When saving or printing, disable Automatic Calculation.

TEST SIGNALS

Francesco Mezzanino

The subscript "fs" is an acronym and indicates that the variable refers to the file "Fourier series.xmcd"
The subscript "dp" is an acronym and indicates that the variable refers to the file "Dirac Pulse - formulas.xmcd"

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REFERENCES

- Reference:C:\new folder\global data.xmcd
- Reference:C:\new folder\Fourier Series.xmcd
- Reference:C:\new folder\Dirac Pulse - formulas.xmcd
- Reference:C:\new folder\rf pulse data.xmcd
- Reference:C:\new folder\sawtooth pulse data.xmcd
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INTRODUCTION

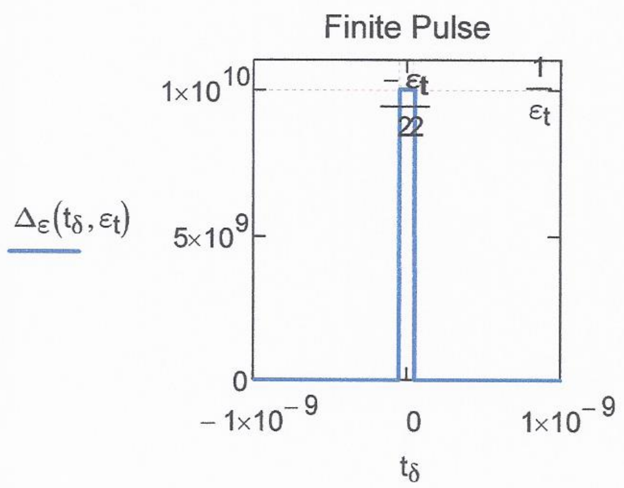
In this worksheet are treated deterministic signals only. It isn't a rational - deductive demostration of new concepts. It was created only for practical purposes(For this reason the name test signal) and therfore it can be very useful.
It is a collection of some common (and not), signals used in electronics.
To get the signal spectra (magnitude and phase, autocorrelation etc.), open the worksheet "Signal Analysis.xmcd".
To simplify the realization of this file (this worksheet) and to make more agile and immediate viewing of signal's graphics, the data, for some signal, are defined in other worksheets as listed in the references before this introduction. (Staircase, Pulse train, FM and PM signal, ect).
Hence, to change the data, open the relative file and modify them. Save and close both (relative file data and test signal) files to store the data variations done, than reopen test signal.
Fourier transforms and Fourier series are found in "Signal Analysis.xmcd" worksheet,
See references for data.

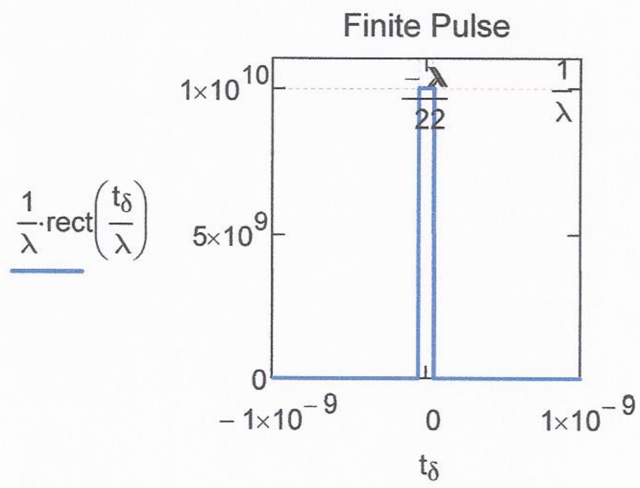
Voltage Pulses

1 Dirac Pulse Approximation

For data and definition see the worksheet "Dirac Pulse - formulas.xmcd"

$\varepsilon_t = 0.1 \cdot ns \qquad t_\delta := -10 \cdot \varepsilon_t , -10 \cdot \varepsilon_t + \frac{20 \cdot \varepsilon_t}{2000} .. 10 \cdot \varepsilon_t \qquad \lambda := \varepsilon_t$





Voltage Pulses

2 Voltage step

Some text in electrical engineering indicate the unitary step with the symbol: $\mathbf{u_{\_1}}(t) = \int_{-\infty}^{t} \mathbf{u_0}(\xi)\, d\xi = \Phi(t)$,

therfore: $\mathbf{u_0}(t) = \frac{d}{dt}\mathbf{u_{\_1}}(t)$.

Other definition are $\Phi(t) = \lim_{\lambda \to 0} \int_{-\infty}^{t} \frac{1}{\lambda} \cdot \Pi\left(\frac{\xi}{\lambda}\right) d\xi = \int_{-\infty}^{t} \mathbf{u_0}(\xi)\, d\xi = \int_{-\infty}^{t} \Delta(\xi)\, d\xi$

Voltage step $V_{stp}(t, V_{pp}) := V_{pp} \cdot \Phi(t)$

Discrete time Unitary step (Unitary pulse: $\delta(\nu, k)$):

$$v_2(t) := \frac{V_{stp}(t, V_{pp})}{V}$$

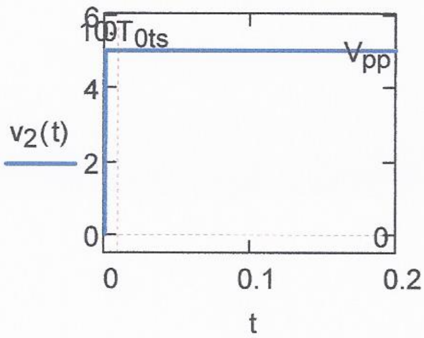
$$\mathbf{u_\delta}(\nu) := \begin{cases} \sum_{j=0}^{\nu} \delta(\nu, j) & \text{if } \nu \ge 0 \\ 0 & \text{otherwise} \end{cases}$$

T_{0ts} and V_{pp} are defined in "global data.xmcd "

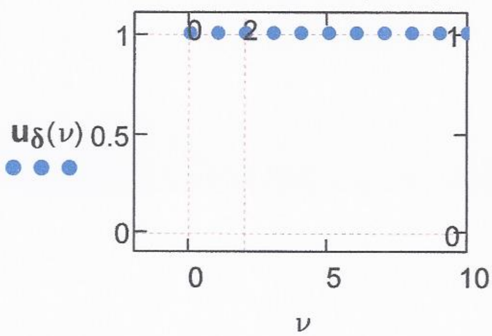
$$t := -1 \cdot T_{0ts}, -1 \cdot T_{0ts} + \frac{201 \cdot T_{0ts}}{5000} .. 200 \cdot T_{0ts}$$

$$\nu := 0 .. 20$$

Unitary step



Discrete Unitary step



Voltage Pulses

3 Ramp with slope V_l/T

Some text of electrical engineering indicate the ramp function, use the symbol: $u_2(t) := t \cdot \Phi(t)$
T_{0ts} and V_{pp} are defined in "global data.xmcd "

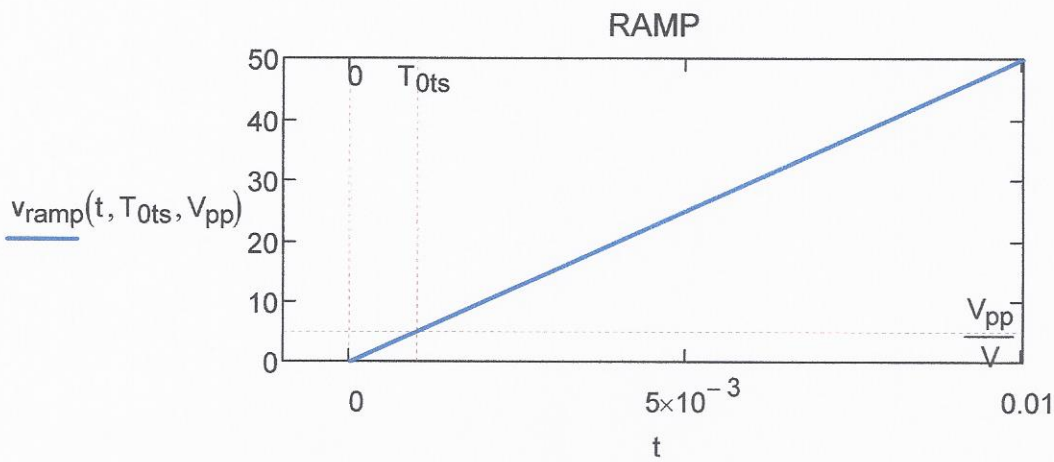
$$\mathbf{u_2}(t) = t \cdot \Phi(t) = \int_{-\infty}^{t} \Phi(\xi)\, d\xi$$

Voltage ramp:

$T_{0ts} := T_{0ts}$ $\qquad V_{pp} := V_{pp}$ $\qquad \mathbf{u_2}(t) := \frac{V_{pp}}{T_{0ts}} \cdot \int_0^t \Phi(\tau_{ts})\, d\tau_{ts} \rightarrow \frac{V_{pp} \cdot t}{T_{0ts}}$

$$\boxed{v_{ramp}(t, T_{0ts}, V_{pp}) := \frac{V_{pp}}{T_{0ts}} \cdot \frac{t \cdot \Phi(t)}{V}}$$

$$t := 0 \cdot T_{0ts}, 0 \cdot T_{0ts} + \frac{10 \cdot T_{0ts}}{5000} .. 10 \cdot T_{0ts}$$



Voltage Pulses

4 Voltage Pulse

Description of the Function's parameters: Adimensional_amplitude· rect1(time ,risingedge ,width)

Data in file " pulse train data.xmcd"

Pulse width: $\tau_{pwts} := \tau_{ptd}$, $\tau_{pwts} = 250 \cdot \mu s$, $T_{0ts} = 1 \times 10^3 \cdot \mu s$,

Amplitude: $V_{pp} = 5 \cdot volt$

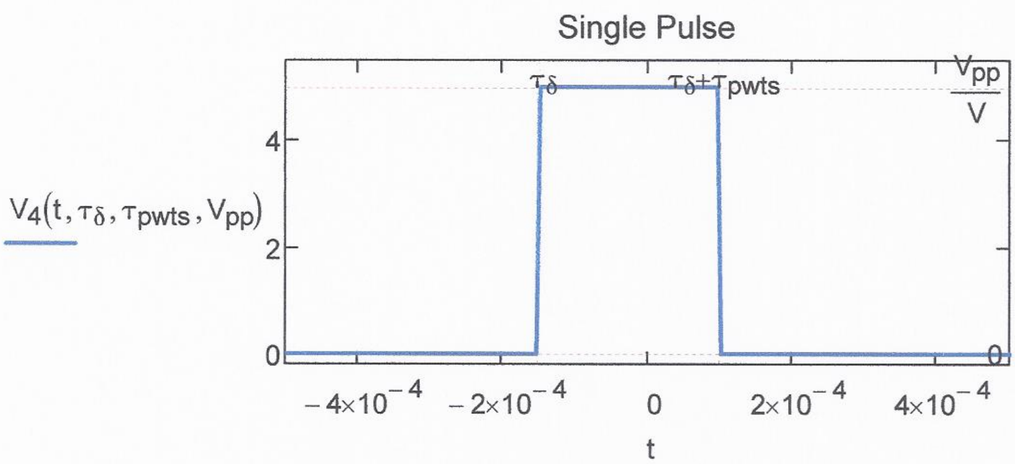
Pulse displacement from the origin: $\xi_{ts} := 0.40$, $\tau_{pwts} = \tau_{pwts} \cdot (1 - \xi_{ts}) + \xi_{ts} \cdot \tau_{pwts}$,

Time delay from the origin: $\tau_\delta := -\tau_{pwts} \cdot (1 - \xi_{ts})$, risingedge $:= \tau_\delta$, width $:= \tau_{pwts}$.

Generic pulse definition defined in "Fourier Series.xmcd": rect1 (t ,risingedge ,width)

$\tau_\delta = -150 \cdot \mu s$ $\qquad t := -2 \cdot \tau_{pwts}, -2 \cdot \tau_{pwts} + \frac{102 \cdot \tau_{pwts}}{5000} .. 100 \cdot \tau_{pwts}$ $\qquad \tau_{pwts} = 250 \cdot \mu s$

$\tau_{pwts} = 250 \cdot \mu s$ $\qquad V_4(t, \tau_\delta, \tau_{pwts}, V_{pp}) := \frac{V_{pp}}{V} \cdot rect1(t, \tau_\delta, \tau_{pwts})$ $\qquad V_4(0 \cdot sec, \tau_\delta, \tau_{pwts}, V_{pp}) = 5$



Voltage Pulses

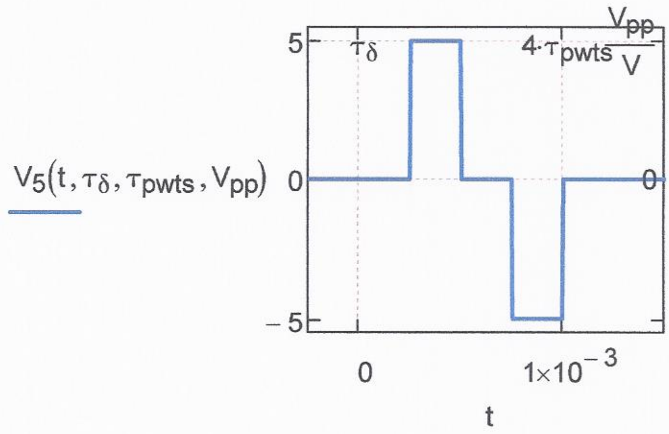
-5 Doublet Voltage Pulse

Description of the Function's parameters: $V_4(t, risingedge, width, pulse\_amplitude)$,
$V_5(t, risingedge, width, pulse\_amplitude)$

Data in file " pulse train data.xmcd"

$\tau_\delta := 0$ $\tau_{pwts} = 250 \cdot \mu s$ $V_4(t, \tau_\delta, \tau_{pwts}, V_{pp}) = \frac{V_{pp}}{V} \cdot rect1(t, \tau_\delta, \tau_{pwts})$

$$V_5(t, \tau_\delta, \tau_{pwts}, V_{pp}) := V_4(t - \tau_{pwts}, \tau_\delta, \tau_{pwts}, V_{pp}) - V_4(t - 3 \cdot \tau_{pwts}, \tau_\delta, \tau_{pwts}, V_{pp})$$



Voltage Pulses

-6 Staircase 1 Voltage Pulse

Data in file "staircase pulse data.xmcd"

Number of steps: $m1_{steps} = 8$

Signal amplitude: $V_{stcs} = 8 \times 10^{-3} \cdot \text{volt}$ $\qquad \dfrac{V_{stcs}}{m1_{steps}} = 1 \times 10^{-3} \cdot V$

