qquad (2.4.61)$$

where the convective (h_{cw}), radiative (h_{rw}), and evaporative (h_{ew}) heat-transfer coefficients are given in Eqs. (2.4.32), (2.4.40), and (2.4.56) respectively.

   The energy fraction of each mode gives a superior understanding of all three modes of internal heat transfers. Heat transfer by way of each mode is expressed as a fraction of the total heat transfer in order to understand the relative significance or

contribution of the magnitudes of each mode of heat transfer [2, 7]. Thus, the evaporative, convective, and radiative energy fractions can be obtained as follows:

$$F_c = \frac{\dot{Q}_{cw}}{Q_1}; F_e = \frac{\dot{Q}_{ew}}{Q_1} \text{ and } F_r = \frac{\dot{Q}_{rw}}{Q_1} \qquad (2.4.62)$$

where

$$\dot{Q}_1 = \dot{Q}_{cw} + \dot{Q}_{rw} + \dot{Q}_{ew}(W) \qquad (2.4.63)$$

Figure 2.2 shows the variation of these three energy fractions with respect to water temperature for indoor simulation as determined by Cooper [12, 13].

Total external heat transfer

(a) **Top-loss coefficient**

A fraction of the incident solar radiation is absorbed by the small thickness of the condensing cover of the solar-distillation system. Consequently, the heat transfer from the outer surface of the top condensing cover to the atmosphere takes place by way of convection and radiation. The total rate of external heat transfer from the top surface can be expressed as follows:

$$\dot{q}_g = \dot{q}_{rg} + \dot{q}_{cg} \qquad (2.4.64)$$

Fig. 2.2 Variation of evaporative, convective and radiative energy fraction with respect to the water temperature as found by the Cooper (1973) (2.1/14/SDB)
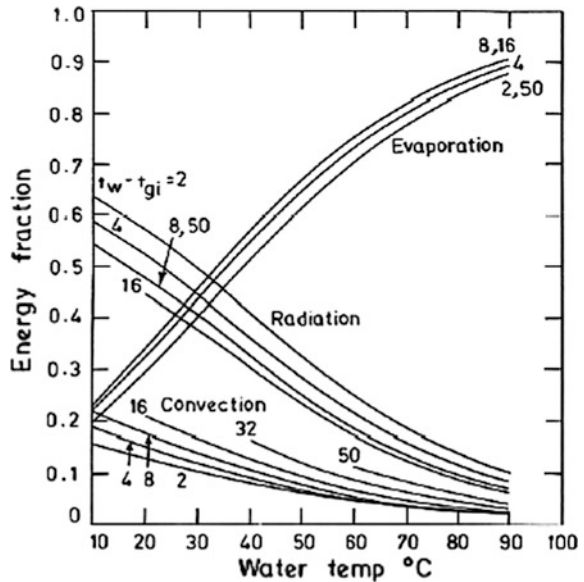
The rate of radiative heat transfer from the condensing surface to the atmosphere is given as follows:

$$\dot{q}_{rg} = \sigma\epsilon_g\left[\left(T_g + 273\right)^4 - \left(T_{sky} + 273\right)^4\right] \tag{2.4.65}$$

Or

$$\dot{q}_{rg} = h_{rg}\left(T_g - T_a\right) \tag{2.4.66}$$

where

$$h_{rg} = \frac{\sigma\epsilon_g\left[\left(T_g + 273\right)^4 - \left(T_{sky} + 273\right)^4\right]}{\left(T_g - T_a\right)} \tag{2.4.67}$$

The rate of convective heat transfer from the condensing surface to the ambient air is expressed as:

$$\dot{q}_{cg} = h_{cg}\left(T_g - T_a\right) \tag{2.4.68}$$

On substituting $\left(\dot{q}_{rg}\right)$ and $\left(\dot{q}_{cg}\right)$ in Eq. (2.4.64), one can obtain the following:

$$\dot{q}_g = h_{1g}\left(T_g - T_a\right) \tag{2.4.69}$$

where

$$h_{1g} = h_{rg} + h_{cg} \tag{2.4.70}$$

The empirical relation for the top-loss coefficient $\left(h_{1g}\right)$ is discussed later in the text. Several workers analyzed the heat transfer from a flat plate exposed to outside winds. The following equation for the convective heat-transfer coefficient was reported by McAdams [30]:

$$h_c = 5.7 + 3.8\,V \quad \text{for} \quad 0 \leq V \leq 5\,\text{m/s} \tag{2.4.71}$$

where V is the wind speed (m/s).

For zero wind speed, the above-mentioned Eq. 2.4.71 gives heat loss by natural convection. Here, the case is very complex because wind may not always flow parallel to the heating/cooling surface.

Watmuff et al. [31] modified Eq. 2.4.71 by including the effect of free convection and radiation and reported that this equation should be as follows:

$$h_c = 2.8 + 3V \quad \text{for} \quad 0 \leq V \leq 7\,\text{m/s} \tag{2.4.72}$$

(b) Bottom- and side-loss coefficient

Heat loss from the basin water to the atmosphere also takes place through the insulating body of the distillation system such that it occurs by way of convection and radiation from the bottom and side surfaces of the system. The bottom-loss coefficient (U_b) can be expressed as follows:

$$U_b = \left[\frac{1}{h_w} + \frac{1}{h_b}\right]^{-1} = \left[\frac{1}{h_w} + \frac{L_i}{K_i} + \frac{1}{(h_{cb} + h_{rb})}\right]^{-1} \quad (2.4.73)$$

where K_i and L_i are the thermal conductivity and thickness of insulation respectively.

The side heat–loss coefficient (U_e) can be estimated as

$$U_e = U_b \left(\frac{A_{ss}}{A_s}\right) \quad (2.4.74)$$

where, A_s is the area of the basin of the solar-distillation system and A_{ss} is the side area in contact with water. In above equation, U_e can be neglected for small water depth ($A_{ss} \ll A_s$).

The rate of heat loss (per m^2) from the basin liner to the ambient air can be expressed as:

$$\dot{q}_b = h_b(T_b - T_a) \quad (2.4.75)$$

where

$$h_b = \left[\frac{L_i}{K_i} + \frac{1}{(h_{cb} + h_{rb})}\right]^{-1} \quad (2.4.76)$$

Example 2.9 Estimate the external radiative heat-transfer coefficient of a single-slope solar distiller unit for the condensing cover and the ambient temperature of 45 and 30 °C, respectively.

