1. Design Data Input

1.1 Steel Properties (For steel reinforcement, ASTM A615M Grade 420 MPa)

$f_y := 420\ MPa = 60.916\ ksi$ $E_s := 2 \cdot 10^5\ MPa$

1.2 Concrete Properties

$\gamma_{concrete} := 25\ \frac{kN}{m^3} = (2.549 \cdot 10^3)\ \frac{kgf}{m^3}$

$f_{ck} := 32 \cdot MPa$: characteristic cylinder strength of concrete

$f_{ctm} := 3.02\ MPa$: mean value of the tensile strength of the concrete (Eurocode 2, Table 3.1)

$f_{ct.eff} := f_{ctm}$: mean value of the tensile strength of the concrete effective at the time when the cracks may first be expected to occur

$E_{cm} := 33.3\ GPa$: secant modulus of elasticity of concrete (Eurocode 2, Table 3.1)

$E_{c28} := 1.05 \cdot E_{cm} = 34.965\ GPa$: modulus of elasticity of concrete

$\varphi := 1.5$: creep coefficient at relevant loading time and duration

$E_{eff} := \frac{E_{c28}}{1+\varphi} = 13.986\ GPa$: effective modulus of concrete

$\alpha_e := \frac{E_s}{E_{eff}} = 14.3$

1.3 Geometric Properties

$b := 1000\ mm$: breath of section

$h := 900\ mm$: overal depth of section

$c_c := 75\ mm$: clear cover of reinforcement

$c_{c2} := 75\ mm$: clear cover of compression reinforcement

$w_{limit} := 0.15\ mm$: maximum allowed crack width

$Dia := [10\ mm \quad 12\ mm \quad 16\ mm \quad 20\ mm \quad 25\ mm \quad 32\ mm]^T$

$Area := [0.7854\ cm^2 \quad 1.131\ cm^2 \quad 2.011\ cm^2 \quad 3.142\ cm^2 \quad 4.909\ cm^2 \quad 8.042\ cm^2]^T$

$Dn := 6$: Rebar Selection D10=1, D12=2, D16=3, D20=4, D25=5, D32=6

$s := 150\ mm$ $\phi := Dia_{Dn} = 32\ mm$

$A_s := Area_{Dn} \cdot \frac{b}{s} = (5.361 \cdot 10^3)\ mm^2$: area of tension reinforcement

$d := h - c_c = 0.825\ m$ $d_c := c_c + \frac{\phi}{2} = 91\ mm$

$Dn2 := 5$: Rebar Selection D10=1, D12=2, D16=3, D20=4, D25=5, D32=6

$s_2 := 200\ mm$ $\phi_2 := Dia_{Dn2} = 25\ mm$

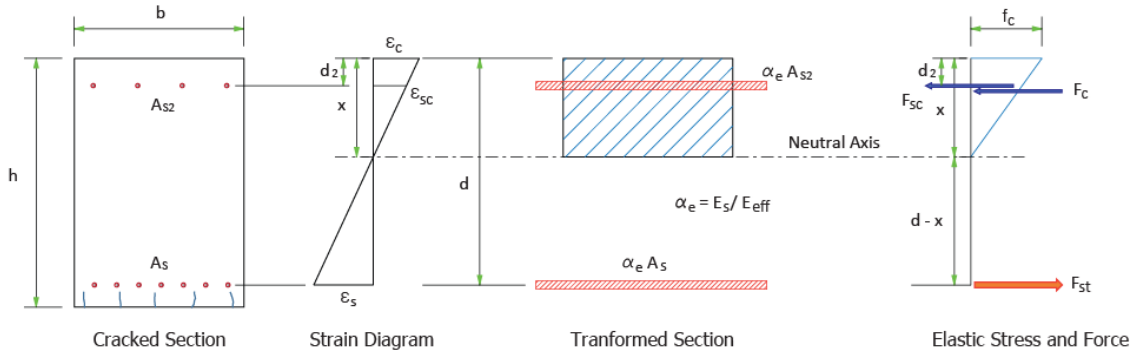
$A_{s2} := Area_{Dn2} \cdot \frac{b}{s_2} = (2.455 \cdot 10^3)\ mm^2$: area of compression reinforcement

$d_2 := c_{c2} + \frac{\phi_2}{2} = 0.088\ m$

1.4 Moment due to Quasi-permanent Actions

$M_{QP} := 400\ kN \cdot m$

2. Calculation of Tensile Stress in Reinforcement at Service Loads (Doubly Reinforced)



2.1 Cracking Moment and Moment at Service Loads

$$x_u := \frac{\frac{b \cdot h^2}{2} + (\alpha_e - 1) \cdot (A_s \cdot d + A_{s2} \cdot d_2)}{b \cdot h + (\alpha_e - 1) \cdot (A_s + A_{s2})} = 0.465\ m$$: neutral axis depth of uncracked section

$$I_u := \frac{b \cdot h^3}{12} + b \cdot h \cdot \left(\frac{h}{2} - x_u\right)^2 + (\alpha_e - 1) \cdot \left(A_s \cdot (d - x_u)^2 + A_{s2} \cdot (x_u - d_2)^2\right) = 0.075\ m^4$$: moment inertia of of uncracked section transformed to concrete

$$M_{cr} := f_{ctm} \cdot I_u \cdot \frac{1}{h - x_u} = 519.437\ kN \cdot m$$: cracking moment