Step Amplitude: $V_{stcstp0} = 1 \times 10^{-3} \cdot V$

Step length: $T_{1stpl\_} = 0.37 \cdot \mu s$

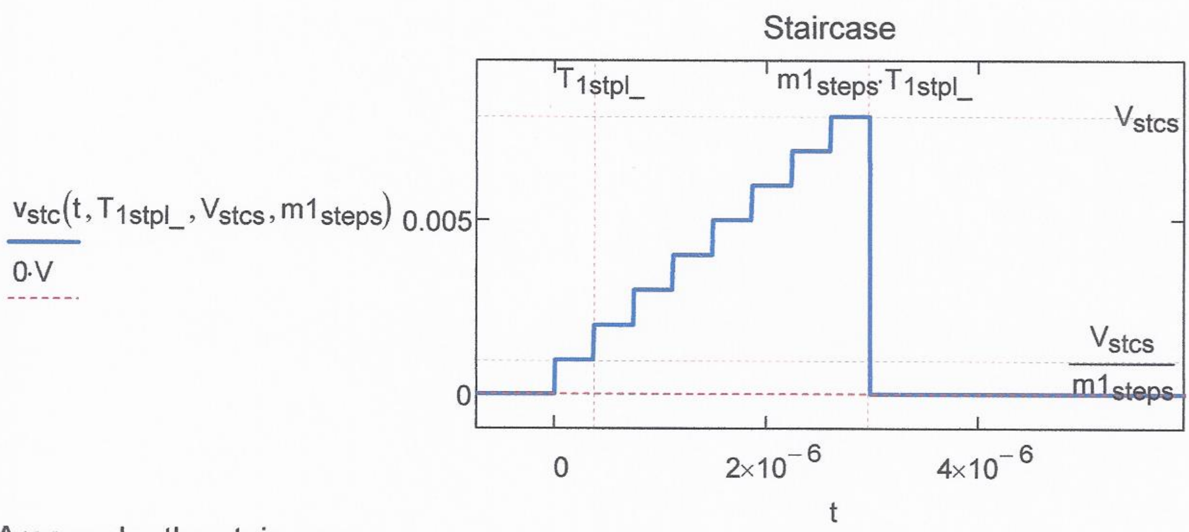
Description of the Function's parameters: $v_{stc}(t, \text{step\_length}, \text{signal\_amplitude}, \text{number\_of\_steps})$

Test signal:

$$v_{stc}(t, T_{1stpl\_}, V_{stcs}, m1_{steps}) := \frac{V_{stcs}}{m1_{steps}} \cdot \left[\sum_{k=0}^{m1_{steps}-1} \left(\Phi(t - k \cdot T_{1stpl\_})\right) - m1_{steps} \cdot \Phi(t - m1_{steps} \cdot T_{1stpl\_}) \right]$$

$$t := -10 \cdot T_{1stpl\_}, -10 \cdot T_{1stpl\_} + \frac{100 \cdot T_{1stpl\_} + 10 \cdot T_{1stpl\_}}{5000} .. 100 \cdot T_{1stpl\_}$$

$m1_{steps} = 8$



Area under the staircase:

$$A_{stc} := T_{1stpl\_} \cdot \frac{V_{stcs}}{m1_{steps}} \cdot \sum_{k=1}^{m1_{steps}} (m1_{steps} - k + 1) \qquad A_{stc} = 13.333 \cdot \text{volt} \cdot \text{ns}$$

$$\int_0^{(m1_{steps}+1) \cdot T_{1stpl\_}} v_{stc}(t, T_{1stpl\_}, V_{stcs}, m1_{steps})\, dt = 13.345 \cdot \text{volt} \cdot \text{ns}$$

Voltage Pulses

- 7 Staircase 2 Voltage Pulse

Data file "staircase 2 pulse data.xmcd"

$$V_{stc} = 5 \cdot V$$

$$m2_{steps} = 8$$

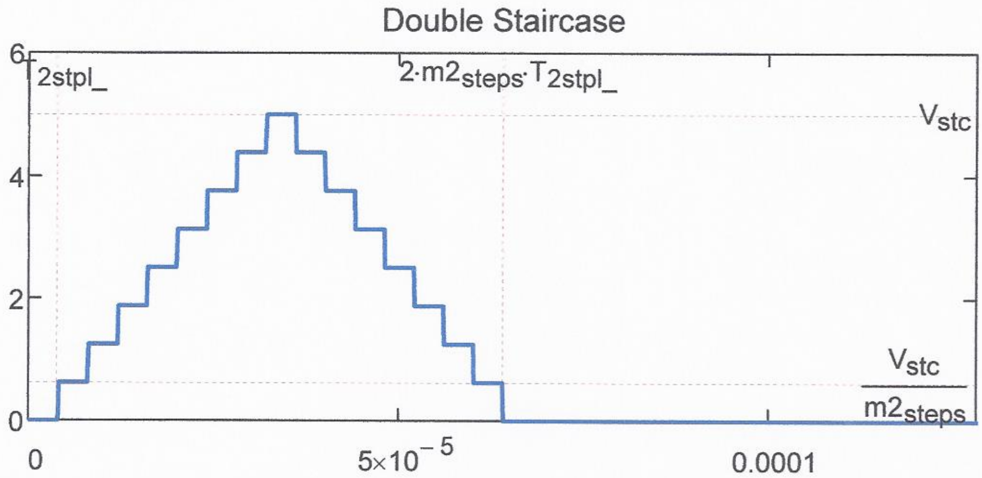
$$T_{2stpl\_} = 4 \cdot \mu s$$

Description of the Function's parameters: $v_{stcc}(t, step\_length, signal\_amplitude, number\_of\_steps)$

Voltage step: $V_{stcstp} = 0.625 \cdot V$

$$v_{stcc}(t, T_{2stpl\_}, V_{stc}, m2_{steps}) := \frac{V_{stc}}{m2_{steps}} \cdot \left[\sum_{k=1}^{m2_{steps}} \left(\Phi\left(t - k \cdot T_{2stpl\_}\right)\right) - \sum_{k=1}^{m2_{steps}} \Phi\left[t - T_{2stpl\_} \cdot \left(k + m2_{steps}\right)\right]\right]$$

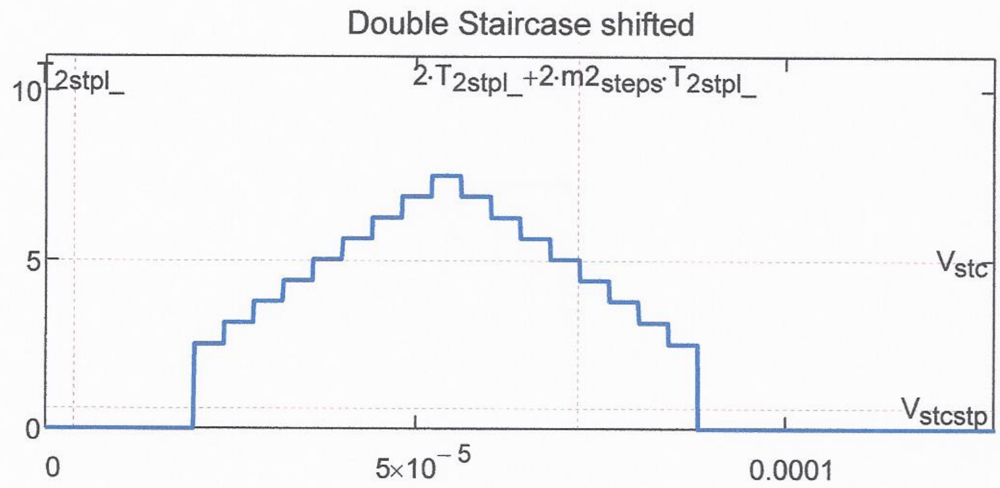
$$t := 0 \cdot T_{2stpl\_}, 0 \cdot T_{2stpl\_} + \frac{100 \cdot T_{2stpl\_}}{5000} .. 100 \cdot T_{2stpl\_}$$



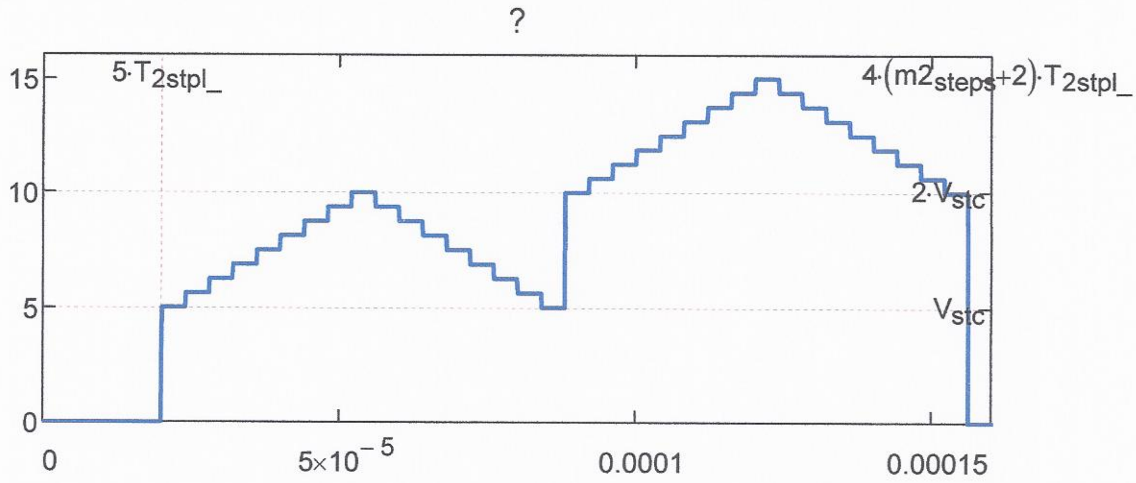
Area under the staircase $A_{stcc} := 2 \cdot T_{2stpl\_} \cdot \frac{V_{stc}}{m2_{steps}} \cdot \sum_{k=1}^{m2_{steps}} \left(\left(m2_{steps} - k + 1\right)\right) - V_{stc} \cdot T_{2stpl\_}$

$$A_{stcc} = 0.16 \cdot volt \cdot ms$$

$$\int_{T_{2stpl\_}}^{2 \cdot m2_{steps} \cdot T_{2stpl\_}} v_{stcc}(t, T_{2stpl\_}, V_{stc}, m2_{steps})\, dt = 160 \cdot volt \cdot \mu s$$



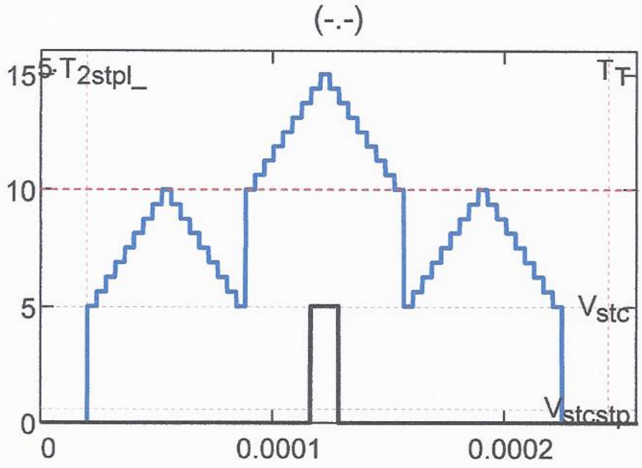
$$v_{2stcc}(t, T_{2stpl\_}, V_{stc}, m2_{steps}) := v_{stcc}(t - 5 \cdot T_{2stpl\_}, T_{2stpl\_}, V_{stc}, m2_{steps}) \ldots$$
$$+ V_{stc} \cdot \text{rect1}[t - 5 \cdot T_{2stpl\_}, 0 \cdot T_{2stpl\_}, (2 \cdot m2_{steps} + 1) \cdot T_{2stpl\_}] \ldots$$
$$+ \left[v_{stcc}[t - 5 \cdot T_{2stpl\_} - (2 \cdot m2_{steps} + 1) \cdot T_{2stpl\_}, T_{2stpl\_}, V_{stc}, m2_{steps}] \ldots \right.$$
$$\left. + 2 \cdot V_{stc} \cdot \text{rect1}[t - 5 \cdot T_{2stpl\_} - (2 \cdot m2_{steps} + 1) \cdot T_{2stpl\_}, 0 \cdot T_{2stpl\_}, (2 \cdot m \right.$$

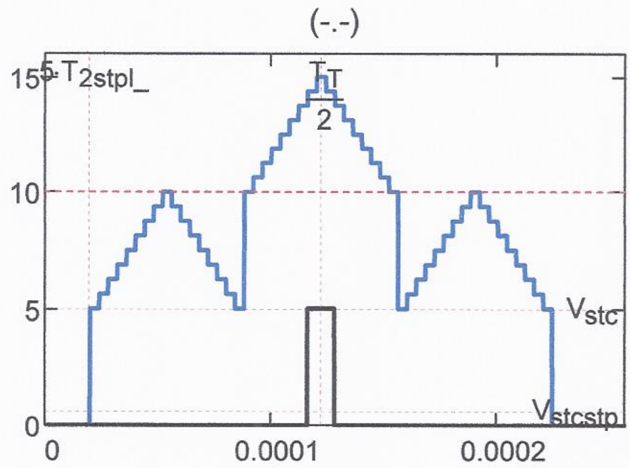


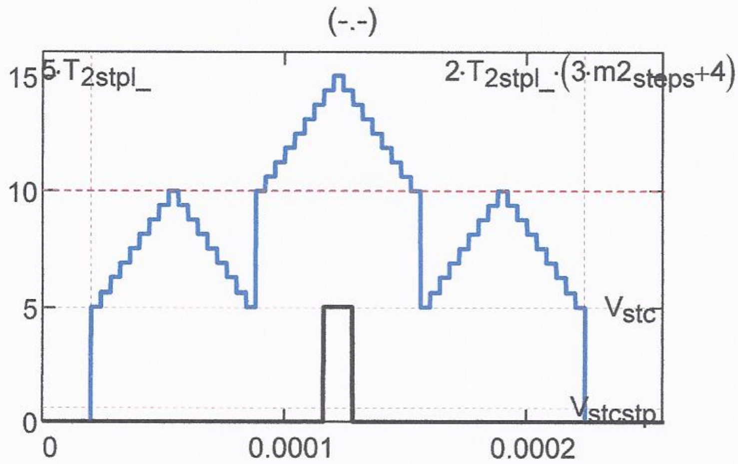
$$v_H(t, T_{2stpl\_}, V_{stc}, m2_{steps}) := v_{2stcc}(t, T_{2stpl\_}, V_{stc}, m2_{steps}) \ldots$$
$$+ \left[v_{stcc}[t - T_{2stpl\_} \cdot (4 \cdot m2_{steps} + 7), T_{2stpl\_}, V_{stc}, m2_{steps}] \ldots \right.$$
$$\left. + V_{stc} \cdot \text{rect1}[t - T_{2stpl\_} \cdot (4 \cdot m2_{steps} + 7), 0 \cdot T_{2stpl\_}, (2 \cdot m2_{steps} + 1) \cdot T_{2stpl\_} \right.$$

$$T_T := 2 \cdot T_{2stpl\_} \cdot (3 \cdot m2_{steps} + 4) + 5 \cdot T_{2stpl\_}$$

$$v_{HDoor}(t, T_{2stpl\_}, V_{stc}, m2_{steps}) := V_{stc} \cdot \text{rect1}\left(t, \frac{T_T - 3 \cdot T_{2stpl\_}}{2}, 3 \cdot T_{2stpl\_}\right)$$







$$\int_{5\cdot T_{2stpl\_}}^{2\cdot T_{2stpl\_}\cdot(3\cdot m2_{steps}+4)} v_H(t, T_{2stpl\_}, V_{stc}, m2_{steps})\, dt \;\ldots \qquad = 1.84 \cdot V \cdot ms$$

$$+ (-1) \cdot \int_{1\cdot T_{2stpl\_}+\frac{1\cdot(2\cdot m2_{steps}+1)\cdot T_{2stpl\_}}{6.6}}^{1\cdot T_{2stpl\_}+\frac{1\cdot(2\cdot m2_{steps}+1)\cdot T_{2stpl\_}}{6.6}+3\cdot T_{2stpl\_}} v_{HDoor}(t, T_{2stpl\_}, V_{stc}, m2_{steps})\, dt$$

Voltage Pulses

-8 Staircase 3 Voltage Pulse

Data in file "staircase 3 pulse data.xmcd"

Description of the Function's parameters: vstc1(t,step_length,signal_amplitude,number_of_steps)