Solution
Here, $T_{sky} = T_a - 6 = 24\,°C$

The radiative heat-transfer coefficient from the condensing cover to the ambient air is:

$$h_{rg} = \frac{\sigma \epsilon_g \left[(T_g + 273)^4 - (T_{sky} + 273)^4\right]}{(T_g - T_a)}$$

$$= \frac{(5.64 \times 10^{-8}) \times 0.95 \times \left[(45 + 273)^4 - (24 + 273)^4\right]}{15} = 27.43\ \text{W/m}^2\,\text{K}$$

Example 2.10 Calculate the radiative heat-transfer coefficient from the condensing cover at 32 °C an ambient temperature at 20 °C as well s the convective heat-transfer coefficient when the wind is blowing at a velocity of 3 m/s. Compare the result with the combined radiative and convective heat-transfer coefficient.

Solution

Here, $T_{sky} = T_a - 6 = 24\,°C$

The radiative heat-transfer coefficient from the condensing cover to the ambient air is:

$$h_{rg} = \frac{\sigma \epsilon_g \left[(T_g + 273)^4 - (T_{sky} + 273)^4 \right]}{(T_g - T_a)}$$

$$= \frac{(5.64 \times 10^{-8}) \times 0.95 \times \left[(32 + 273)^4 - (14 + 273)^4 \right]}{12} = 7.95\,\text{W/m}^2\,\text{K}$$

$$h_{cg} = 2.8 + 3\,V = 11.8\,\text{W/m}^2\,\text{K}$$

$$h_{cg} = 2.8 + 3\,V = 11.8\,\text{W/m}^2\,\text{K}$$

The total heat-transfer coefficient from the glass to the ambient air is $h_{1g} = 19.75\,\text{W/m}^2\,\text{K}$. The combined radiative and convective heat-transfer coefficient from Eq. (2.4.71) is

$$h_{cg} = 5.7 + 3.8\,V = 17.1\,\text{W/m}^2\,\text{K}.$$

2.5   Overall Heat Transfer

The overall heat-transfer coefficient from different components of the solar still is given as follows.

(a) **Top-loss coefficient**

The top-loss coefficient (U_t) from the water surface to the ambient air (atmosphere) can be written as follows:

$$U_t = \left[\frac{1}{h_{1g}} + \frac{1}{h_{1w}} \right]^{-1} \tag{2.5.77}$$

Therefore, rate of heat loss in an upward direction of distillation system is as follows:

$$\dot{q}_t = U_t (T_w - T_a) \tag{2.5.78}$$

(b) **Bottom-loss coefficient**

The rate of het loss from the water to the ambient air through the bottom of the insulation is as follows:

$$\dot{q}_b = U_b(T_w - T_a) \tag{2.5.79}$$

$$U_b = \left[\frac{L_i}{K_i} + \frac{1}{h_w} + \frac{1}{(h_{cb} + h_{rb})}\right]^{-1}$$

where U_b is the bottom heat–loss coefficient.

The rate of total heat loss per m^2 from the water surface to the ambient air through the bottom and the top of the system can be determined by adding Eqs. (2.5.78) and (2.5.79), and it can be expressed as follows:

$$\dot{q}_{loss} = U_L(T_w - T_a) \tag{2.5.80}$$

where $U_L = U_t + U_b$ is the overall heat transfer.

Example 2.11 Calculate the top-loss coefficient from a water surface at 20 °C to the ambient air at 8.5 °C. Given the temperature of glass, 12 °C, $\varepsilon_g = \varepsilon_w = 0.9$.

Solution

Here, $\epsilon_{eff} = \left[\frac{1}{0.9} + \frac{1}{0.9} - 1\right]^{-1} = 0.82$

The radiative, convective, and evaporative heat-transfer coefficients from the water to the glass are given as:

$$h_{rw} = 0.82 \times \left(5.67 \times 10^{-8}\right)\left[(20 + 273)^2 + (12 + 273)^2\right](20 + 12 + 546)$$
$$h_{rw} = 4.49 \, \text{W/m}^2\,°\text{C}$$

From Eq. (2.4.33),

$$P_w = \exp\left[25.317 - \left(\frac{5144}{273 + 12}\right)\right] = 2367.69 \, \text{N/m}^2$$
$$P_g = \exp\left[25.317 - \left(\frac{5144}{273 + 12}\right)\right] = 1447.18 \, \text{N/m}^2$$
$$h_{cw} = 0.844\left[(8) + \frac{(2367.69 - 1447.18)(20 + 273)}{(2.689 \times 10^3 - 2367.69)}\right]^{\frac{1}{3}} = 1.84 \, \text{W/m}^2\,°\text{C}$$
$$h_{ew} = (0.016273 \times 1.84)\left(\frac{920.51}{8}\right) = 3.44 \, \text{W/m}^2\,°\text{C}$$

The total internal heat-transfer coefficient from the water surface to the internal surface of the condensing cover is as follows:

$$h_{1w} = 3.445 + 1.84 + 4.49 = 9.775 \text{ W/m}^2 \,^\circ\text{C}$$

$$h_{cg} = 5.7 + 3.8V = 17.1 \text{ W/m}^2 \,^\circ\text{C}$$

The top-loss coefficient from the water surface to the ambient air can be obtained as:

$$U_t = \left[\frac{1}{9.775} + \frac{1}{17.1} \right]^{-1} = 6.25 \text{ W/m}^2 \,^\circ\text{C}$$

Example 2.12 Calculate the bottom-loss coefficient from a solar still with 5 mm–thick insulation ($K = 0.04 \text{ W/m}^2 \,^\circ\text{C}$) Given that $h_w = 100 \text{ W/m}^2 \,^\circ\text{C}$.

Solution
Here, $h_{cb} + h_{rb} = 5.7 \text{ W/m}^2 \,^\circ\text{C}$
The bottom-loss coefficient is given as follows:

$$U_b = \left[\frac{1}{100} + \frac{0.005}{0.04} + \frac{1}{5.7} \right]^{-1} = 3.23 \text{ W/m}^2 \,^\circ\text{C}$$

2.6   Dunkle's Relation

The heat within the space between the water surface and the condensing cover is transported by of free convection across the bulk of humid air inside the solar-distillation system. During the mechanism of free convection, the buoyancy force arises due to the density variation in the space, which triggers the fluid motion. Furthermore, the density variation exists whenever there is a temperature gradient in the fluid. In contrast, the forced convection arises due to externally applied pressure gradient.

