Ex.5 Convolution (Response of electric circuits)
R=3Ω,L=1 H, C=0.5 F

$R := 3 \qquad L := 1 \qquad C := 0.5 \qquad Z(s) := R + s \cdot L + \frac{1}{s \cdot C}$

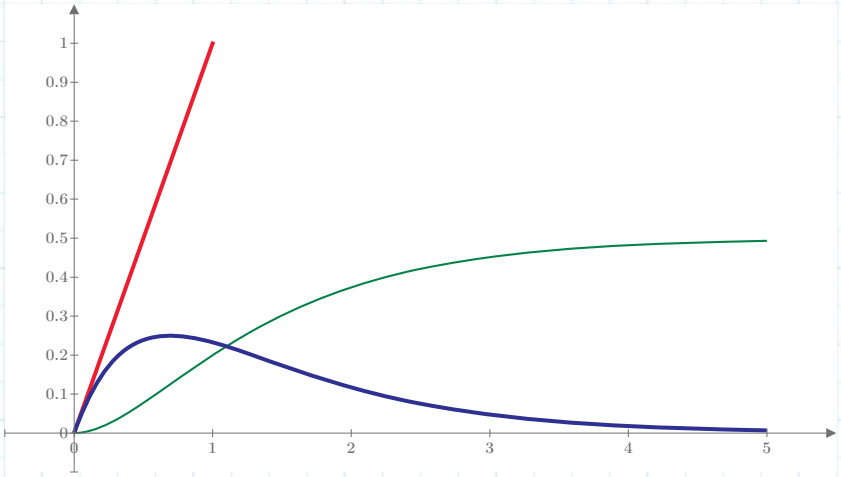
$\frac{1}{Z(s)} \xrightarrow{invlaplace} 2 \cdot e^{-2 \cdot t} - e^{-t}$ impulse response

$e(t) := t \xrightarrow{laplace} \frac{1}{s^2} \qquad E(s) := \frac{1}{s^2} \qquad e(t) := t$ *input*

$I(s) := \frac{E(s)}{Z(s)} \rightarrow \frac{1}{s^2 \cdot \left(s + \frac{2.0}{s} + 3\right)} \xrightarrow{invlaplace} \frac{\left(e^{-t} - 1\right)^2}{2}$

$i(t) := \frac{\left(e^{-t} - 1\right)^2}{2} \xrightarrow{expand} \frac{e^{-2 \cdot t}}{2} - e^{-t} + \frac{1}{2}$

$v_L(t) := L \cdot \frac{d}{dt} i(t) \rightarrow e^{-t} - e^{-(2 \cdot t)}$ *output*



$i(t)$

$e(t)$

$L \cdot \frac{d}{dt} i(t)$

推定値 $R := 3 \qquad L := 1 \qquad C := 0.5$

制約条件 $\frac{1}{C} \cdot q(t) + R \cdot \frac{d}{dt} q(t) + L \cdot \frac{d^2}{dt^2} q(t) = t \cdot \Phi(t) \qquad q(0) = 0 \qquad q'(0) = 0$

ソルバー $q := \text{odesolve}(q(t), 5)$

$i(t) := \frac{d}{dt} q(t)$

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input *output*

impulse response

$$\frac{d}{dt}\Phi(t) \xrightarrow{laplace} 1 \qquad e^{-t}-e^{-(2\cdot t)} \qquad \frac{1}{(s+1)\cdot(s+2)} \xrightarrow{invlaplace} e^{-t}-e^{-(2\cdot t)}$$

indicial response
(unit step response)

$$\Phi(t) \xrightarrow{laplace} \frac{1}{s} \qquad \frac{(e^{-t}-1)^2}{2} \qquad \frac{\frac{1}{s}}{(s+1)\cdot(s+2)} \xrightarrow{invlaplace} \frac{(e^{-t}-1)^2}{2}$$

ramp response

$$\int_0^t \Phi(t)\,dt \xrightarrow{laplace} \frac{1}{s^2} \qquad \frac{t}{2}+e^{-t}-\frac{e^{-2\cdot t}}{4}-\frac{3}{4}$$

$$\frac{\frac{1}{s^2}}{(s+1)\cdot(s+2)} \xrightarrow{invlaplace} \frac{t}{2}+e^{-t}-\frac{e^{-2\cdot t}}{4}-\frac{3}{4}$$

exponential response

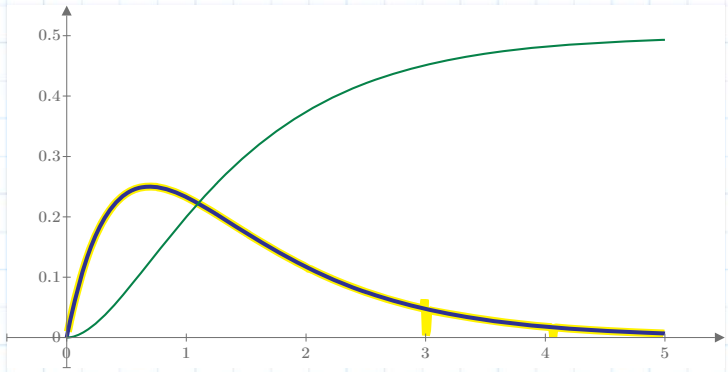
$$e^{-2\cdot t} \xrightarrow{laplace} \frac{1}{s+2} \qquad e^{-t}-e^{-(2\cdot t)}-t\cdot e^{-2\cdot t}$$

$$\frac{\frac{1}{s+2}}{(s+1)\cdot(s+2)} \xrightarrow{invlaplace} e^{-t}-e^{-(2\cdot t)}-t\cdot e^{-2\cdot t}$$

$h(t) := e^{-t}-e^{-(2\cdot t)}$ impulse response

$h_i(t) := \int_0^t h(t)\,dt \rightarrow \frac{e^{-2\cdot t}}{2}-e^{-t}+\frac{1}{2}$ indicial response (unit step response)

$t := 0, 0.01 .. 5$



$L\cdot\frac{d}{dt}i(t)$

$h(t)$

$h_i(t)$

t

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