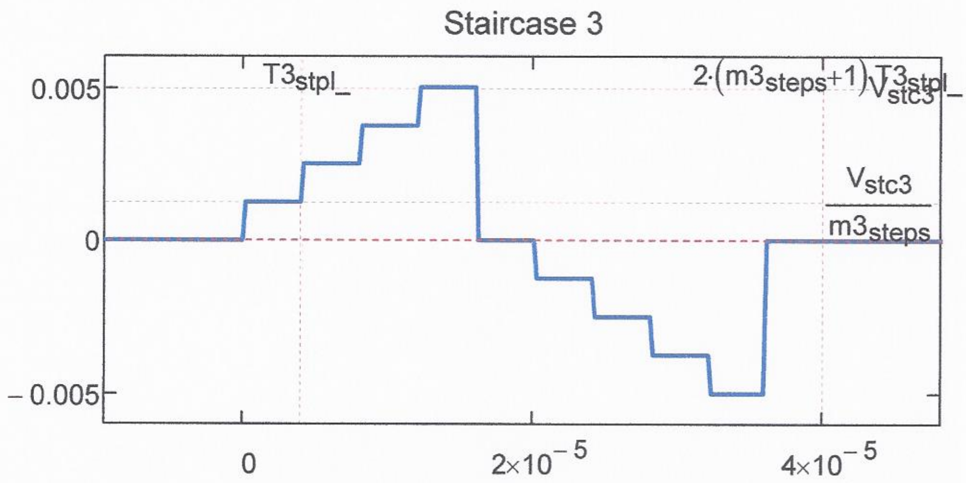
$m3_{steps} := 8$ $\quad V_{stc3} = 5 \times 10^{-3} \cdot V$

$m3_{steps} = 4$

$T3_{stpl\_} = 4 \cdot \mu s$

$$vstc1(t, T3_{stpl\_}, V_{stc3}, m3_{steps}) := v_{stc}(t, T3_{stpl\_}, V_{stc3}, m3_{steps}) \ldots + (-1) \cdot v_{stc}\left[t - (m3_{steps} + 1) \cdot T3_{stpl\_}, T3_{stpl\_}, V_{stc3}, m3_{steps}\right]$$

$m3_{steps} = 4$ $\quad t := -1 \cdot T3, -1 \cdot T3 + \dfrac{8 \cdot m3_{steps} \cdot T3 + 1 \cdot T3}{5000} .. 8 \cdot m3_{steps} \cdot T3$



$$v_{12}(t, T3_{stpl\_}, V_{stc3}, m3_{steps}) := \frac{vstc1(t, T3_{stpl\_}, V_{stc3}, m3_{steps})}{V}$$

Voltage Pulses

-9 Triangular Voltage Pulse

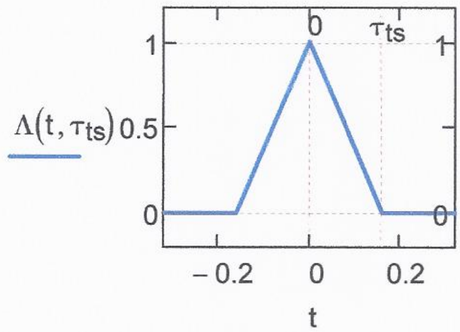
Data in file " global data.xmcd"

Definition of the Triangle function: $\Lambda(t,\tau_{ts}) := \begin{cases} 1-\left|\dfrac{t}{\tau_{ts}}\right| & \text{if } \left|\dfrac{t}{\tau_{ts}}\right| < 1 \\ 0 & \text{if } \left|\dfrac{t}{\tau_{ts}}\right| > 1 \end{cases}$ $\tau_{ts} = 1.592\times 10^{5}\cdot \mu s$

Altenative definition using the function rect(t) or ($\Pi(\tau_{ts})$) both defined in "Dirac Pulse - formulas.xmcd" :

$$\Lambda(t,\tau_{ts}) := \left(\int_{-\infty}^{\infty} \Pi(\tau_{ts})\cdot\Pi(t-\tau_{ts})\, d\tau_{ts}\right)$$

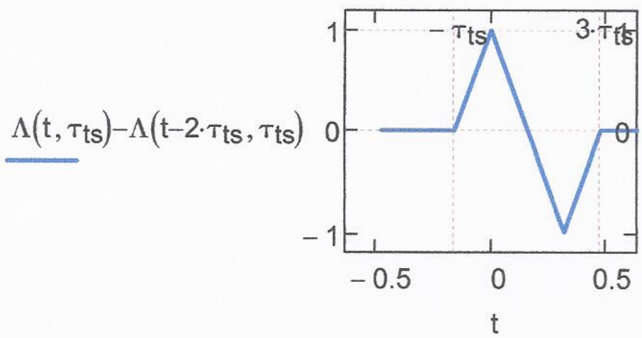
$$t := -\tau_{ts}\cdot 3, -\tau_{ts}\cdot 3 + \frac{20\cdot\tau_{ts}+\tau_{ts}\cdot 3}{40000} .. 20\cdot\tau_{ts} \qquad v_{14}(t) := \Lambda(t,\tau_{ts})$$



Voltage Pulses

-10 Bipolar Triangular Voltage Pulse

Data in file " global data.xmcd"



$$v_{15}(t) := \Lambda(t,\tau_{ts}) - \Lambda(t-2\cdot\tau_{ts},\tau_{ts})$$

TEST SIGNALS

Voltage Pulses

-11 Sawtooth Voltage Pulse with positive slope

Data in file " sawtooth pulse data.xmcd"

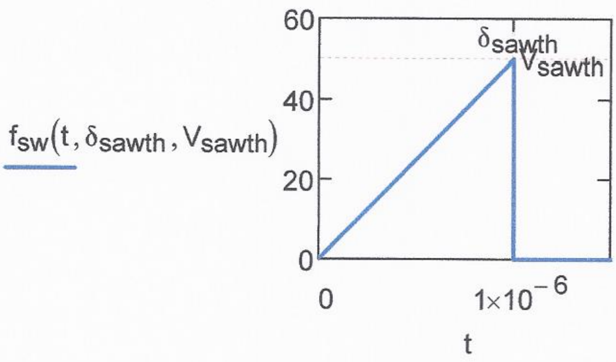
Signal amplitude: $V_{sawth} = 50 \cdot volt$

Slope: $p_{sawth} = 50 \cdot \frac{V}{\mu s}$

$$f_{sw}(t, \delta_{sawth}, V_{sawth}) := \frac{V_{sawth}}{\delta_{sawth}} \cdot t \cdot rect1(t, 0.0 \cdot \delta_{sawth}, \delta_{sawth})$$

$\alpha := atan\left(p_{sawth} \cdot \frac{sec}{volt}\right)$ $\alpha = 1.571$ $\delta_{sawth} = 1 \cdot \mu s$

$$t := -\delta_{sawth} \cdot 0, -\delta_{sawth} \cdot 0 + \frac{5 \cdot \delta_{sawth} + \delta_{sawth} \cdot 0}{5000} .. 5 \cdot \delta_{sawth}$$



TEST SIGNALS

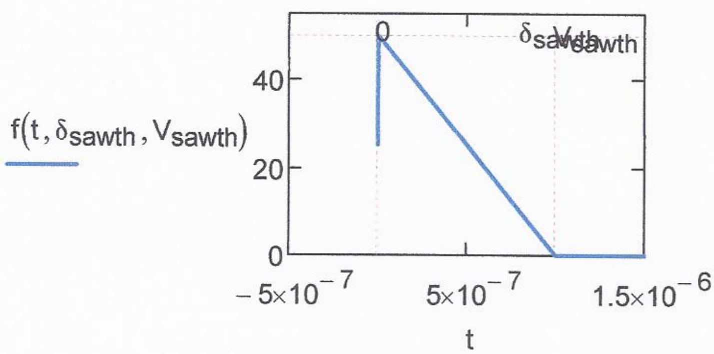
Voltage Pulses

-12 Sawtooth Voltage Pulse with negative slope

Data in file " sawtooth pulse data.xmcd"

$$f(t, \delta_{sawth}, V_{sawth}) := V_{sawth} \cdot \left(\frac{-t}{\delta_{sawth}} + 1\right) \cdot (\Phi(t) - \Phi(t - \delta_{sawth}))$$

$$f_{12} := \frac{1}{\delta_{sw}}$$



Voltage Pulses

13 Raised-Cosine (RC) Pulse

α is called the excess bandwidth factor since the bandwidth of this pulse is $Bw_{rc} = \dfrac{1+\alpha_{rc}}{2 \cdot T_{rc}}$

$$v_{rc}(t,\alpha_{rc},T_{rc}) := V_{pp} \cdot \mathrm{sinc}\left(\frac{\pi \cdot t}{T_{rc}}\right) \cdot \frac{\cos\left(\frac{\alpha_{rc} \cdot \pi \cdot t}{T_{rc}}\right)}{1.0 - \left(\frac{2 \cdot \alpha_{rc} \cdot t}{T_{rc}}\right)^2}$$

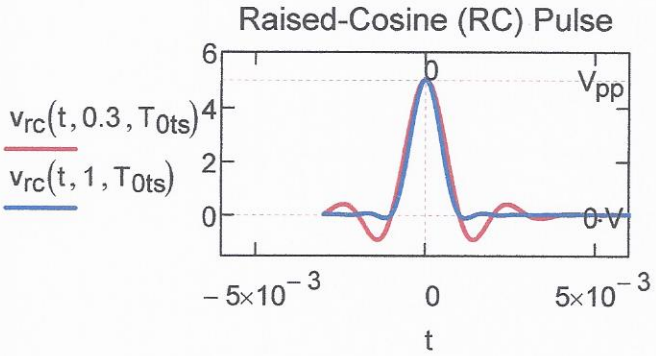
$$V_{rc}(\omega,\alpha_{rc},T_{rc}) := V_{pp} \cdot \begin{cases} 1.0 & \text{if } 0.0 \le |\omega| \le \frac{(1-\alpha_{rc}) \cdot \pi}{T_{rc}} \\ \cos\left[\frac{\pi \cdot T_{rc}}{2.0 \cdot \alpha_{rc}} \cdot \left(\left|\frac{\omega}{2 \cdot \pi}\right| - \frac{1-\alpha_{rc}}{2.0 \cdot T_{rc}}\right)\right]^2 & \text{if } \frac{(1-\alpha_{rc}) \cdot \pi}{T_{rc}} \le |\omega| \le \frac{(1+\alpha_{rc}) \cdot \pi}{T_{rc}} \\ 0.0 & \text{otherwise} \end{cases}$$

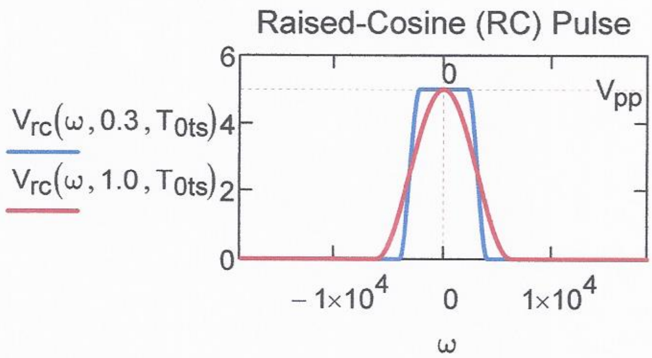
Example $\alpha_{rc} := 0.3$ $T_{0ts} = 1 \times 10^{6} \cdot ns$ $Bw_{rc} := \dfrac{1+\alpha_{rc}}{2 \cdot T_{0ts}}$ $Bw_{rc} = 6.5 \times 10^{-4} \cdot MHz$

$T_{rc} := T_{0ts}$

$$t := -T_{0ts} \cdot 3, -T_{0ts} \cdot 3 + \frac{10 \cdot T_{0ts} + T_{0ts} \cdot 3}{5000} .. 10 \cdot T_{0ts}$$

$$\omega := -\frac{6 \cdot \pi}{T_{0ts}}, -\frac{6 \cdot \pi}{T_{0ts}} + \frac{12 \cdot \pi}{T_{0ts} \cdot 5000} .. \frac{6 \cdot \pi}{T_{0ts}}$$





Adimensional function: $V_{rc}(t,\alpha_{rc},T_{rc}) := \dfrac{v_{rc}(t,\alpha_{rc},T_{rc})}{V}$

$$V_{rc}\left(\frac{T_{rc}}{3},\alpha_{rc},T_{rc}\right) = 4.096$$

Voltage Pulses

14 Root Raised-Cosine (RC) Pulse

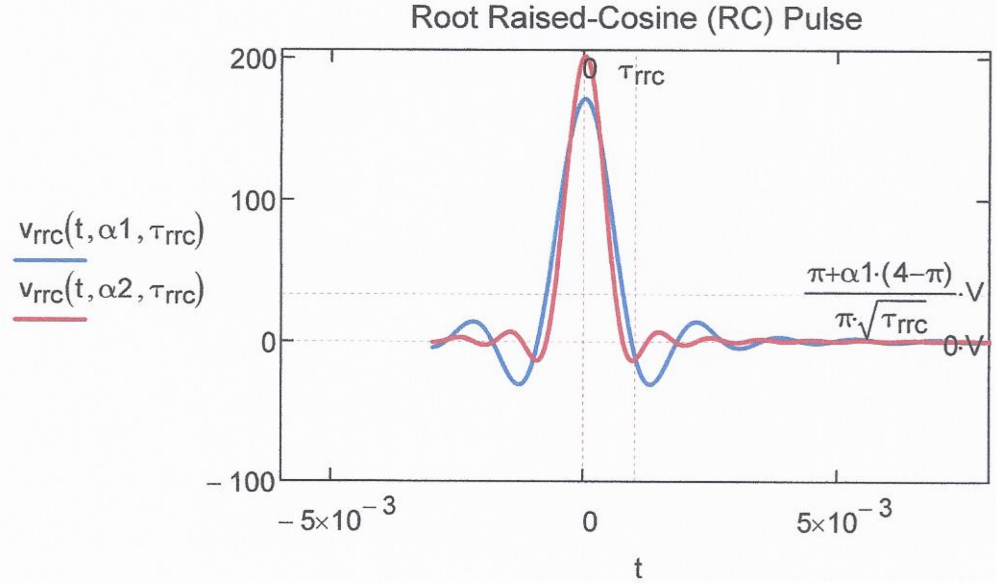
$$\tau_{rrc} := T_{0ts}$$

$$f1_{rrc}(t,\alpha_{rrc},\tau_{rrc}) := \frac{1}{\sqrt{\tau_{rrc}}}\cdot\frac{\sin\left[\pi\cdot(1-\alpha_{rrc})\cdot\frac{t}{\tau_{rrc}}\right]+\frac{4\cdot\alpha_{rrc}\cdot t}{\tau_{rrc}}\cdot\cos\left[\pi\cdot\frac{(1+\alpha_{rrc})\cdot t}{\tau_{rrc}}\right]}{\left(\frac{\pi\cdot t}{\tau_{rrc}}\right)\cdot\left[1-\left(\frac{4\cdot\alpha_{rrc}\cdot t}{\tau_{rrc}}\right)^2\right]}$$

$$f2_{rrc}(t,\alpha_{rrc},\tau_{rrc}) := \frac{\alpha_{rrc}}{\sqrt{2\cdot\tau_{rrc}}}\cdot\left[\left(1+\frac{2}{\pi}\right)\cdot\sin\left(\frac{\pi}{4\cdot\alpha_{rrc}}\right)+\left(1-\frac{2}{\pi}\right)\cdot\cos\left(\frac{\pi}{4\cdot\alpha_{rrc}}\right)\right]$$

$$v_{rrc}(t,\alpha_{rrc},\tau_{rrc}) := V_{pp}\cdot\left|\begin{array}{ll} f1_{rrc}(t,\alpha_{rrc},\tau_{rrc}) & \text{if } t\neq 0.0\wedge t\neq\frac{\tau_{rrc}}{4\cdot\alpha_{rrc}}\vee t\neq-\frac{\tau_{rrc}}{4\cdot\alpha_{rrc}} \\ \frac{1}{\sqrt{\tau_{rrc}}}\cdot\left(1-\alpha_{rrc}+\frac{4\cdot\alpha_{rrc}}{\pi}\right) & \text{if } t=0.0 \\ f2_{rrc}(t,\alpha_{rrc},\tau_{rrc}) & \text{if } \left(t=\frac{\tau_{rrc}}{4\cdot\alpha_{rrc}}\right)\vee\left(t=-\frac{\tau_{rrc}}{4\cdot\alpha_{rrc}}\right) \end{array}\right.$$