This humid air (conveyed the heat) release its enthalpy at the surface of glass cover, which is a comparatively at lower temperature. In general, the *Nusselt* number depends on the heat-transfer coefficients in the case of forced convection, whereas it is a function of the *Grashof* and *Prandtl* numbers in the case of natural convection,

$$Nu = f(GrPr) \tag{2.6.81}$$

By correlating the experimental data of Mull and Reiher for the flow of heat against the forces of gravity such that there is an upward flow of heat from the water surface, Jakob [14, 15] proposed the following relationship:

$$Nu = C(GrPr)^n \tag{2.6.82}$$

Using Eq. (2.4.19), Eq. 2.6.82 can be extended as given here:

$$Nu = \frac{h_{cw} \times d}{k} = C(GrPr)^n \tag{2.6.83}$$

where C and n are the constants. It becomes necessary to use a special *Grashof* number (Gr') for the case of convective heat transfer by humid air in the presence of the mass transfer of a fluid of lower molecular weight (i.e., water vapor). For non-isothermal evaporation, it has been shown by Sharpley and Boelter [16] as follows:

$$Gr' = \left(\frac{x_1^3 \rho_f^2 g}{\mu_f^2}\right)\left[\frac{M_\infty T_o'}{M_o T_\infty'} - 1\right] \tag{2.6.84}$$

where the subscript o refers to the condition at the surface at which the evaporation originates, and ∞ refers to a point far away from this surface. The subscript f refers to air saturated with water vapor. For $M_0 = M_\infty$, the term in brackets reduces to:

$$Gr' = \beta'\Delta T$$

$$\text{where } \beta' = \frac{1}{T_\infty'}$$

Let subscripts w and a refer to the diffusing (water vapor) and inert (air) gases, respectively. Assuming these to be ideal gases, which is a necessary condition for the modified Eq. (2.6.84), and remembering that:

$$P_{w\infty} + P_{a\infty} = P_{wo} + P_{ao} = P_T \text{ (Total gas pressure)},$$

the term in the square bracket in Eq. (2.6.84) can be expressed as follows:

$$
\begin{aligned}
\left[\frac{M_\infty T_o'}{M_o T_\infty'} - 1\right] &= \left[\frac{M_\infty P_{w\infty} T_o' + M_a P_{a\infty} T_o'}{M_w P_{wo} T_\infty' + M_a P_{ao} T_\infty'} - 1\right] \\
&= \frac{M_\infty\left(P_{w\infty} T_o' - P_{wo} T_\infty'\right) + M_a\left(P_{aw} T_o' - P_{ao} T_\infty'\right)}{T_\infty'\left(M_w P_{wo} + M_a P_{ao}\right)} \\
&= \frac{[M_\infty P_{w\infty} + M_a(P_T - P_{wo})]\left(T_o' - T_\infty'\right) + (M_w - M_a)(P_{a\infty} - P_{wo})T_o'}{T_\infty'[M_w P_{wo} + M_a(P_T - P_{wo})]}
\end{aligned}
$$

or

$$\left[\frac{M_\infty T_o'}{M_o T_\infty'} - 1\right] = \frac{T_o' - T_\infty'}{T_\infty'} + \frac{(M_w - M_a)(P_{a\infty} - P_{wo})T_o'}{T_\infty'[M_w P_{wo} + M_a(P_T - P_{wo})]} \tag{2.6.85}$$

For an air/water-vapor system at normal atmospheric pressure,

$$M_w = 18, M_w = 28.96, P_T = 98.07 \times 10^3 \text{ pa}$$

and with the above-mentioned substitution in Eq. (2.6.85) gives:

$$\left[\frac{M_\infty T'_o}{M_o T'_\infty} - 1 \right] = \frac{T'_o - T'_\infty}{T'_\infty} + \frac{(P_{a\infty} - P_{wo})T'_o}{T'_\infty [268.91 \times 10^3 - P_{wo}]}$$
$$= \beta' \left[\Delta T + \frac{(P_{a\infty} - P_{ao})T'_o}{[268.91 \times 10^3 - P_{wo}]} \right]$$

or

$$\left[\frac{M_\infty T'_o}{M_o T'_\infty} - 1 \right] = \beta' \Delta T' \tag{2.6.86}$$

where $\Delta T'$ is the equivalent temperature difference. Therefore,

$$Gr' = \frac{x_1^3 \rho_f^2 g \beta' \Delta T'}{\mu_f^2} \tag{2.6.87}$$

The above-mentioned relationship is used by Dunkle [11]. For an equivalent temperature difference and a mean air temperature of 50 and 17 °C, respectively, assuming saturated air, the special *Grashof number* (Gr') in Eq. (2.6.87) can be expressed as $(2.81 \times 10^7)x_1^3$. The special *Grashof number* (Gr') may be seen to lie in the range for which $C = 0.075$ and $n = 1/3$ over the normal operating temperature range for values (not too small) of the mean water-to-cover spacing (x_1).

The length parameter conveniently cancels out for this value of the exponent. Consequently, the heat-transfer coefficient becomes substantially independent of the spacing (x_1). Dunkle [11] derived the following relationship by choosing the physical-parameter values occurring in the dimensionless variables that are relevant over the normal range of operation of the solar-distillation system,

$$\dot{q}_{cw} = 0.844 \left[(T_w - T_{gi}) + \frac{(P_w - P_{gi})(T_w + 273)}{(2.69 \times 10^3 - P_w)} \right]^{\frac{1}{3}} (T_w - T_{gi}) \tag{2.6.88}$$

$$= h_{cw}(T_w - T_{gi}) \tag{2.6.89}$$

where,

$$h_{cw} = 0.844 \left[(T_w - T_{gi}) + \frac{(P_w - P_{gi})(T_w + 273)}{(2.69 \times 10^3 - P_w)} \right]^{\frac{1}{3}} \tag{2.6.90}$$

2.7 New Model

The internal convective and evaporative heat transfers occur simultaneously in a solar-distillation system. The expressions derived for the convective heat-transfer coefficient (h_{cw}) (Eq. 2.6.90) and the evaporative heat-transfer coefficient (h_{ew}) (Eq. 2.4.56) have many limitations, and this model with limitations is known as the "*Dunkle* model [11] with constants" as $C = 0.075$ and $n = 1/3$. Most of the researchers in this field generally use these relationships, even in non-suitable conditions under the model limitations. Therefore, it is believed essential to inspect the validity of the *Dunkle* model at least for those conditions not falling within its limitations.

