Ex.9 Convolution (Response of rectangular function)

when the impulse response is also rectangular function

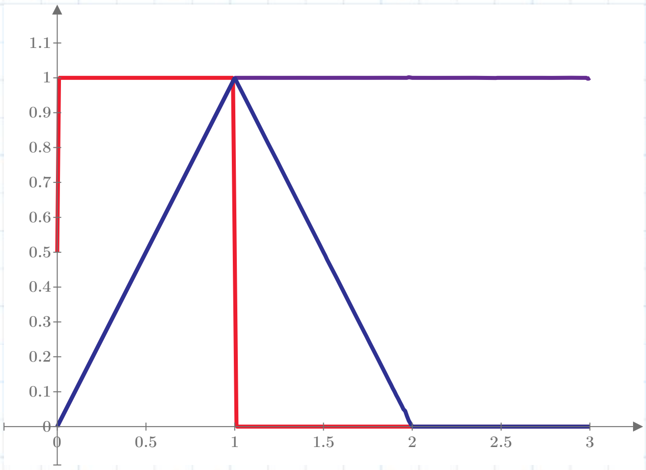
$t := 0, 0.01 .. 3$

$rec(t) := \Phi(t) - \Phi(t-1)$

$x(t) := rec(t)$

$h(t) := rec(t)$

$y(t) := \int_0^t x(\tau) \cdot h(t-\tau)\, d\tau$



$x(t)$ *input*

impulse response

$\int_0^t h(t)\, dt$ indicial response

$y(t)$ *output*

t

$y(t) := t \cdot \Phi(t) + 2 \cdot \Phi(t-1) - 2 \cdot \Phi(t-2) - 2 \cdot t \cdot \Phi(t-1) + t \cdot \Phi(t-2)$

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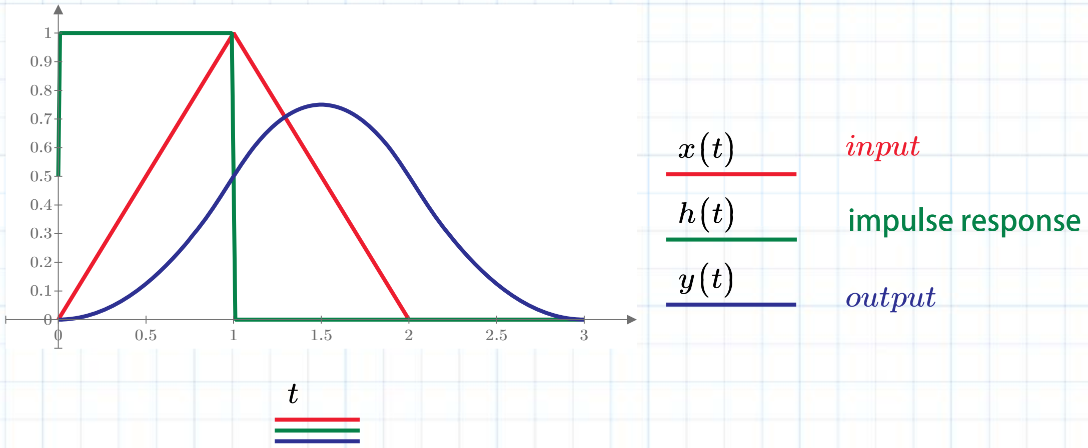
The one of triangular function input.

$$x(t) := t \cdot (\Phi(t) - \Phi(t-1)) + (2-t) \cdot (\Phi(t-1) - \Phi(t-2))$$

$$h(t) := (\Phi(t) - \Phi(t-1))$$

$$y(t) := \int_0^t h(\tau) \cdot x(t-\tau)\, d\tau$$

$$y(t) := \frac{t^2}{2} \cdot \Phi(t) - \frac{3 \cdot \Phi(t-1) \cdot (t-1)^2}{2} + \frac{3 \cdot \Phi(t-2) \cdot (t-2)^2}{2} - \frac{\Phi(t-3) \cdot (t-3)^2}{2}$$

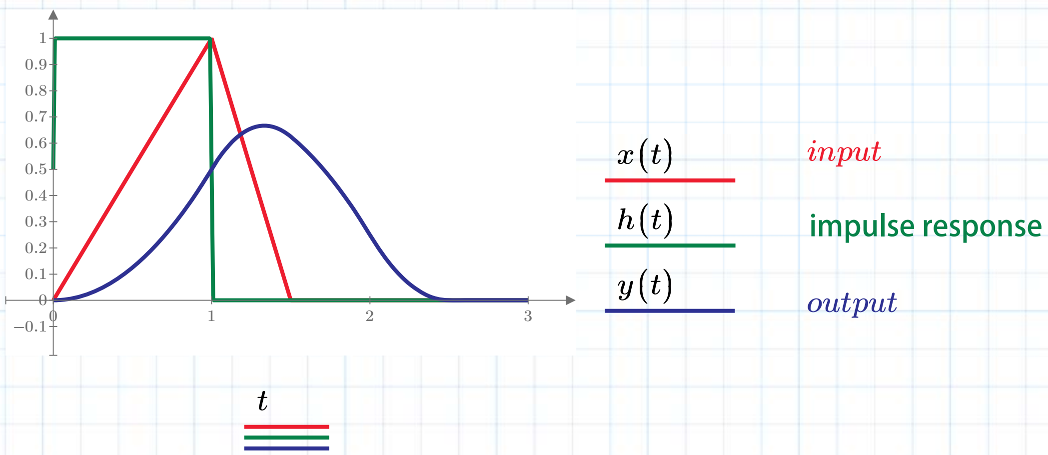


$$x(t) := t \cdot (\Phi(t) - \Phi(t-1)) + (3 - 2 \cdot t) \cdot \left(\Phi(t-1) - \Phi\left(t - \frac{3}{2}\right)\right)$$

$$h(t) := (\Phi(t) - \Phi(t-1))$$

$$y(t) := \int_0^t h(\tau) \cdot x(t-\tau)\, d\tau$$

$$y(t) := \frac{t^2}{2} \cdot \Phi(t) - 2 \cdot \Phi(t-1) \cdot (t-1)^2 + \frac{3 \cdot \Phi(t-2) \cdot (t-2)^2}{2} + \frac{\Phi\left(t - \frac{3}{2}\right) \cdot (2 \cdot t - 3)^2}{4} - \frac{\Phi\left(t - \frac{5}{2}\right) \cdot (2 \cdot t - 5)^2}{4}$$



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