Example $\alpha1 := 0.3$ $\alpha2 := 1.0$ $\alpha_{rrc} := \alpha1$ $T_{rrc} := 6\cdot T_{0ts}$



The pulse is ortogonal: $\int_{-\infty}^{\infty} v_{rrc}(t,\alpha_{rrc},T)\cdot v_{rrc}(t-n\cdot T,\alpha_{rrc},T)\,dt = 0$

Adimensional function $Vrrc(t, \alpha_{rrc}, T_{rrc}) := v_{rrc}(t, \alpha_{rrc}, T_{rrc}) \cdot \frac{sec^{0.5}}{V}$ $\qquad f_{14} := \frac{1}{T_{rrc}}$

$$Vrrc\left(\frac{T_{rrc}}{3}, \alpha_{rrc}, T_{rrc}\right) = 55.205$$

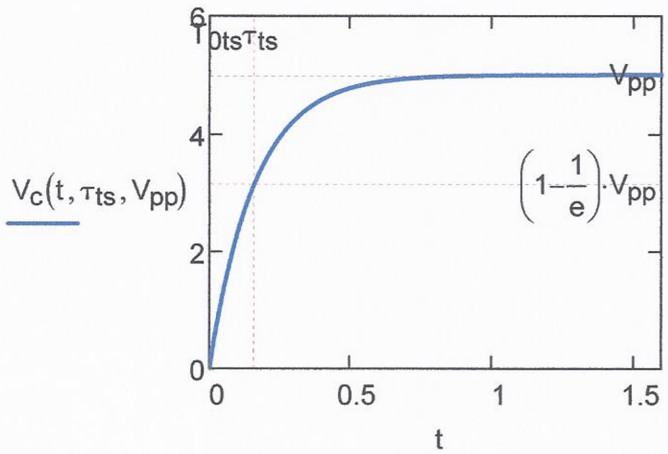
TEST SIGNALS

Voltage Pulses

-15 Rising Exponential Voltage Pulse

$$\tau_{ts} = 1.592 \times 10^{5} \cdot \mu s \qquad V_c(t,\tau_{ts},V_{pp}) := \left(1 - e^{\frac{-t}{\tau_{ts}}}\right) \cdot V_{pp} \cdot \Phi(t) \qquad V_{pp} = 5 \cdot V$$

$$t := -T_{0ts} \cdot 3, -T_{0ts} \cdot 3 + \frac{5000 \cdot T_{0ts} + T_{0ts} \cdot 3}{5000} .. 5000 \cdot T_{0ts}$$



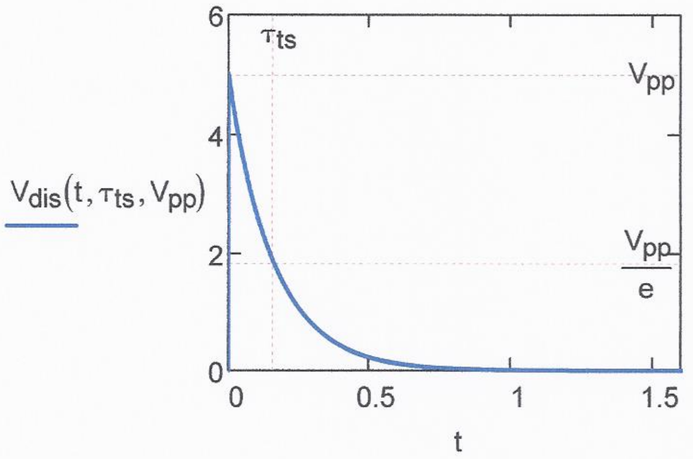
Adimensional function: $\quad V_{cad}(t,\tau_{ts},V_{pp}) := \dfrac{V_c(t,\tau_{ts},V_{pp})}{V}$

TEST SIGNALS

Voltage Pulses

-16 Decaying Exponential Voltage Pulse

$$\tau_{ts} = 1.592 \times 10^{5} \cdot \mu s \qquad V_{dis}(t,\tau_{ts},V_{pp}) := e^{\frac{-t}{\tau_{ts}}} \cdot V_{pp} \cdot \Phi(t) \qquad V_{pp} = 5 \cdot V$$



Adimensional function $V_{disad}(t,\tau_{ts},V_{pp}) := \dfrac{V_{dis}(t,\tau_{ts},V_{pp})}{V}$

TEST SIGNALS

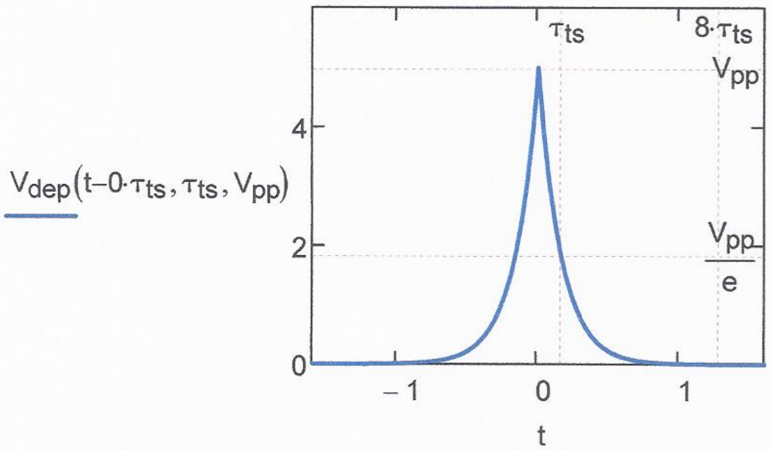
Voltage Pulses

- 17 Double Exponential Pulse

$$V_{dep}(t,\tau_{ts},V_{pp}) := \left| \begin{array}{l} \text{return "}\tau\text{.ts less or =0.0"} \ \ \text{if} \ \ \tau_{ts} \le 0 \\ \text{return "Vpp=0.0"} \ \ \text{if} \ \ V_{pp} = 0.0 \cdot V \ \ \text{otherwise} \\ e^{\frac{-t}{\tau_{ts}}} \cdot V_{pp} \ \ \text{if} \ \ t > 0.0 \\ V_{pp} \ \ \text{if} \ \ t = 0.0 \\ e^{\frac{t}{\tau_{ts}}} \cdot V_{pp} \ \ \text{if} \ \ t < 0.0 \end{array} \right.$$

$$V_{dep}(t,\tau_{ts},V_{pp}) := \left| \begin{array}{l} \text{return "}\tau\text{.ts less or =0.0"} \ \ \text{if} \ \ \tau_{ts} \le 0 \\ \text{return "Vpp=0.0"} \ \ \text{if} \ \ V_{pp} = 0.0 \cdot V \ \ \text{otherwise} \\ V_{pp} \cdot e^{\frac{-|t|}{\tau_{ts}}} \end{array} \right.$$

$$t := -200 \cdot \tau_{ts}, -200 \cdot \tau_{ts} + \frac{200 \cdot \tau_{ts} + 200 \cdot \tau_{ts}}{5000} \,..\, 200 \cdot \tau_{ts}$$



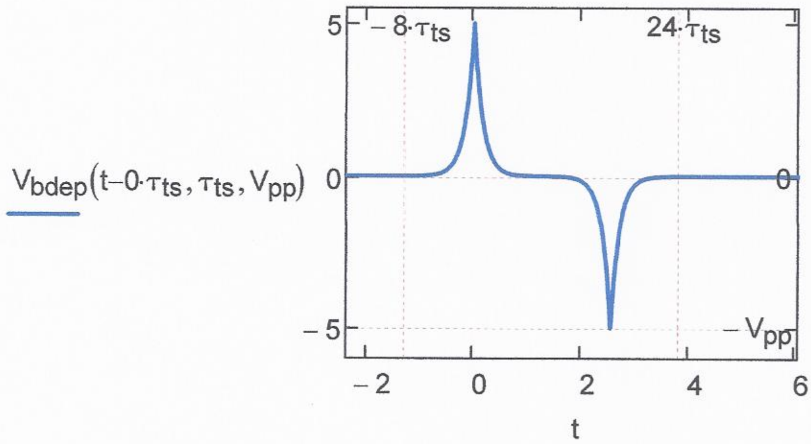
Adimensional function: $$V_{depad}(t, \tau_{ts}, V_{pp}) := \frac{V_{dep}(t, \tau_{ts}, V_{pp})}{V}$$

TEST SIGNALS

Voltage Pulses

- 18 Bipolar Double Exponential Pulse

$$V_{bdep}(t, \tau_{ts}, V_{pp}) := V_{dep}(t, \tau_{ts}, V_{pp}) \cdot \text{rect1}(t, -8 \cdot \tau_{ts}, 16 \cdot \tau_{ts}) \; ... \\ + (-1) \cdot V_{dep}(t - 16 \cdot \tau_{ts}, \tau_{ts}, V_{pp}) \cdot \text{rect1}(t, 8 \cdot \tau_{ts}, 16 \cdot \tau_{ts})$$

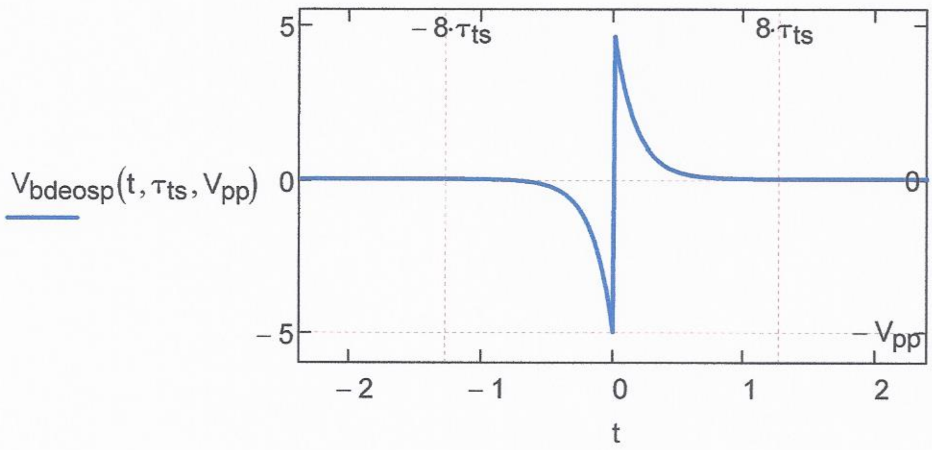


Adimensional function: $$V_{bdepad}(t, \tau_{ts}, V_{pp}) := \frac{V_{bdep}(t, \tau_{ts}, V_{pp})}{V}$$

TEST SIGNALS

- 19 Bipolar Double Exponential Odd symmetrc Pulse

$$V_{bdeosp}(t,\tau_{ts},V_{pp}) := \begin{array}{|l} \text{return "}\tau\text{.ts less or =0.0"} \quad \text{if} \quad \tau_{ts} \le 0 \\ \text{return "Vpp=0.0"} \quad \text{if} \quad V_{pp} = 0.0 \cdot V \quad \text{otherwise} \\ e^{\frac{-t}{\tau_{ts}}} \cdot V_{pp} \quad \text{if} \quad t > 0.0 \\ 0 \cdot V_{pp} \quad \text{if} \quad t = 0.0 \\ -e^{\frac{t}{\tau_{ts}}} \cdot V_{pp} \quad \text{if} \quad t < 0.0 \end{array}$$

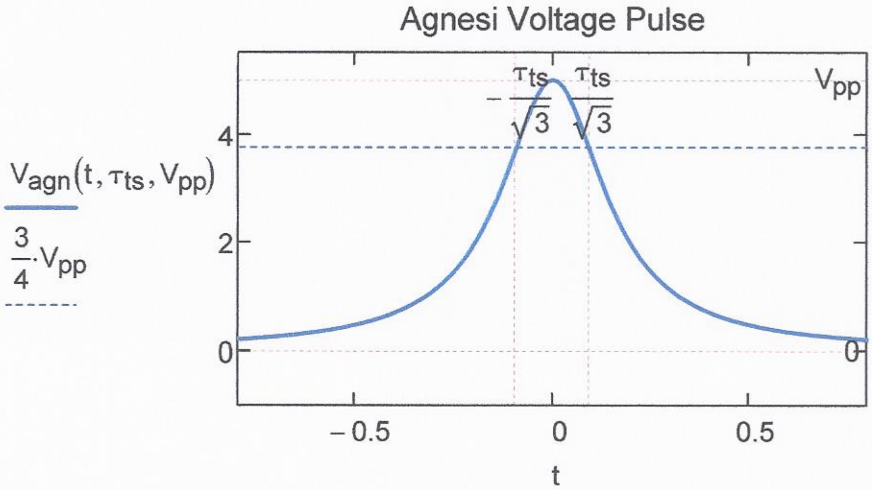


Adimensional function: $V_{bdeospad}(t,\tau_{ts},V_{pp}) := \frac{V_{bdeosp}(t,\tau_{ts},V_{pp})}{V}$

Voltage Pulses

- 20 Agnesi Profile Voltage Pulse

$$V_{agn}(t,\tau_{ts},V_{pp}) := \frac{V_{pp}}{\tau_{ts}} \cdot \frac{\tau_{ts}^3}{t^2+\tau_{ts}^2}$$

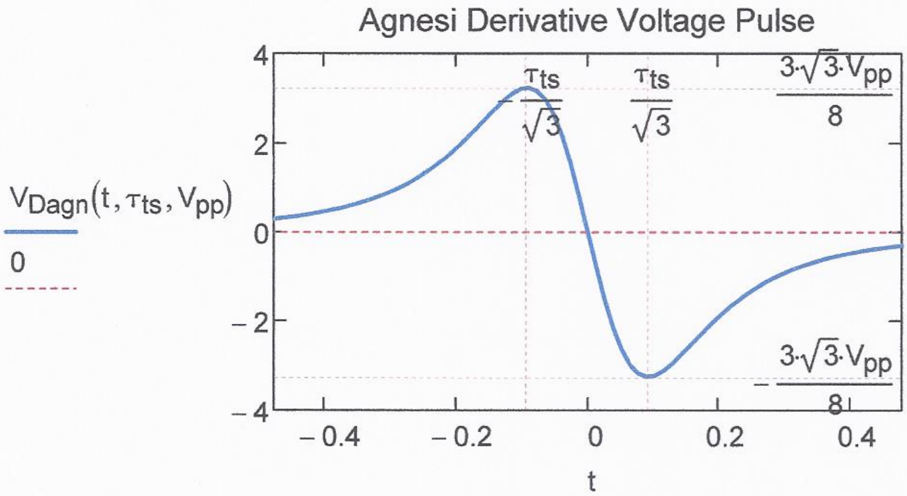


Adimensional function: $V_{agnad}(t,\tau_{ts},V_{pp}) := \frac{V_{agn}(t,\tau_{ts},V_{pp})}{V}$

Voltage Pulses

- 21 Agnesi Derivative Voltage Pulse

$$V_{Dagn}(t,\tau_{ts},V_{pp}) := -\tau_{ts} \cdot \frac{2 \cdot V_{pp} \cdot t \cdot \tau_{ts}^2}{\left(t^2+\tau_{ts}^2\right)^2}$$