In this aspect, *Clark* [17], amongst others, made attempts by conducting experiments (steady-state indoor) on the basin-type solar-distillation system using a solar simulator. He pointed out that the *Dunkle* model of heat transfer for such a solar distiller is effective. However, an additional factor of "1/2" (to account for saturation) must be introduced in the expression of the evaporative heat-transfer coefficient (h_{ew}). The purpose for introducing this factor is because the rates of evaporation and condensation are equal under an ideal condition. This state is attained by cooling the condensing cover using a fan. It is impracticable for solar-distillation systems operating in actual field conditions. The following values of **C** and **n** were proposed for different *Grashof number* ranges on the basis of steady-state experimental results:

(i) $$C = 0.21, n = 1/4 \text{ for } 10^4 < Gr < 10^5$$

(ii) $$C = 0.1255, n = 1/3 \text{ for } 2.51 \times 10^5 < Gr < 10^7$$

To develop an empirical relation, Shawaqfeh and Farid [18] performed simulation studies based on the best fit. The obtained correlations delivered expressions for the convective heat-transfer coefficients, which are independent of the solar-distiller geometry. *Dunkle* suggested a value of $n = 1/3$ and a reduced value of $C = 0.067$ to account for the turbulent flow inside a solar-distillation system. All of these correlations are developed for indoor simulations and steady-state conditions.

Kumar and Tiwari [19, 20] suggested the values of $C = 0.0322, n = 0.4144$ for $1.794 \times 10^6 < Gr < 5.724 \times 10^6$ using regression analysis by performing experiments on a single-slope solar-distillation system under actual field conditions.

In the literature, it is reported that the same values of C and n are used for the analysis of a double-slope solar-distillation system as well. However, the suggested values of C and n for a single-slope solar-distillation system are not valid for a double-slope solar still due to the following reasons:

(i) There are two condensing surfaces for one evaporating surface in double-slope solar still.

(ii)   The temperatures of both of the condensing surfaces are different at any given time interval.

(iii)   The solar intensity falling on both condensing surfaces is also different at any given time interval.

Taking these facts into account, Agrawal [21] developed a thermal model and experimentally validated it for evaluating the values of C and n with the help of regression analysis for the following:

(a)   A conventional single-slope solar-distillation system,
(b)   A conventional double-slope solar-distillation system,
(c)   A double condensing-chamber solar-distillation system, and
(d)   An active single-slope solar-distillation system.

Agrawal and Tiwari [22] and Kumar and Tiwari [19, 20] used the following methodology for evaluating C and n:

On substituting h_{cw} from Eq. (2.6.83) into Eq. (2.4.56), one can obtain the following:

$$h_{ew} = \left(16.273 \times 10^{-3}\right)\left(\frac{P_w - P_{gi}}{T_w - T_{gi}}\right)\left(\frac{k}{d}\right)[C(GrPr)^n] \qquad (2.7.91)$$

Substitute the above Eq. in (2.4.55), one can obtain:

$$\dot{q}_{ew} = \left(16.273 \times 10^{-3}\right)\left(P_w - P_{gi}\right)\left(\frac{k}{d}\right)[C(GrPr)^n]$$

Finally, Eq. (2.4.59) can be expressed as follows:

$$\dot{m}_{ew} = \left(16.273 \times 10^{-3}\right)\left(P_w - P_{gi}\right)\left(\frac{k}{d}\right)\left(\frac{3600}{L}\right)[C(Ra)^n] \qquad (2.7.92)$$

where

$$Ra = GrPr. \qquad (2.7.93)$$

The above-mentioned equation can be rewritten as follows:

$$\dot{m}_{ew} = RC(Ra)^n \quad \text{or} \quad \frac{\dot{m}_{ew}}{R} = C(Ra)^n \qquad (2.7.94)$$

where

$$R = \left(16.273 \times 10^{-3}\right)\left(P_w - P_{gi}\right)\left(\frac{k}{d}\right)\left(\frac{3600}{L}\right) \qquad (2.7.95)$$

Equation (2.7.94) can be expressed in the following form:

$$y = ax^b \tag{2.7.96}$$

where $y = \frac{\dot{m}_{ew}}{R}$; $x = Ra$; $a = C$, and $b = n$

Equation (2.7.96) can be reduced into a straight-line equation by taking log on both sides:

$$\ln(y) = \ln(a) + b\ln(x) \tag{2.7.97}$$

or

$$Y = A + BX \tag{2.7.98}$$

where

$$Y = \ln(y); A = \ln(a); B = b, \text{ and } X = \ln(x) \tag{2.7.99}$$

From Eq. (2.7.99), the values of coefficients A and B are computed using linear-regression analysis. The expression for A and B are given by:

$$B = \frac{N(\sum XY) - (\sum X)(\sum Y)}{N(\sum X^2) - (\sum X)^2} \tag{2.7.100}$$

$$A = \frac{\sum Y}{N} - B\frac{\sum X}{N} \tag{2.7.101}$$

where N is the number of experimental observations. After calculating the values of coefficients A and B from Eqs. (2.7.100) and (2.7.101), respectively, one can obtain C and n by the following expressions:

$$C = \exp(A) \text{ and } n = B \tag{2.7.102}$$

They considered the following facts for the proposed and tested the values of C and n, which have not been considered explicitly in previous models:

  (i) the effect of the solar-still cavity,
 (ii) the operating temperature range, and
(iii) the orientations of the condensing covers.

By using new values of C and n within an average accuracy of 19% (except for some extreme cases), a reasonable agreement is observed between the experimental and theoretical values.