2.2 Calculation of Tensile Stress in Reinforcement

$$x := \left(\sqrt{\left((A_s \cdot \alpha_e + A_{s2} \cdot (\alpha_e - 1))\right)^2 + 2\ b \cdot (A_s \cdot d \cdot \alpha_e + A_{s2} \cdot d_2 \cdot (\alpha_e - 1))} - (A_s \cdot \alpha_e + A_{s2} \cdot (\alpha_e - 1))\right) \cdot \frac{1}{b} = 0.27\ m$$

$$I_{cr} := \frac{b \cdot x^3}{3} + \alpha_e \cdot A_s \cdot (d - x)^2 + (\alpha_e - 1) \cdot A_{s2} \cdot (d_2 - x)^2 = (3.126 \cdot 10^{10})\ mm^4$$

$$f_{st} := \frac{M_{QP}}{I_{cr}} \cdot \alpha_e \cdot (d - x) = 101.474\ MPa$$

$$f_c := \frac{M_{QP}}{I_{cr}} \cdot x = 3.459\ MPa$$

$$f_{sc} := \frac{M_{QP}}{I_{cr}} \cdot \alpha_e \cdot (x - d_2) = 33.459\ MPa$$

- verificaton -

$$T := A_s \cdot f_{st} = 544.038\ kN \qquad C := 0.5 \cdot b \cdot x \cdot f_c - A_{s2} \cdot f_c \cdot \frac{x - d_2}{x} + A_{s2} \cdot f_{sc} = 544.038\ kN \qquad : T = C$$

$$M := T \cdot (d - x) + 0.5 \cdot b \cdot x \cdot f_c \cdot \frac{c}{2} x + \left(A_{s2} \cdot f_{sc} - A_{s2} \cdot f_c \cdot \frac{x - d_2}{x}\right) \cdot (x - d_2) = 400\ kN \cdot m \qquad M_{QP} = 400\ kN \cdot m \qquad : M = M_{srv}$$

2.3 Calculation of Crack Width (Eurocode 2)

$\sigma_s := f_{st}$

$k_t := 0.4$: factor dependent on the duration of the load (0.6 for short term, 0.4 for long term)

$h_{c.ef} := min\left(2.5 \cdot (h - d), \frac{(h - x)}{3}, \frac{h}{2}\right) = 187.5\ mm$: depth of effective tension area

$A_{c.eff} := h_{c.ef} \cdot b - A_s = \left(1.821 \cdot 10^5\right)\ mm^2$: effective tension area

$\rho_{p.eff} := \frac{A_s}{A_{c.eff}} = 0.029$: effective steel ratio for effective tension area

$k_1 := 0.8$: coefficient of rebar bond property (0.8 for high bond bar, 1.6 for prestressing tendons)

$k_2 := 0.5$: coefficient of strain distribution (0.5 for bending, 1.0 for pure tension)

$k_3 := 3.4$ $k_4 := 0.425$

$$\varepsilon_{sm\_\varepsilon cm} := \frac{\sigma_s - k_t \cdot \frac{f_{ct.eff}}{\rho_{p.eff}} \cdot \left(1 + \alpha_e \cdot \rho_{p.eff}\right)}{E_s} = 2.158 \cdot 10^{-4}$$

: variance in mean strains between reinorcement and concrete

check values

$0.6 \cdot \frac{\sigma_s}{E_s} = 3.044 \cdot 10^{-4}$

$$\varepsilon_{sm\_\varepsilon cm} := \left\| \begin{array}{l} \varepsilon_{sm\_\varepsilon cm} \leftarrow \varepsilon_{sm\_\varepsilon cm} \\ \text{if } \varepsilon_{sm\_\varepsilon cm} < 0.6 \cdot \frac{\sigma_s}{E_s} \\ \quad \varepsilon_{sm\_\varepsilon cm} \leftarrow 0.6 \cdot \frac{\sigma_s}{E_s} \\ \varepsilon_{sm\_\varepsilon cm} \end{array} \right. = 3.044 \cdot 10^{-4}$$

$s_{r.max} := k_3 \cdot c_c + \frac{k_1 \cdot k_2 \cdot k_4 \cdot \phi}{\rho_{p.eff}} = 439.811\ mm$

: maximum crack spacing

check values

$s = 150\ mm$, $5 \cdot \left(c_c + \frac{\phi}{2}\right) = 455\ mm$

$1.3 \cdot (h - x) = 818.512\ mm$

$$s_{r.max} := \left\| \begin{array}{l} s_{r.max} \leftarrow s_{r.max} \\ \text{if } s > 5 \cdot \left(c_c + \frac{\phi}{2}\right) \\ \quad s_{r.max} \leftarrow 1.3 \cdot (h - x) \\ s_{r.max} \end{array} \right. = 439.811\ mm$$

$w_k := s_{r.max} \cdot \left\langle \varepsilon_{sm\_\varepsilon cm} \right\rangle = 0.134\ mm$: crack width

$$Judge := \left\| \begin{array}{l} \text{if } M_{QP} \le M_{cr} \\ \quad \text{“Not cracked”} \\ \text{else if } w_k \le w_{limit} \\ \quad \text{“Lower than limit”} \\ \text{else} \\ \quad \text{“NG”} \end{array} \right. = \text{“Not cracked”}$$