$V_{pp} = 5 \cdot V$ $\quad \frac{V_{pp}}{\tau_{ts}} = 31.416 \cdot \frac{V}{sec}$ $\quad \frac{V_{pp}}{e} = 1.839 \cdot V$

Adimensional function: $V_{Dagnad}(t,\tau_{ts},V_{pp}) := \frac{V_{Dagn}(t,\tau_{ts},V_{pp})}{V}$

TEST SIGNALS

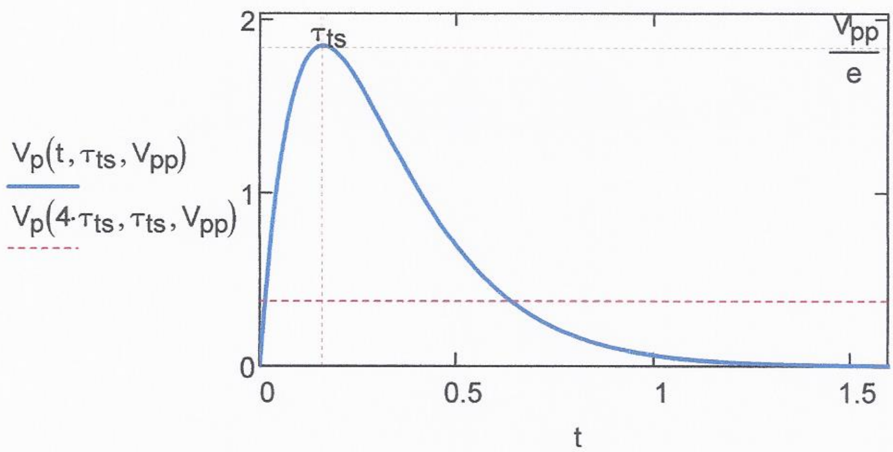
Voltage Pulses

- 22 Poisson Profile Voltage Pulse

$$t := 0 \cdot \tau_{ts}, 0 \cdot \tau_{ts} + \frac{200 \cdot \tau_{ts}}{5000} .. 200 \cdot \tau_{ts}$$

$\tau_{ts} = 0.159\,s$

$$V_p(t,\tau_{ts},V_{pp}) := \frac{V_{pp}}{\tau_{ts}} \cdot t \cdot e^{\frac{-t}{\tau_{ts}}}$$



Adimensional function: $V_{pad}(t, \tau_{ts}, V_{pp}) := \dfrac{V_p(t, \tau_{ts}, V_{pp})}{V}$

TEST SIGNALS

Voltage Pulses

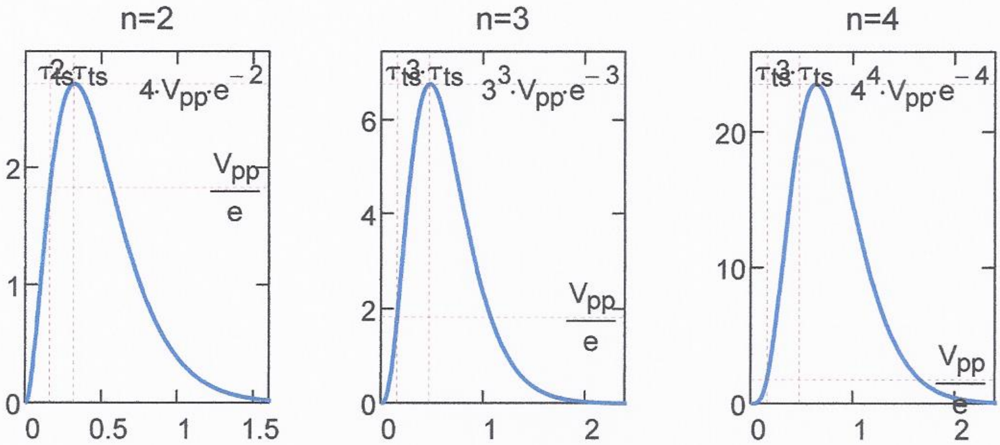
- 23 n^{th} Poisson   Profile Voltage Pulse

$V_{pp} := V_{pp} \quad \tau_{ts} := \tau_{ts} \quad t := t$

$n := 2 \qquad maxy := V_{pp} \cdot e^{-n} \cdot n^n \qquad maxy = 2.707 \cdot V \quad maxx := n \cdot \tau_{ts}$

$$V_{2p}(t, \tau_{ts}, V_{pp}) := \frac{V_{pp}}{\tau_{ts}^{2}} \cdot t^2 \cdot e^{\frac{-t}{\tau_{ts}}} \qquad V_{3p}(t, \tau_{ts}, V_{pp}) := \frac{V_{pp}}{\tau_{ts}^{3}} \cdot t^3 \cdot e^{\frac{-t}{\tau_{ts}}} \qquad V_{4p}(t, \tau_{ts}, V_{pp}) := \frac{V_{pp}}{\tau_{ts}^{4}} \cdot t^4 \cdot e^{\frac{-t}{\tau_{ts}}}$$

$$t := 0 \cdot \tau_{ts}, 0 \cdot \tau_{ts} + \frac{200 \cdot \tau_{ts}}{5000} .. 200 \cdot \tau_{ts}$$



Adimensional function: $V_{2pad}(t, \tau_{ts}, V_{pp}) := \dfrac{V_{2p}(t, \tau_{ts}, V_{pp})}{V}$

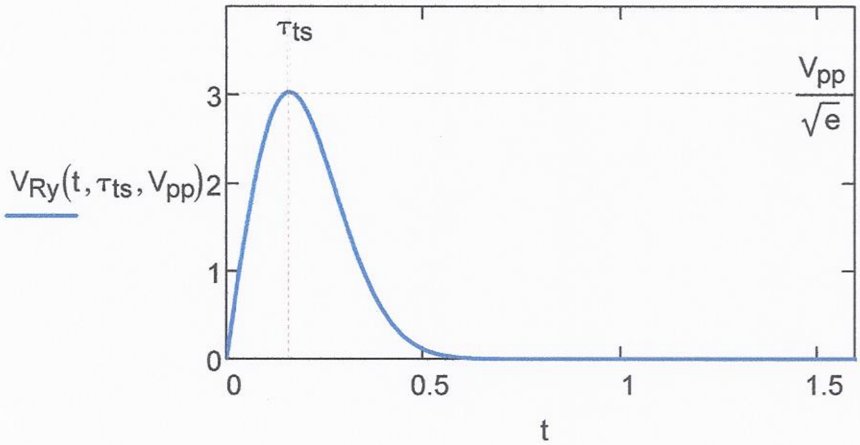
TEST SIGNALS

Voltage Pulses

- 24 Rayleigh Profile Voltage Pulse

$$V_{Ry}(t,\tau_{ts},V_{pp}) := \frac{V_{pp}}{\tau_{ts}}\cdot t\cdot e^{\frac{-t^2}{2\cdot\tau_{ts}^2}}$$

$$t := 0\cdot\tau_{ts}, 0\cdot\tau_{ts} + \frac{200\cdot\tau_{ts}}{5000} .. 200\cdot\tau_{ts}$$



TEST SIGNALS

Periodic Signals Periodic Signals

1 Half wave

Data in file " global data.xmcd"

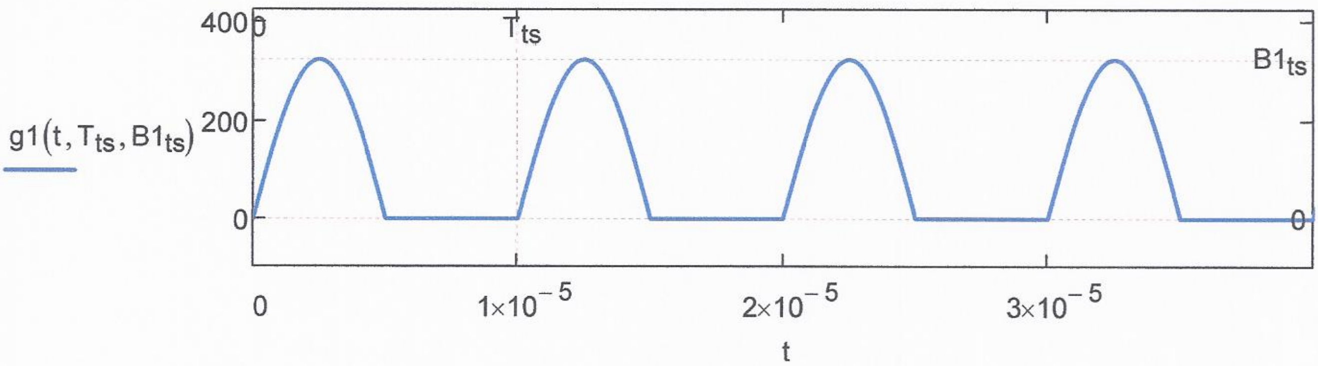
Amplitude: $B1_{ts} := 230 \cdot \sqrt{2} \cdot V$

Period: $T_{ts} := 10 \cdot \mu s$ Angular frequency: $\omega_{ts} := \dfrac{2 \cdot \pi}{T_{ts}}$

$$t := -T_{ts} \cdot 1, -T_{ts} \cdot 1 + \frac{8 \cdot T_{ts} + T_{ts} \cdot 1}{5000} .. 8 \cdot T_{ts}$$

$$g1(t, T_{ts}, B1_{ts}) := \frac{B1_{ts}}{V} \cdot \sum_{k=0}^{N1} \left(\mathrm{rect1}\left(t - k \cdot T_{ts}, -1 \cdot T_{ts}, \frac{T_{ts}}{2}\right) \cdot \sin\left(\frac{2 \cdot \pi}{T_{ts}} \cdot t\right)\right)$$

Half wave



TEST SIGNALS

Periodic Signals

2 Half wave filtered (Capacitive)

Max half wave amplitude: $B1_{ts} = 325.269 \cdot V$,
Amplitude of the decresing exponential fo t=0: V_{pp},
Exponential Time constant: $\tau_{tsf} := 2 \cdot T_{ts}$,
Period: $T_{ts} = 10 \cdot \mu s$,
Pulsation: $\omega_{ts} = 0.628 \cdot \dfrac{Mrads}{sec}$,
Intersection abscissa between half wave and exponential: ζ (scalar),
Tangent points abscissas between half wave and exponential: τ0 (vector)

Tangent point abscissa: $\tau 0_k := \dfrac{\operatorname{atan}(-\omega_{ts} \cdot \tau_{tsf}) + k \cdot \pi}{\omega_{ts}}$ $\qquad \tau 0_1 = 2.626 \times 10^{-6}\,s$

$\tau 0^T =$

	0	1	2	3	4
0	$-2.374 \cdot 10^{-6}$	$2.626 \cdot 10^{-6}$	$7.626 \cdot 10^{-6}$	$1.263 \cdot 10^{-5}$	...

s

Tangent point voltage: $V_{tpp} := B1_{ts} \cdot \sin(\omega_{ts} \cdot \tau 0_1) \cdot e^{\frac{\tau 0_1}{\tau_{tsf}}}$ $\qquad V_{tpp} = 369.746 \cdot V$

$$Z0(\tau_{tsf}, \omega_{ts}, B1_{ts}, V_{tpp}) := \left| \begin{array}{l} \xi \leftarrow \dfrac{2 \cdot \pi}{\omega_{ts} \cdot 1} \\ f(\xi) \leftarrow B1_{ts} \cdot \sin(\omega_{ts} \cdot \xi) \\ g(\xi) \leftarrow e^{\frac{-\xi}{\tau_{tsf}}} \cdot V_{tpp} \\ S \leftarrow \operatorname{root}(f(\xi) - g(\xi), \xi) \\ \text{return } S \end{array} \right.$$

Intersection abscissa: $\qquad \zeta := Z0(\tau_{tsf}, \omega_{ts}, B1_{ts}, V_{tpp})$

$\tau 0_1 = 2.626 \cdot \mu s \qquad \zeta = 11.13 \cdot \mu s$

$$g02(t, \tau_{tsf}, \tau 0, \zeta, \omega_{ts}, B1_{ts}, V_{tpp}) := \left| \begin{array}{l} Y \leftarrow \dfrac{B1_{ts}}{V} \cdot \sin(\omega_{ts} \cdot t) \ \text{ if } \ 0 \le t \le \tau 0_1 \\ \text{otherwise} \\ \quad \left| \begin{array}{l} Y \leftarrow e^{\frac{-t}{\tau_{tsf}}} \cdot \dfrac{V_{tpp}}{V} \ \text{ if } \ \tau 0_1 < t < \zeta \\ Y \leftarrow \dfrac{B1_{ts}}{V} \cdot \sin(\omega_{ts} \cdot \zeta) \ \text{ if } \ t = \zeta \ \text{ otherwise} \end{array} \right. \\ \text{return } Y \end{array} \right.$$

$$g03(t,\tau_{tsf},\tau 0,\zeta,\omega_{ts},B1_{ts},V_{tpp}) := \left| \begin{array}{l} X \leftarrow \frac{B1_{ts}}{V}\cdot\sin(\omega_{ts}\cdot t) \ \text{ if } \ \zeta \le t \le \tau 03 \\ X \leftarrow e^{\frac{-\left(t-\frac{2\cdot\pi}{\omega_{ts}}\right)}{\tau_{tsf}}}\cdot\frac{V_{tpp}}{V} \ \text{ if } \ \tau 03 < t \le \zeta + \frac{2\cdot\pi}{\omega_{ts}} \ \text{ otherwise} \\ \text{return } X \end{array} \right.$$

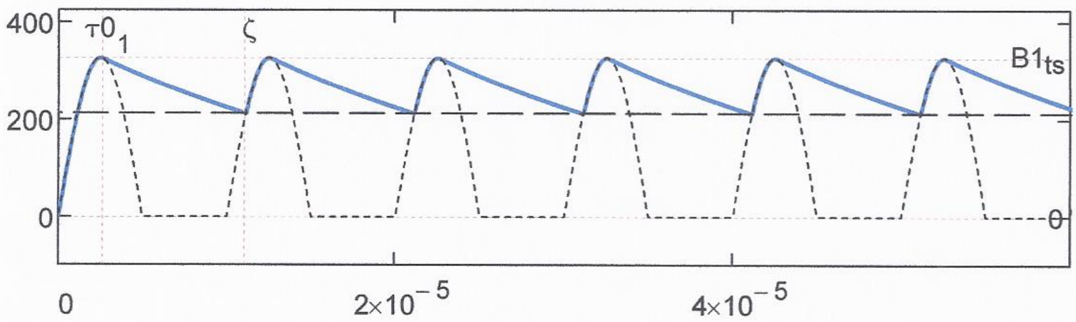
$$rip0 := \frac{\frac{B1_{ts}}{V} - \frac{B1_{ts}}{V}\cdot\sin(\omega_{ts}\cdot\zeta)}{\frac{B1_{ts}}{V}}$$

$$g2(t,\tau_{tsf},\tau 0,\zeta,\omega_{ts},B1_{ts},V_{tpp}) := g02(t,\tau_{tsf},\tau 0,\zeta,\omega_{ts},B1_{ts},V_{tpp})\cdot rect1\left(t,0\cdot\frac{2\cdot\pi}{\omega_{ts}},\zeta\right) \ \ldots$$
$$+ \sum_{k=0}^{N1}\left(g03\left(t-k\cdot\frac{2\cdot\pi}{\omega_{ts}},\tau_{tsf},\tau 0,\zeta,\omega_{ts},B1_{ts},V_{tpp}\right)\cdot rect1\left(t,\zeta+k\cdot\frac{2\cdot\pi}{\omega_{ts}},\right.\right.$$

$\tau 0_1 = 2.626\cdot\mu s$ $\quad\zeta = 11.13\cdot\mu s$ $\quad$ Ripple: $rip0 = 34.839\cdot\%$ $\quad 8\cdot T_{ts} = 80\cdot\mu s$

Increase the time constant to reduce the ripple.