They performed an extensive year-round experiment for a year under actual field conditions for all four above-mentioned solar-distillation systems. To evaluate

C and n, the average values of temperatures and the hourly distillate output were used. During experimentation, the hourly observations of distillate outputs and various temperatures were obtained and classified in the following three operating temperature ranges:

Case I	24−55 °C	(Dunkle's operating temperature range)
Case II	24−42 °C	(Lower operating temperature range)
Case III	55−75 °C	(Greater operating temperature range)

The temperature ranges of cases I and II (for a conventional solar still) and case I (for a double condensing–chamber solar-distillation system) are close to the winter conditions for New Delhi. The values of are significantly different for case III because both the stills belong to monthly summer conditions ($C = 0.1265$ and $n = 0.284$ for the months of August and September) than Dunkle's ($C = 0.075$ and $n = 0.333$) cases. Hence, this verifies that without confirming the validity of the values of C and n for different designs of solar-distillation systems under different operating temperature ranges, the values proposed by *Dunkle* cannot be used. The investigators observed that under similar climatic conditions:

- $C = 0.8100$ and $n = 0.3572$ for a double-slope solar-distillation system (east side).
- $C = 0.0845$ and $n = 0.3432$ for a double-slope solar-distillation system (west side).
- $C = 0.0278$ and $n = 0.3513$ for an active single-slope solar-distillation system.

These are different from of the values for a conventional single-slope solar-distillation system. They studied all of the three cases of temperature ranges regarding the double-condensing solar-distillation system. They perceived that their suggested thermal model to evaluate C and n is in close agreement with experimental results within an accuracy of 20%. However, the Dunkle's model has more deviation— $\leq 48\%$—as shown in Figs. 2.3 and 2.4, respectively, for the east and west condensing surfaces of the system. However, the deviation from Dunkle's model goes up to 68% in case of an active single-slope solar-distillation system.

Now, from the above-mentioned thermal modeling, experimental results, and analysis, one can conclude that it is essential to evaluate C and n values in order to precisely predict the performance of solar stills. Under simulated conditions, the calculated values of C and n are no longer valid for stills operating under actual field conditions. This is due to the fact that the variations of climatic parameters are under human control in the laboratory conditions, whereas the same is not true for actual operation condition of the solar-distillation system at the user's end. One can conclude that Dunkle's C and n values can be used only for an operating temperature range of 50 (or 55 °C at the maximum) for a conventional single-slope, double-condensing chamber and an active single-slope solar-distillation system.

These values of Dunkle for other types of solar-distillation systems and temperature ranges cannot be used.

The analysis of Agrawal [22] strongly supports that it is essential for the estimation of convective mass transfer to predict the performances of each type of solar-distillation system under actual field conditions. Different values of operating temperature range and shape of the condensing cover gives different values of *C* and *n* . Therefore, incorrect results will be obtained on using the values of *C* and *n* estimated under simulation conditions. Hence, it is recommended that experiments be performed on a specific model of solar-distillation system for given climatic conditions to estimate the values of *C* and *n* from the thermal model before predicting the performance theoretically. Thus, the obtained values of *C* and *n* will have no limitations. Consequently, the convective heat-transfer (h_{cw}) and radiative

Fig. 2.3 Comparison of experimental outputs with theoretical outputs of other models for east condensing surface
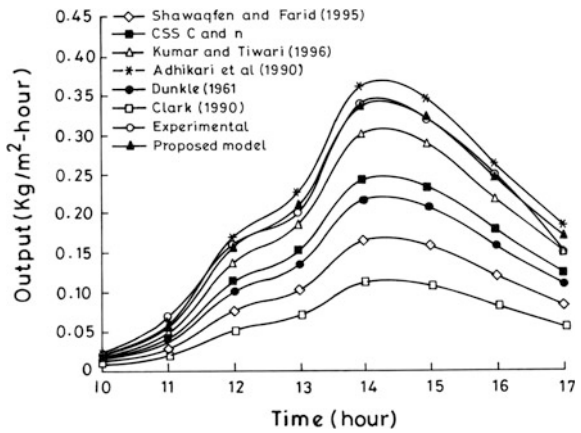

Fig. 2.4 Comparison of experimental outputs with theoretical outputs of other models for west condensing surface
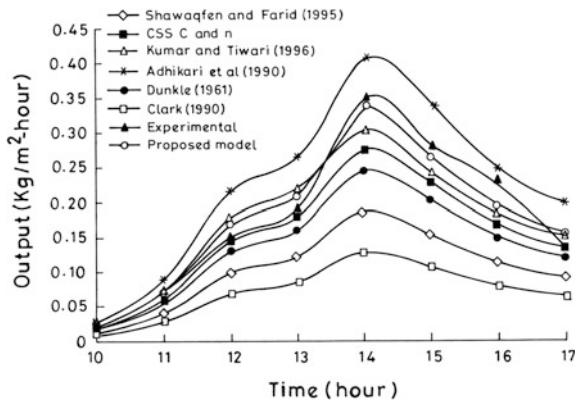
Table 2.11 Values of C, n and heat transfer coefficients (h_{cw} and h_{ew}) for different inclinations of the condensing cover

Values obtained	15°	30°	35°
Value of C	1.418	2.536	0.968
Value of n	0.148	0.158	0.209
Average, h_{cw} (W/m² °C)	1.67	2.44	2.01
Average, h_{ew} (W/m² °C)	13.36	16.93	12.84

heat-transfer coefficients (h_{ew}) can be obtained by evaluating C and n experimentally from Eq. (2.7.102) without any limitations by using the following expressions:

$$h_{cw} = \left(\frac{k}{d}\right)[C(GrPr)^n].\tag{2.7.103}$$

$$h_{ew} = \frac{\dot{q}_{ew}}{3600 \times (T_w - T_{gi})}\tag{2.7.104}$$

The values of C and n for a single-slope solar-distillation system for different inclinations of condensing cover for New Delhi summer climatic conditions are listed in Table 2.11. This table clearly indicates the substantial change in the values of C and n for different inclinations of the condensing cover.

Evaporative heat-transfer (h_{ew}) and convective heat-transfer coefficients (h_{cw}) have been evaluated from Eqs. (2.7.104) and (2.7.103), respectively. These coefficients are evaluated for the values of C and n for a 15 °C inclination of condensing cover as well as for positive values of temperature difference between the water surface and the inner surface of the condensing cover. The results are shown in Fig. 2.5a, b. The values of convective and evaporative heat-transfer coefficients obtained by Dunkle's model are also shown. Significant difference in the results has been perceived from the present model as well as Dunkle's model.

Example 2.13 Evaluate the evaporative heat-transfer coefficient using the new model for a single-slope solar-distillation system having heat flow in an upward direction for example 2.3.

Solution

From Example 2.3; $Gr = 0.29 \times 10^7 \cong 3 \times 10^6$ and $Pr = 0.695$

Equation (2.7.91) gives:

$$h_{ew} = (16.273 \times 10^{-3})\left(\frac{7795.21}{20}\right)\left(\frac{0.0275}{0.30}\right)\left[0.54(2.085 \times 10^6)^{0.25}\right]$$
$$= 11.92 \text{ W/m}^2 \text{ K}$$

Fig. 2.5 a Variation of
convective heat transfer
coefficient (h_{cw}) evaluated by
the present and Dunkle's
models for positive values of
ΔT and 15° cover inclination.
b Variation of evaporative
heat transfer coefficient (h_{ew})
evaluated by present and
Dunkle's models for positive
values of ΔT and 15° cover
inclination
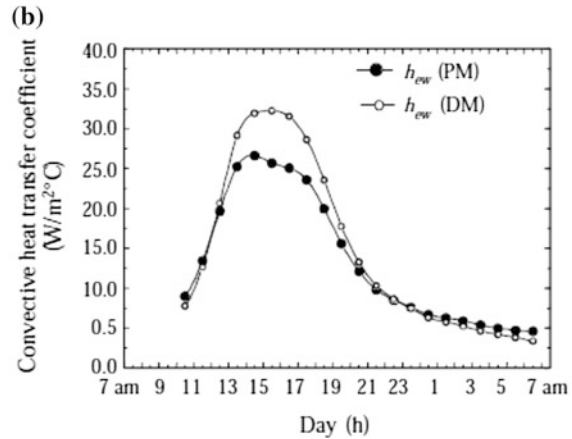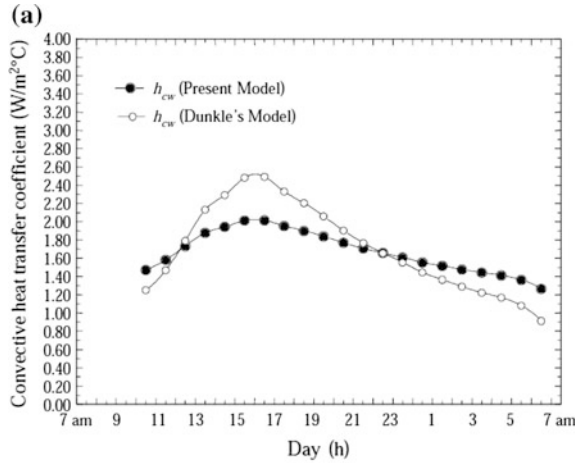
2.8 Thermal Efficiency

The thermal efficiency of a solar-distillation system can be defined as the ratio of the
amount of thermal energy used to obtain a certain amount of distilled water to the
incident solar energy within a given time span.

2.8.1 Instantaneous Efficiency

All of the absorbed solar radiation inside a solar distiller is consumed for the
evaporation of water and thermal losses (top, bottom, and side) when considering
the evaporation process to be an isobaric (at atmospheric pressure) process at
thermal equilibrium.

The energy balance around the water basin for steady-state conditions can be expressed as [23]:

$$\left[\alpha'_w + \alpha'_b\right]I(t)A_s = \dot{Q}_{ew} + \dot{Q}_{loss} \tag{2.8.105}$$

$$\text{Rate of energy in} = \text{Rate of energy out}$$

where $\dot{Q}_{ew} = \dot{m}_w L$ and $\dot{Q}_{loss} = U'_L(T_w - T_a)A_s$

U'_L is the overall heat-transfer coefficient from the water surface to the ambient air through the top, bottom, and sides of the solar-distiller unit. It is assumed that:

$$\alpha'_w + \alpha'_b = (\alpha\tau)_w$$

where $\dot{Q}_{loss}$, i.e., the losses in the *Tamini* model, does not consider the evaporative heat loss. On comparing the analysis with that of a flat-plate collector, it has been observed that the upward losses should be minimum in case of a flat-plate collector, whereas the upward losses should be maximum in a solar-distiller unit in order to obtain a greater yield. As discussed previously, in the conventional solar still, radiative, convective, and evaporative losses are grouped together and considered as the total internal heat-transfer coefficient from the water surface to the inner surface of the condensing cover [2, 24].

Equation (2.8.105) can be expressed as follows:

$$\dot{Q}_{ew} = \dot{m}_w L = (\alpha\tau)_w I(t)A_s - U'_L(T_w - T_a)A_s \tag{2.8.106}$$

The expression for instantaneous efficiency (η_i) can be expressed as shown:

$$\eta_i = \frac{\dot{m}_w L}{I(t)A_s} = (\alpha\tau)_w - \frac{U'_L(T_w - T_a)}{I(t)} \tag{2.8.107}$$

The plot of η_i versus $\frac{(T_w - T_a)}{I(t)}$ represents a straight line with $(\alpha\tau)_w$ and $-U'_L$ as the intercept and the slope, respectively.

where U'_L can be considered as a constant [25]. The expression of instantaneous efficiency (η_i) is exactly similar to that of a conventional flat-plate collector excluding the heat-removal factor. An actual variation of η_i versus $\frac{(T_w - T_a)}{I(t)}$ (known as the "characteristic curve") is shown in Fig. 2.6. According to Tamini [23], the solar-distillation system or distiller unit can be deliberated as a special type of flat-plate collector that produces distilled water on trapping and using (harvesting) the incident solar energy.