Ripple

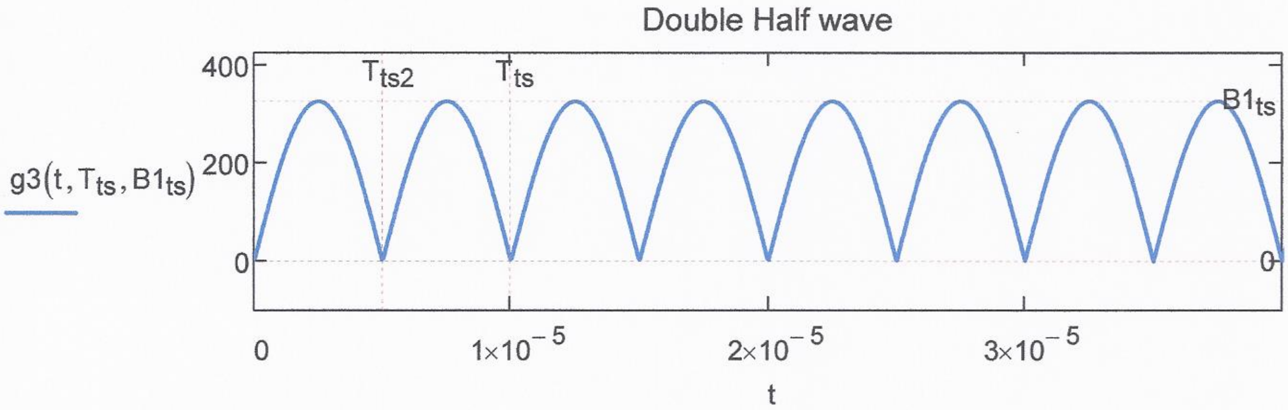


Periodic Signals

3 Double Half wave

$$T_{ts2} := \frac{T_{ts}}{2} \qquad \omega_{ts2} := \frac{\pi}{T_{ts2}} \qquad g3(t, T_{ts}, B1_{ts}) := \frac{B1_{ts}}{V} \cdot \left|\sin\left(\frac{2\cdot\pi}{T_{ts}}\cdot t\right)\right|$$

$$t := -T_{ts}\cdot 1, -T_{ts}\cdot 1 + \frac{8\cdot T_{ts} + T_{ts}\cdot 1}{2000} \,..\, 8\cdot T_{ts}$$



TEST SIGNALS

Periodic Signals

4 Double Half wave filtered (Capacitive)

Max half wave amplitude: $B1_{ts} = 325.269 \cdot V$,

Amplitude of the decresing exponential fo t=0: $V_{pp} = 5 \cdot V$,

Exponential Time constant: $\tau_{tsf} := 40 \cdot \mu s$,

Period: $\frac{T_{ts}}{2} = 5 \cdot \mu s$,

Pulsation: $\omega_{ts} = 0.628 \cdot \frac{Mrads}{sec}$,

Intersections between half wave and exponential: τ0 (vector),
Tangent points between half wave and exponential: θ (scalar)

Tangent point abscissa: $\tau0_k := \dfrac{\operatorname{atan}(-\omega_{ts}\cdot\tau_{tsf}) + k\cdot\pi}{\omega_{ts}}$

$$V_{ppt} := B1_{ts}\cdot\sin(\omega_{ts}\cdot\tau0_1)\cdot e^{\frac{\tau0_1}{\tau_{tsf}}} \qquad V_{pp} = 5\cdot V$$

Intersection

$$Z1(\tau_{tsf}, \omega_{ts}, B1_{ts}, V_{ppt}) := \left|\begin{array}{l} \xi \leftarrow \dfrac{\pi}{\omega_{ts}} \\ f(\xi) \leftarrow B1_{ts}\cdot\left|\sin(\omega_{ts}\cdot\xi)\right| \\ g(\xi) \leftarrow e^{\frac{-\xi}{\tau_{tsf}}}\cdot V_{ppt} \\ S \leftarrow \operatorname{root}(f(\xi) - g(\xi), \xi) \\ \text{return } S \end{array}\right.$$

$$\theta := Z1(\tau_{tsf}, \omega_{ts}, B1_{ts}, V_{ppt})$$

$$\tau0_1 = 2.563\cdot\mu s \qquad \tau0_2 = 7.563\cdot\mu s \qquad \theta = 6.779\cdot\mu s$$

$$h02\_(t, \tau_{ts}, \tau0, \theta, \omega_{ts}, B1_{ts}, V_{ppt}) := \left|\begin{array}{l} Y \leftarrow \dfrac{B1_{ts}}{V}\cdot\left|\sin(\omega_{ts}\cdot t)\right| \quad \text{if } \ 0 \le t \le \tau0_1 \\ \text{otherwise} \\ \quad \left|\begin{array}{l} Y \leftarrow e^{\frac{-t}{\tau_{ts}}}\cdot\dfrac{V_{ppt}}{V} \quad \text{if } \ \tau0_1 < t < \theta \\ Y \leftarrow \dfrac{B1_{ts}}{V}\cdot\left|\sin(\omega_{ts}\cdot\theta)\right| \quad \text{if } \ t = \theta \ \ \text{otherwise} \end{array}\right. \\ \text{return } Y \end{array}\right.$$

$$h03\_(t, \tau_{ts}, \tau0, \theta, \omega_{ts}, B1_{ts}, V_{ppt}) := \left|\begin{array}{l} X \leftarrow \dfrac{B1_{ts}}{V}\cdot\left|\sin(\omega_{ts}\cdot t)\right| \quad \text{if } \ \theta \le t \le \tau0_2 \\ X \leftarrow e^{\frac{-\left(t - \frac{\pi}{\omega_{ts}}\right)}{\tau_{ts}}}\cdot\dfrac{V_{ppt}}{V} \quad \text{if } \ \tau0_2 < t \le \theta + \dfrac{\pi}{\omega_{ts}} \ \ \text{otherwise} \\ \text{return } X \end{array}\right.$$

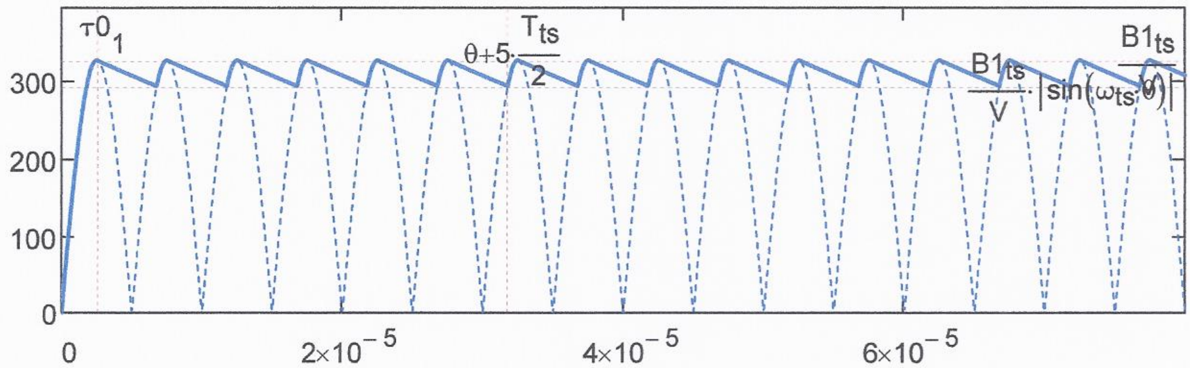
$$rip1 := \frac{\frac{B1_{ts}}{V} - \frac{B1_{ts}}{V} \cdot \left|\sin(\omega_{ts} \cdot \theta)\right|}{\frac{B1_{ts}}{V}}$$

$$g4(t, \tau_{tsf}, \tau 0, \theta, \omega_{ts}, B1_{ts}, V_{ppt}) := h02\_(t, \tau_{tsf}, \tau 0, \theta, \omega_{ts}, B1_{ts}, V_{ppt}) \cdot rect1\left(t, 0 \cdot \frac{\pi}{\omega_{ts}}, \theta\right) \ldots$$
$$+ \sum_{k=0}^{N1} \left(h03\_\left(t - k \cdot \frac{\pi}{\omega_{ts}}, \tau_{tsf}, \tau 0, \theta, \omega_{ts}, B1_{ts}, V_{ppt}\right) \cdot rect1\left(t, \theta + k \cdot \frac{\pi}{\omega_{ts}}, \frac{\pi}{\omega}\right.\right.$$

$N1 = 50$ $\tau 0_1 = 2.563 \cdot \mu s$ $\zeta = 11.13 \cdot \mu s$ Ripple: $rip1 = 10.075 \cdot \%$

Increase the time constant to reduce the ripple amplitude.

Ripple



Periodic Signals

5 Voltage Pulse Train

Data in file " pulse train data.xmcd"

Pulse period:	$T_{ptd} = 1 \times 10^{3} \cdot \mu s$		
Pulse Cadence:	$f_{ptd} = 1 \times 10^{-3} \cdot MHz$		
Pulse width:	$\tau_{ptd} = 250 \cdot \mu s$		
Duty Cycle:	$\delta_{ptd} = 0.25$	$\tau_{ptd} = T_{ptd} \cdot \delta_{ptd}$	$T_{ptd} = \dfrac{\tau_{ptd}}{\delta_{ptd}}$
Amplitude:	$B_{ptd} = 5 \cdot volt$		
Pulse delay from the origin	$\tau_{ptd\delta0} = -250 \cdot \mu s$		
Average value:	$v_{ptm} := B_{ptd} \cdot \delta_{ptd} = 1.25 \cdot volt$		

Generic pulse definition: rect1 (t , risingedge , width)

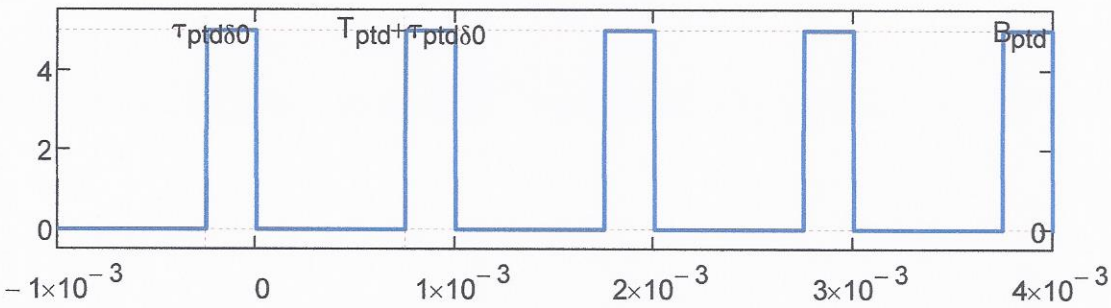
$$v_{ptts}(t, T_{ptd}, \tau_{\delta 0}, \delta_{ts}, B_{ptd}) := B_{ptd} \cdot \sum_{k=0}^{N1} \text{rect1}(t - k \cdot T_{ptd}, \tau_{\delta 0}, T_{ptd} \cdot \delta_{ptd})$$

$$Vip1(t, \tau_{ptd}, \tau_{\delta 0}, \delta_{ts}, B2_{ts}) := \frac{v_{ptts}(t, \tau_{ptd}, \tau_{\delta 0}, \delta_{ts}, B2_{ts})}{volt}$$

$k1\_ := 0 .. N1$ $\qquad \tau_{ptd\delta 0} = -250 \cdot \mu s$

$$t := -T_{ptd} \cdot 1, -T_{ptd} \cdot 1 + \frac{8 \cdot T_{ptd} + T_{ptd} \cdot 1}{5000} .. 8 \cdot T_{ptd}$$

Voltage Pulse Train



Function definition: Vip1(time, Period, Pulse delay from the origin or rising edge, Duty Cycle, pulse amplitude

Periodic Signals

6 RF Pulse Train

Data in file " rf pulse data.xmcd"

Generic pulse definition: $\mathrm{rect1}(t, \mathrm{risingedge}, \mathrm{width}) := \mathrm{rect1}(t, \tau_{\delta rf}, \tau_{ptd})$

Period: $T_{ptd} = 1 \times 10^{3} \cdot \mu s$

Pulse Cadence: $f_{ptd} = 1 \times 10^{-3} \cdot MHz$

Pulse width: $\tau_{ptd} = 250 \cdot \mu s$

Duty Cycle: $\delta_{ptd} = 0.25$ $\quad \tau_{ptd} = T_{ptd} \cdot \delta_{ptd}$

Amplitude: $B_{ptd} = 5 \cdot volt$

Pulse delay from the origin $\tau_{ptd\delta 0} = -250 \cdot \mu s$

Average value: $v_{rfptm} := B_{ptd} \cdot \delta_{ptd} = 1.25 \cdot volt$

$\omega_{rfpt} = 4.398 \times 10^{-4} \cdot \dfrac{Grads}{sec}$ $\quad f_{rfpt}(t, \tau_{\delta rf}, \tau_{ptd}, \omega_{rfpt}) := \mathrm{rect1}(t, \tau_{\delta rf}, \tau_{ptd}) \cdot \cos(\omega_{rfpt} \cdot t)$

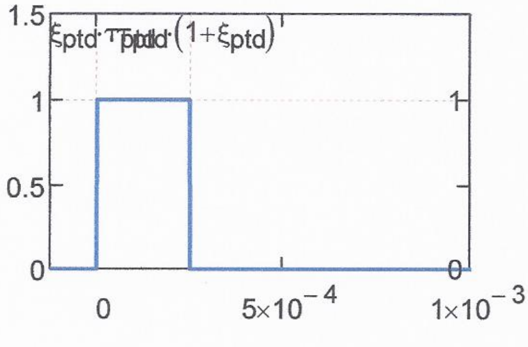
$T_{rf} = 1.429 \times 10^{4} \cdot ns$ $\quad t := -1 \cdot \tau_{ptd}, -1 \cdot \tau_{ptd} + \dfrac{4 \cdot T_{ptd} + \tau_{ptd}}{5000} .. 4 \cdot T_{ptd}$ $\quad \tau_{rf} = 2.274 \times 10^{4} \cdot ns$

$\tau_{\delta rf} = 0$

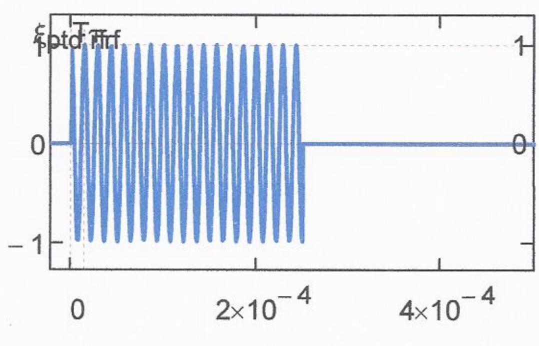
$\xi_{ptd} = 0$ $\quad \xi_{ptd} \cdot \tau_{rf} = 0 \cdot ns$ $\quad -\tau_{rf} \cdot (1 - \xi_{ptd}) = -2.274 \times 10^{4} \cdot ns$

$f_{rf} = 7 \times 10^{-5} \cdot GHz$

Generic Pulse



Generic RF Pulse



(v_{ptrfs} parameters: v_{pt}(time, period, pulse_width, duty_cycle, pulse_amplitude))