Fig. 2.6 Variation of instantaneous efficiency of solar distillation system
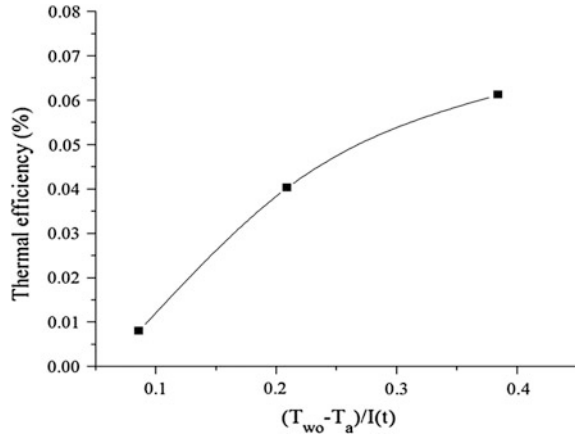
Example 2.14 Calculate (a) the rate of evaporative heat loss for example 2.8, (b) the hourly output (take $L = 2390 \times 10^3$ J/kg), and (c) the instantaneous efficiency of a distillation unit for a solar intensity of 900 W/m².

Solution

(a) $\dot{q}_{ew} = h_{ew}(T_w - T_g) = 3.445 \times (20 - 12) = 27.56 \text{ W/m}^2$

(b) $\dot{m}_{ew} = \left[\dfrac{h_{ew}(T_w - T_g)}{L}\right] \times 3600 = 0.0415 \text{ kg/m}^2 \text{ h}$

(c) $\eta_i = \dfrac{\dot{q}_{ew}}{I(t)} = \dfrac{27.56}{900} = 0.0306 = 3.06\%$

2.8.2 Overall Thermal Efficiency

The overall thermal efficiency of the solar-distillation system in passive and active modes of operation can be mathematically written as follows:

$$\eta_{passive} = \left[\frac{\sum \dot{m}_w L}{A_s \int I(t)dt}\right] \times 100 \qquad\qquad (2.8.108)$$

$$\eta_{active} = \left[\frac{\sum \dot{m}_w L}{A_s \int I(t)dt + A_s \int I'(t)dt}\right] \times 100 \qquad\qquad (2.8.109)$$

where N is the number of collectors connected either in series or in parallel.

The latent heat of vaporization $L(\text{J/kg})$ is a function of temperature and can be expressed as [26, 27]

$$L = 3.1615 \times 10^6 \left[1 - \left(7.616 \times 10^{-4}\right)T\right] \quad \text{for} \quad T > 70\,°\text{C}$$

and

$$L = 2.4935 \times 10^6 \left[1 - \left(9.477 \times 10^{-4}\right)T + \left(1.313 \times 10^{-7}\right)T^2 - \left(4.497 \times 10^{-9}\right)T^3\right]$$

for $T < 70\,°\text{C}$.

Problems and descriptive questions

2.1 Calculate the rate of evaporation for an exposed wetted surface having a temperature of $35\,°\text{C}$ and a relative humidity of 50% when the ambient air temperature is $15\,°\text{C}$. In addition, calculate the evaporative heat-transfer coefficient.

2.2 Calculate the convective heat-transfer coefficient for an insulated hot plate at $60\,°\text{C}$ facing upward with a water column at $40\,°\text{C}$. The dimension of the hot plate is $1\,\text{m} \times 1\,\text{m}$.

2.3 Calculate the convective heat-transfer coefficient from a blackened surface to (a) the water (basefluid) and (b) the nanofluid (TiO_2 nanoparticles) of a passive double-slope solar still for the parameters given in example 2.2. The properties of the TiO_2 nanoparticle are given below;
$C_{pp} = 697(\text{J/kg K})$, $\beta_p = 8.1 \times 10^{-6}(K^{-1})$, $\varphi_p = 0.004\%$, $\rho_p = 4.230 \times 10^3 \left(\text{kg/m}^3\right)$, thermal conductivity of particle $k_p = 17.5(\text{W/m}\,°\text{C})$, diameter of nanoparticle, $(d_p) = 15\,\text{nm}$.
Take the temperature of the water as $(T_w) = 60.25\,°\text{C}$, The temperature of the nanofluid $(T_{nf}) = 61.65\,°\text{C}$.

2.4 For the parameters given in problem 2.3, evaluate the convective heat-transfer coefficient of a passive double-slope solar still from the blackened surface to (a) the water (basefluid) and (b) the nanofluid (TiO_2 nanoparticles) for 0.008 and 0.015% volumetric concentration of nanoparticles.

2.5 Calculate the total solar radiation from the first method for an inclined surface having an inclination of 30° facing south at 9 am for New Delhi on 23 September 2015 $(T_R = 4.75$ and $\rho = 0.2)$.

2.6 Calculate (a) the *Grashof* number (Gr), the *Prandlt* number (Pr), the *Reynolds number* (Re), and the *Rayleigh* number (Ra) for a temperature difference of $10\,°\text{C}$ and a fluid layer thickness of $0.20\,\text{m}$ as well as (b) the *Nusselt* number for free and forced convection for a hot surface facing upward ($C = 0.54$, $n = 0.25$). Use $v_0 = 0.01\,\text{m/s}$ and $X_0 = 1$.

2.7 Evaluate and compare all of the thermophysical properties of vapor listed in Tables 2.5 [2] and 2.6 [28].

2.8 Calculate (a) the convective heat-transfer coefficient from Dunkle's model and (b) the radiative heat-transfer coefficient for a passive single-slope solar-distillation system having a temperature of water surface and an inner surface of the condensing cover at 45 and 35 °C., respectively. Use $\varepsilon_w = \varepsilon_g = 0.9$ and $F_{12} = 1$.

2.9 Calculate the radiative and convective heat-transfer coefficients from the condensing cover at 30 °C. the ambient air at 15 °C when the wind is blowing at a velocity of 2 m/s. Compare the result with the combined radiative and convective heat-transfer coefficients.

2.10 Evaluate (a) the top-loss coefficient from a water-surface at temperature 30 °C to the ambient air at 15 °C. Given the temperature of glass, 20 °C, $\varepsilon_g = \varepsilon_w = 0.9$ and (b) the bottom-loss coefficient from a solar still with an insulation of (i) 2 mm and (ii) 4 mm thickness ($K = 0.05$ W/m^2 °C). Given, $h_w = 80$ W/m^2 °C.

2.11 Evaluate the evaporative heat-transfer coefficient using the new model for (a) free convection and (b) forced convection of a passive single-slope solar distiller unit for a hot surface facing upward ($C = 0.54, n = 0.25$) for the following parameters: $\Delta T = 10$ °C, $d = 0.10$ m, $v_0 = 0.01$ m/s and $X_0 = 1$.