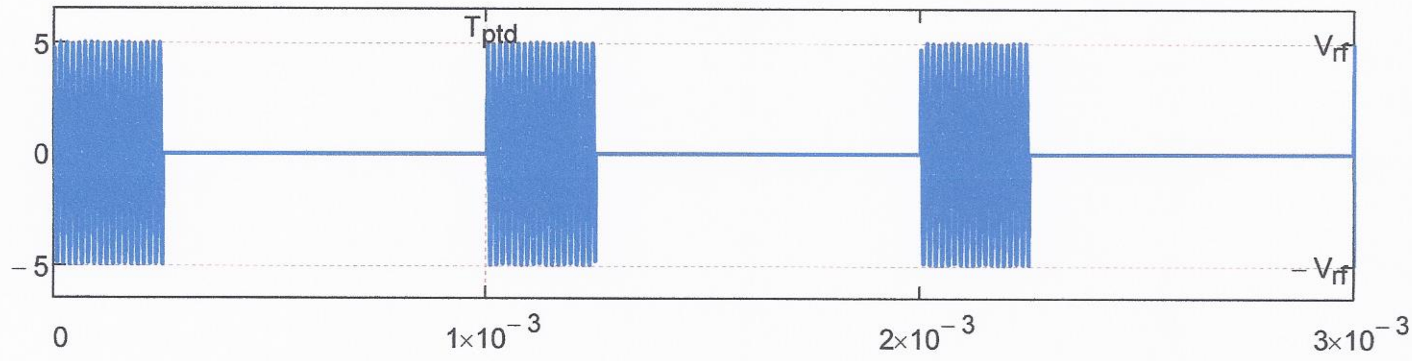
$$v_{ptrf}(t, T_{ptd}, \tau_{\delta rf}, \delta_{ptd}, \omega_{rfpt}, V_{rf}) := V_{rf} \cdot \sum_{k=0}^{N1} f_{rfpt}(t - k \cdot T_{ptd}, \tau_{\delta rf}, T_{ptd} \cdot \delta_{ptd}, \omega_{rfpt})$$

Average value: $v_{ptmrf} := V_{rf} \cdot \delta_{ptd}$ $\quad v_{ptmrf} = 1.25 \cdot volt$ $\quad V_{rf} = 5 \cdot volt$

$\xi_{ptd} \cdot \tau_{rf} = 0 \cdot ns \quad Vi_{pt}(t, T_{ptd}, \tau_{\delta rf}, \delta_{ptd}, \omega_{rfpt}, V_{rf}) := \dfrac{v_{ptrf}(t, T_{ptd}, \tau_{\delta rf}, \delta_{ptd}, \omega_{rfpt}, V_{rf})}{volt} \qquad \delta_{ptd} = 25 \cdot \%$

(v_{ptrfs} parameters: Vi_{pt} (time, period, pulse_width, duty_cycle, pulse_amplitude))

Generic RF Pulse train



Periodic Signals

7 Bipolar Square Wave

Data in file " pulse train data.xmcd"

Signal amplitude: $V_{pp} = 5 \cdot V$

Time constant: $\tau_{ts} = 1.592 \times 10^{8} \cdot ns$

Square wave period: $T_{0ts} = 1 \times 10^{6} \cdot ns$

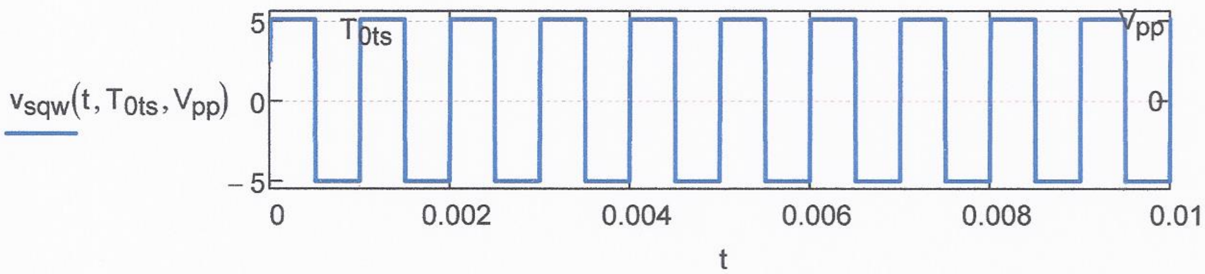
$\omega_{0ts} = 6.283 \times 10^{-6} \cdot \frac{Grads}{sec}$

Test signal: $v_{sqw}(t, T_{0ts}, V_{pp}) := V_{pp} \cdot \sum_{k=0}^{N1} \left[\left[\Phi(t - k \cdot T_{0ts}) - 2 \cdot \Phi\left[t - \left(\frac{2 \cdot k + 1}{2}\right) \cdot T_{0ts}\right] \right] \cdots + \Phi[t - (k+1) \cdot T_{0ts}] \right]$

$N = 50$ $Vsqw(t, T_{0ts}, V_{pp}) := \frac{v_{sqw}(t, T_{0ts}, V_{pp})}{volt}$

$t := 0 \cdot T_{0ts}, 0 \cdot T_{0ts} + \frac{20 \cdot T_{0ts}}{5000} .. 20 \cdot T_{0ts}$

Bipolar Square Wave



Periodic Signals

8 Bipolar Square Wave 1

Data in file " pulse train data.xmcd"

Period: $T_{ptd} := \tau_\delta + 4 \cdot \tau_{pwts}$ $T_{ptd} = 1 \times 10^3 \cdot \mu s$

Pulse Cadence: $f_{ptd} = 1 \times 10^{-3} \cdot MHz$

Pulse width: $\tau_{pwts} :=$ $\tau_{pwts} = 250 \cdot \mu s$

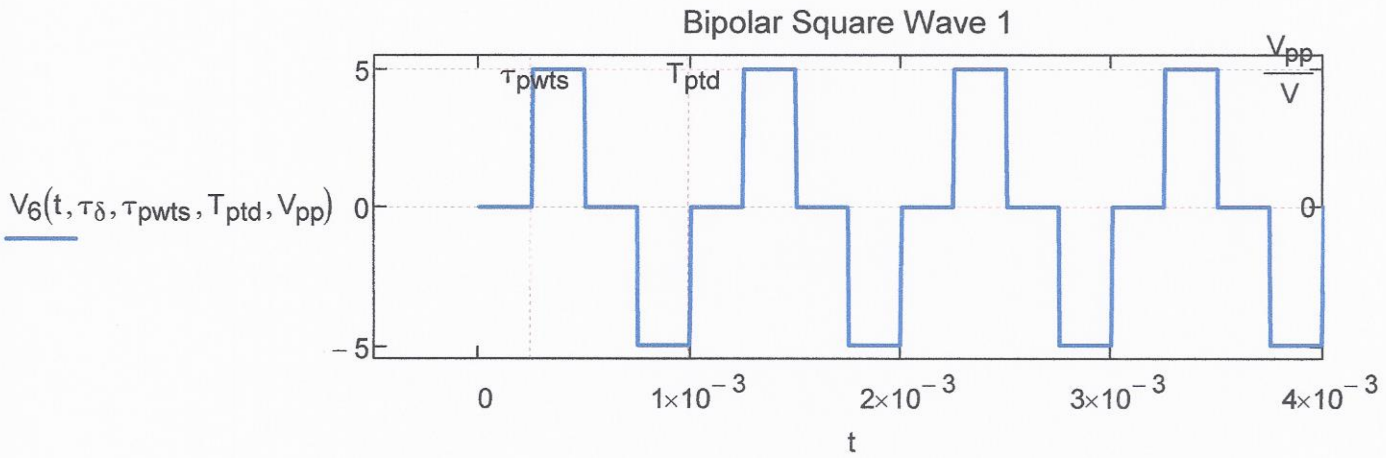
Amplitude: $V_{pp} = 5 \cdot volt$

Pulse delay from the origin: $\tau_\delta = 0 \cdot \mu s$

$$V_6(t, \tau_\delta, \tau_{pwts}, T_6, V_{pp}) := \sum_{k=0}^{N1} V_5(t - k \cdot T_6, \tau_\delta, \tau_{pwts}, V_{pp})$$

$V_5(\ldots)$ is defined in ***-5 Doublet Voltage Pulse***

$$t := 0 \cdot T_{ptd}, 0 \cdot T_{ptd} + \frac{20 \cdot T_{ptd}}{5000} .. 20 \cdot T_{ptd}$$



Periodic Signals

9 Staircase 1 Voltage Pulse Train

Description of the Function's parameters: $v_{stcp}(t, period, signal\_amplitude, number\_of\_steps)$,

: $v_{stc}(t, step\_length, signal\_amplitude, number\_of\_steps)$

For data, see the worksheet *"staircase pulse data.xmcd"*

Period:	$T_{stcpt} = 6.667 \cdot \mu s$	$T_{stcpt} = (m1_{steps} + 1) \cdot T_{1stpl\_} \cdot 2$
Step length:	$T_{1stpl\_} = 0.37 \cdot \mu s$	$T_{1stpl\_} = \dfrac{T_{stcpt}}{2 \cdot (m1_{steps} + 1)}$
Number of steps:	$m1_{steps} = 8$	
Duty Cycle:	$\delta_{ptd} = 0.25$	$\tau_{ptd} = T_{ptd} \cdot \delta_{ptd}$
Step Amplitude:	$V_{stcstp0} = 1 \cdot mV$	
Amplitude:	$V_{stcs} = 8 \cdot mV$	
Pulse delay from the origin	$\tau_{ptd\delta 0} = -250 \cdot \mu s$	

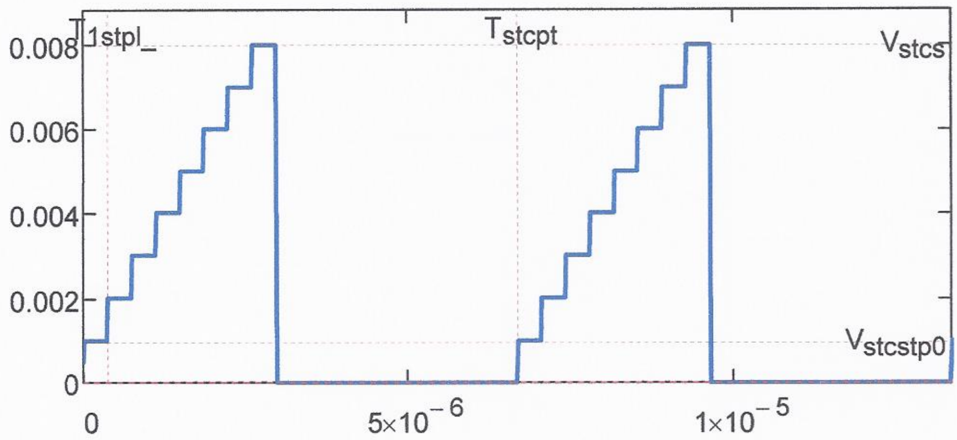
Average value: $$v_{stcpta} := \frac{V_{stcs}}{2 \cdot m1_{steps} \cdot (m1_{steps} + 1)} \cdot \sum_{k=1}^{m1_{steps}} (m1_{steps} - k + 1) = 2 \cdot mV$$

$$\frac{1}{T_{stcpt}} \cdot \int_{T_{1stpl\_}}^{(m1_{steps}+1) \cdot T_{1stpl\_}} v_{stc}(t, T_{1stpl\_}, V_{stcs}, m1_{steps})\, dt = 1.944 \cdot mV$$

Test signal: $$\boxed{v_{stcp}(t, T_{stcpt}, V_{stcs}, m1_{steps}) := \sum_{k=0}^{10} v_{stc}\left[t - k \cdot T_{stcpt}, \frac{T_{stcpt}}{2 \cdot (m1_{steps} + 1)}, V_{stcs}, m1_{steps}\right]}$$

$$t := 0 \cdot T_{stcpt}, 0 \cdot T_{stcpt} + \frac{10 \cdot T_{stcpt}}{5000} .. 10 \cdot T_{stcpt}$$

Periodic Staircase 1



Area under the staircase $A_{stcp} := T_{1stpl\_} \cdot \frac{V_{stcs}}{m1_{steps}} \cdot \sum_{k=1}^{m1_{steps}} \left(m1_{steps} - k + 1\right)$ $\quad A_{stcp} = 13.333 \cdot \text{volt} \cdot \text{ns}$

$$\int_0^{T_{stcpt}} v_{stcp}\left(t, T_{stcpt}, V_{stcs}, m1_{steps}\right) dt = 13.345 \cdot \text{volt} \cdot \text{ns}$$

Adimensional function:

$$\text{Vistcp}\left(t, T_{stcpt}, V_{stcs}, m1_{steps}\right) := \frac{v_{stcp}\left(t, T_{stcpt}, V_{stcs}, m1_{steps}\right)}{\text{volt}}$$

Description of the Function's parameters: Vistcp(t, period, signal_amplitude, number_of_steps)

Periodic Signals

10 Staircase 2 Voltage Pulse Train

Description of the Function's parameters: $v_{stct}(time, period, max\_amplitude, number\_of\_steps),$
$: v_{stcc}(t, step\_length, signal\_amplitude, number\_of\_steps)$

For data, see the worksheet "staircase 2 pulse data.xmcd"

max amplitude: $V_{stc} = 5 \cdot volt$ Period: $T2_{stp\_} = 72 \cdot \mu s$

Number of steps $m2_{steps} = 8$ Step amplitude: $V_{stcstp} = 0.625 \cdot V$

Step length: $T_{2stpl\_} = 4 \cdot \mu s$

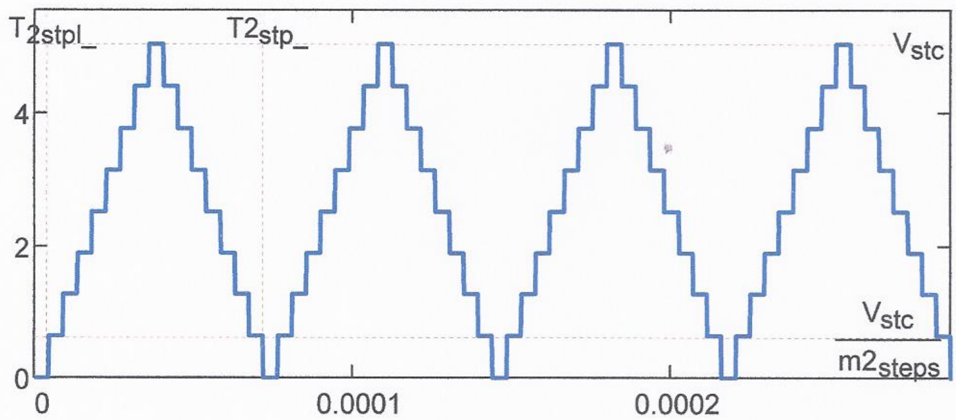
$$v_{stct}(t, T2_{stp\_}, V_{stc}, m2_{steps}) := \sum_{k=0}^{100} v_{stcc}\left(t - k \cdot T2_{stp\_}, \frac{T2_{stp\_}}{2 \cdot m2_{steps}}, V_{stc}, m2_{steps}\right)$$

Adimensional function: $m2_{steps} = 8$

$$Vistct(t, T2_{stp\_}, V_{stc}, m2_{steps}) := \frac{v_{stct}(t, T2_{stp\_}, V_{stc}, m2_{steps})}{V}$$

$$t := 0 \cdot T2_{stp\_}, 0 \cdot T2_{stp\_} + \frac{10 \cdot T2_{stp\_}}{5000} .. 10 \cdot T2_{stp\_}$$

Staircase 2 Wave



Description of the Function's parameters: $Vistct(t, period, max\_amplitude, number\_of\_steps)$

Periodic Signals

11 Staircase 2 Voltage Pulse Train + sinus

Description of the Function's parameters: Vstcsin(t, period, max_amplitude, number_of_steps)
For data, see the worksheet *"staircase 2 pulse data.xmcd"*