2.12 Estimate (a) the convective and evaporative heat-transfer coefficients for a passive single-slope solar-distillation system having a temperature of water surface and inner surface of the condensing cover at 20 and 15 °C, respectively, and (b) the hourly output (take $L = 2250 \times 10^3$ J/kg) and (c) instantaneous efficiency of a distillation unit for a solar intensity of 850 W/m^2.

2.13 Discuss the first and second method to calculate the total solar radiation incident on an inclined surface.

2.14 Explain briefly the heat transfer of (a) convection, (b) conduction, and (c) radiation.

2.15 Derive an expression for Dunkle's relation of convective heat transfer.

2.16 Explain all internal and external heat-transfer coefficients of a passive solar-distillation system.

2.17 Discuss the new model and explain the regression method to obtain the values of C and n.

Objective questions

2.1 The solar constant is measured near:

    (a) the Sun
    (b) the Earth
    (c) the extra-terrestrial region
    (d) the terrestrial region.

2.2   The short-wavelength radiation reaching the Earth measures:

   (a) $30\,\mu m$
   (b) $3-30\,\mu m$
   (c) $0.3-3\,\mu m$
   (d)   None of these

2.3   The value of solar radiation in summer is maximum on the surface having An
      inclination equal to:

   (a)   Latitude
   (b)   Zero
   (c) $45\,^\circ$
   (d) $90\,^\circ$

2.4   The value of the solar constant is:

   (a) $1000\,W/m^2$
   (b)   Zero
   (c) $1367\,W/m^2$
   (d) $500\,W/m^2$.

2.5   The relation between the zenith (θ_z) and the solar-altitude (α) angles is:

   (a) $\theta_z + \alpha = 60\,^\circ$
   (b) $\theta_z + \alpha = 90\,^\circ$
   (c) $\theta_z + \alpha = -90\,^\circ$
   (d) $\theta_z + \alpha = 0\,^\circ$

2.6   The surface-azimuth angle (γ) for a surface sloped toward the south is:

   (a) $90\,^\circ$
   (b) $180\,^\circ$
   (c) $0\,^\circ$
   (d) $-90\,^\circ$

2.7   The radiation from the Sun is governed by:

   (a)   Stefan–Boltzmann law
   (b)   Fourier's law of conduction
   (c)   Faraday's law
   (d)   Wein's displacement law.

2.8   The radiation from the Earth is governed by:

   (a)   Fourier's law of conduction
   (b)   Stefan–Boltzmann law
   (c)   Wein's displacement law
   (d)   None of these.

2.9 Thermal-expansion coefficients depend on:

    (a) Temperature
    (b) Thermal air conductance
    (c) Thermal conductivity
    (d) None of these

2.10 Which of the following represents the correct form of Fourier's law of heat conduction?

    (a) $\dot{Q} = -KA\dfrac{dT}{dx}$
    (b) $\dot{Q} = -\left(\dfrac{K}{A}\right)\dfrac{dT}{dx}$
    (c) $\dot{Q} = KA\dfrac{dT}{dx}$
    (d) $\dot{Q} = \left(\dfrac{K}{A}\right)\dfrac{dT}{dx}$

2.11 Convective heat transfer depends on:

    (a) Physical properties of fluid
    (b) Physical properties of solid
    (c) Characteristics dimension
    (d) All of these.

2.12 The convective heat-transfer coefficient from Dunkle's relation is:

    (a) $h_{cw} = 0.844\left[\left(T_w - T_{gi}\right) - \dfrac{\left(P_w - P_{gi}\right)\left(T_w + 273\right)}{\left(2.69 \times 10^3 - P_w\right)}\right]^{\frac{1}{3}}$

    (b) $h_{cw} = 0.844\left[\dfrac{\left(P_w - P_{gi}\right)\left(T_w + 273\right)}{\left(2.69 \times 10^3 - P_w\right)}\right]^{\frac{1}{3}}$

    (c) $h_{cw} = 0.844\left[\left(T_w - T_{gi}\right) + \dfrac{\left(P_w - P_{gi}\right)\left(T_w + 273\right)}{\left(2.69 \times 10^3 - P_w\right)}\right]^{\frac{1}{3}}.$

    (d) $h_{cw} = 0.844\left[\left(T_w - T_{gi}\right) + \dfrac{\left(P_w + P_{gi}\right)\left(T_w + 273\right)}{\left(2.69 \times 10^3 + P_w\right)}\right]^{\frac{1}{3}}$

2.13 The radiation heat transfer between two surfaces is mainly due to:

    (a) Short wavelength
    (b) Infra-red
    (c) UV
    (d) Long-wavelength radiation

2.14 An expression for a wind-dependent convective heat-transfer coefficient is

    (a) $h_c = 3 + 2.8\,V$
    (b) $h_c = 2.8 + 3\,V$
    (c) $h_c = 3 + 3\,V$
    (d) None of these

2.15   The evaporative heat-transfer coefficient (h_{ew}) is:

    (a)   Proportional to h_{cw} (convective heat-transfer coefficient)
    (b)   Inversely proportional to h_{cw}
    (c)   Independent of convective heat-transfer coefficient
    (d)   None of these

2.16   The evaporative heat-transfer coefficient (h_{ew}) depends on the convective heat-transfer coefficient due to:

    (a)   Lewis relation
    (b)   Newton's law
    (c)   Fourier's law
    (d)   None of these

2.17   Which of the following form represents an expression for instantaneous efficiency (η_i) ?

    (a) $\eta_i = (\alpha\tau)_w + \dfrac{U'_L(T_w - T_a)}{I(t)}$
    (b) $\eta_i = (\alpha\tau)_w - \dfrac{U'_L(T_w - T_a)}{I(t)}$
    (c) $\eta_i = (\alpha\tau)_w - \dfrac{U'_L}{(T_w - T_a)I(t)}$
    (d) $\eta_i = (\alpha\tau)_w - \dfrac{(T_w - T_a)}{U'_L I(t)}$

Answers

2.1 (c) 2.2 (c) 2.3 (b) 2.4 (c) 2.5 (b) 2.6 (c) 2.7 (a) 2.8 (b) 2.9 (a) 2.10 (a) 2.11(a) &(c) 2.12 (c) 2.13(d) 2.1 4(c) 2.15 (a) 2.16(a) 2.17 (b)

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