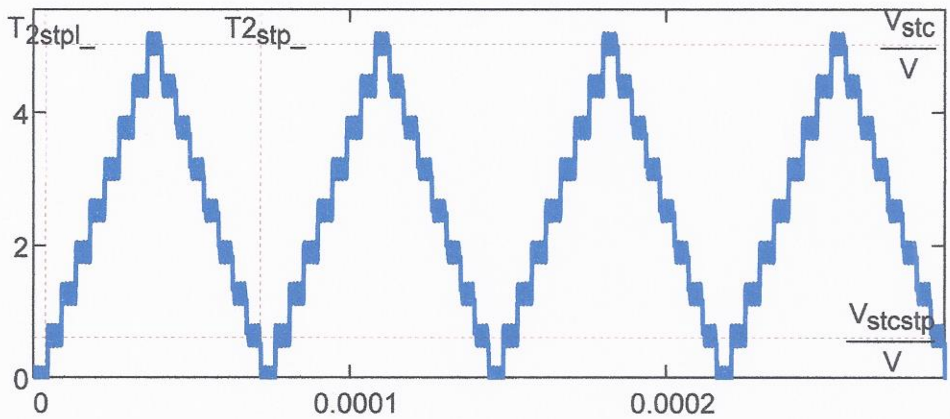
max amplitude: $V_{stc} = 5 \cdot volt$ Period: $T2_{stp\_} = 72 \cdot \mu s$

Number of steps $m2_{steps} = 8$ Step amplitude: $V_{stcstp} = 0.625 \cdot V$

Step length: $T_{2stpl\_} = 4 \cdot \mu s$

$$Vstcsin(t, T2_{stp\_}, V_{stc}, m2_{steps}) := Vistct(t, T2_{stp\_}, V_{stc}, m2_{steps}) + \frac{V_{stc}}{4 \cdot m2_{steps} \cdot V} \cdot \sin\left(\frac{2 \cdot \pi \cdot m2_{steps}}{T2_{stp\_}} \cdot 8 \cdot t\right)$$

Staircase 2 Wave + sinus (adimensional)



Periodic Signals

12 Staircase 3 Voltage Pulse Train

Description of the Function's parameters: v_{stct}(t, period, step_amplitude, number_of_steps),
: v_{stctA0}[t, (period, step_amplitude, number_of_steps)]

Data in "*staircase 3 pulse data.xmcd*"

$m3_{steps} = 4$

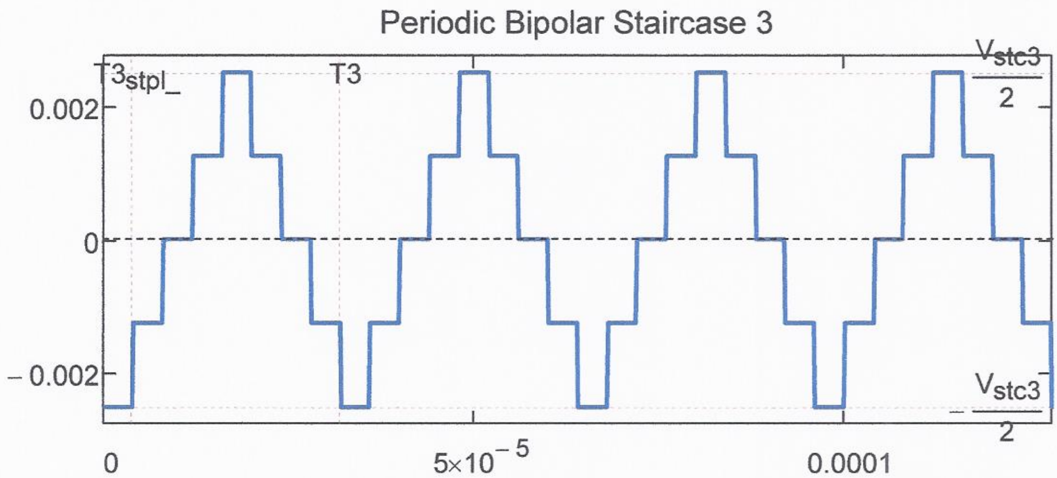
$$T3 = 32 \cdot \mu s \qquad f33 = \frac{1}{T3} \quad f33 = 0.031 \cdot MHz$$

$$V_{stc3} = 5 \cdot mV$$

$$v_{stctA0}(t, T3, V_{stc3}, m3_{steps}) := v_{stct}(t, T3, V_{stc3}, m3_{steps}) - \frac{V_{stc3}}{2}$$

Adimensional function:

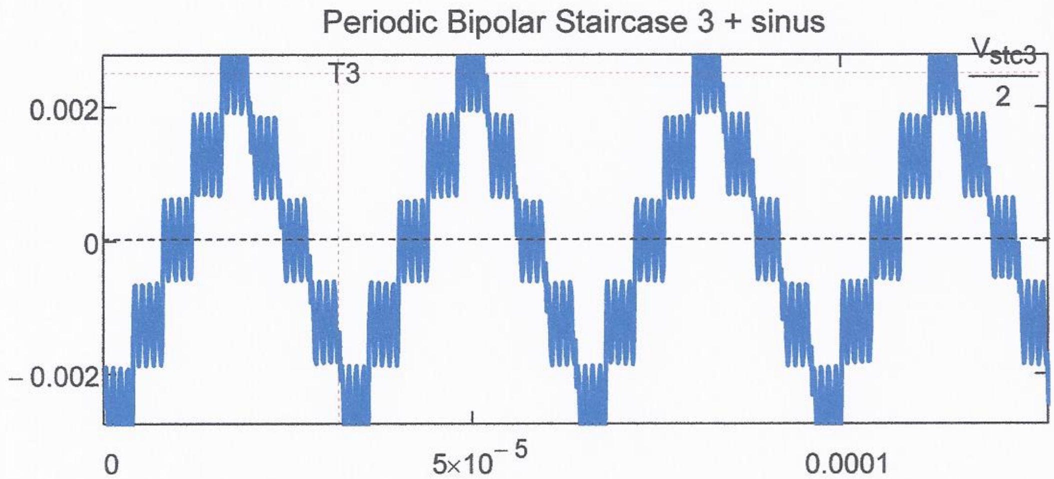
$$VistctA0(t, T3, V_{pbds}, m3_{steps}) := \frac{v_{stctA0}(t, T3, V_{pbds}, m3_{steps})}{V}$$



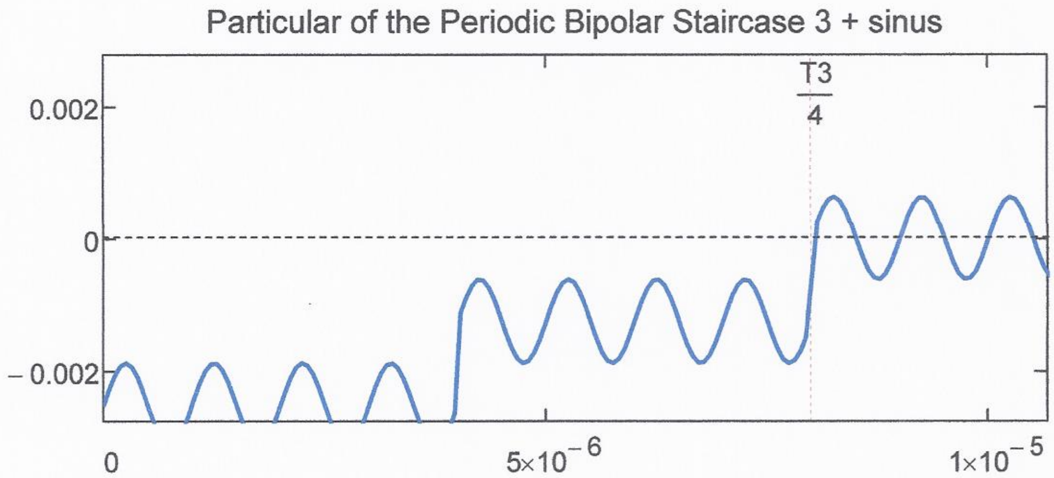
Periodic Signals

13 Staircase 3 Voltage Pulse Train + sinus

$$\mathrm{VistctA0sin}\left(t, T3, V_{pbds}, m3_{steps}\right) := \mathrm{VistctA0}\left(t, T3, V_{pbds}, m3_{steps}\right) \ldots + \frac{V_{pbds}}{2 \cdot m3_{steps} \cdot V} \cdot \sin\left(\frac{2 \cdot \pi}{T3} \cdot 8 \cdot m3_{steps} \cdot t\right)$$



$$t_1 := 0 \cdot T3, 0 \cdot T3 + \frac{10 \cdot T3 - 0 \cdot T3}{5000} .. 10 \cdot T3$$



Periodic Signals

14 Staircase 4 Voltage Pulse Train

Description of the Function's parameters : vstc1p(time, step length, step amplitude, number of steps)

For data, see the worksheet *"staircase 4 pulse data.xmcd"*

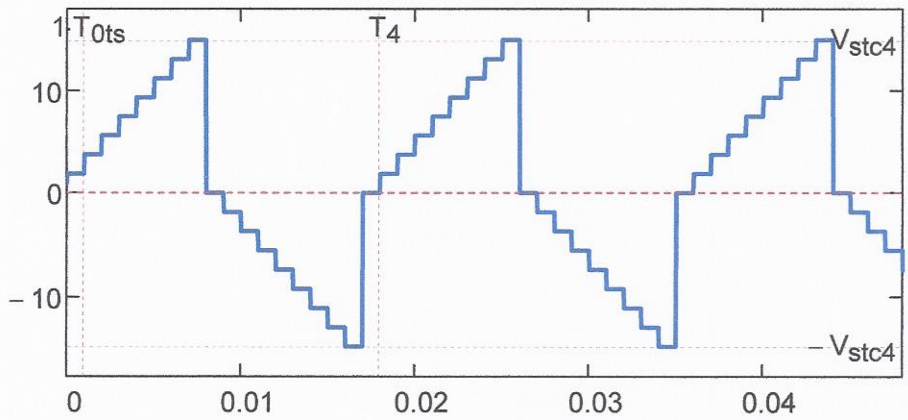
$$f_{44} = 5.556 \times 10^{-5} \cdot \mathrm{MHz} \qquad \frac{1}{T_4} = 5.556 \times 10^{-5} \cdot \mathrm{MHz}$$

$$\omega_{44} = 3.491 \times 10^{-4} \cdot \frac{\mathrm{Mrads}}{\mathrm{sec}} \qquad 2 \cdot \pi \cdot f_{44} = 3.491 \times 10^{-4} \cdot \frac{\mathrm{Mrads}}{\mathrm{sec}}$$

$$\mathrm{vstc1p}\left(t, T_{0ts}, V_{stc4}, m4_{steps}\right) := \sum_{k=0}^{100} \mathrm{vstc1}\left[t - k \cdot 2 \cdot \left(m4_{steps} + 1\right) \cdot T_{0ts}, T_{0ts}, V_{stc4}, m4_{steps}\right]$$

$$t := -T_4 \cdot 0, -T_4 \cdot 0 + \frac{4 \cdot T_4 + T_4 \cdot 0}{5000} \,..\, 4 \cdot T_4$$

Periodic Bipolar Staircase 4



$$m4_{steps} = 8$$

Adimensional function:

$$\mathrm{vstc1p1}\left(t, T_{0ts}, V_{stc4}, m8_{steps}\right) := \frac{\mathrm{vstc1p}\left(t, T_{0ts}, V_{stc4}, m4_{steps}\right)}{V}$$

Periodic Signals

15 Bipolar Triangular Voltage Wave

Description of the Function's parameters : Λ_V(time, triangle half base, triangle amplitude)

For data, see the worksheet *"Dirac Pulse - formulas.xmcd"*

$N = 50$

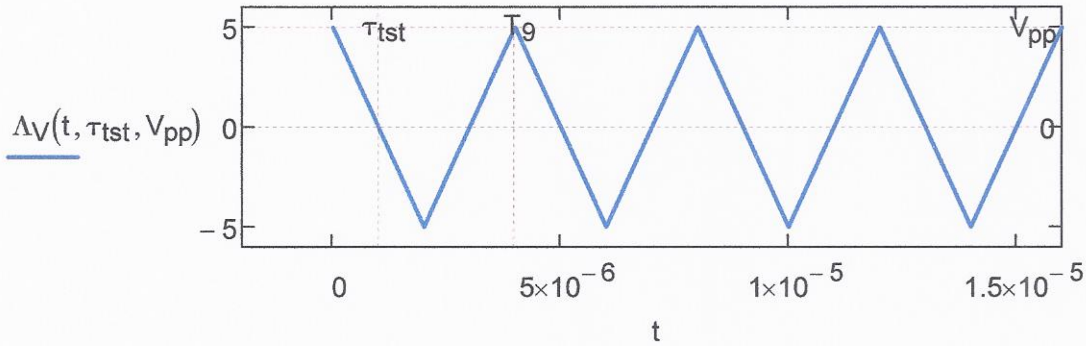
Signal amplitude: $V_{pp} = 5 \cdot V$

Time constant: $\tau_{tst} := 1 \cdot \mu s$

Period: $T_9 := 4 \cdot \tau_{tst}$ $\qquad f_9 := \frac{1}{T_9}$

$$\Lambda_V(t, \tau_{tst}, V_{pp}) := V_{pp} \cdot \sum_{k=-N}^{N} \left[(-1)^k \cdot \Lambda(t - 2 \cdot k \cdot \tau_{tst}, \tau_{tst})\right]$$

$$t := -T_9 \cdot 0, -T_9 \cdot 0 + \frac{4 \cdot T_9 + T_9 \cdot 0}{2000} .. 4 \cdot T_9 \qquad T_9 = 4 \cdot \mu s$$



Bipolar Triangular Voltage Wave Built using the Step Function

Signal amplitude: $V_{pp} = 5 \cdot V$

Time constant: $\tau_{tst} = 1 \cdot \mu s$

Period: $T_9 = 4 \cdot \mu s$

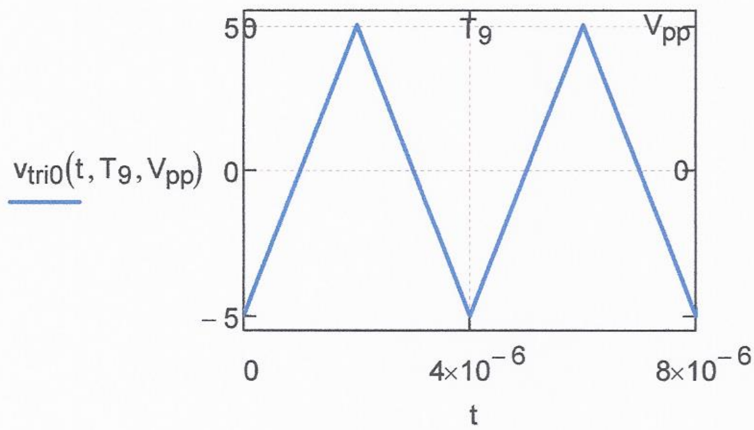
$\omega_9 := 2 \cdot \pi \cdot f_9$ $\qquad \omega_9 = 1.571 \times 10^6 \cdot \dfrac{rad}{sec}$

$N = 50$

$$v_{tri0}(t, T_9, V_{pp}) := \left[\frac{4 \cdot V_{pp}}{T_9} \cdot \sum_{k=0}^{N} \left[\begin{array}{l} (t - k \cdot T_9) \cdot \Phi(t - k \cdot T_9) \ldots \\ + (-1) \cdot \left[2 \cdot \left[t - \left(k + \frac{1}{2}\right) \cdot T_9 \right] \cdot \Phi\left[t - \left(k + \frac{1}{2}\right) \cdot T_9 \right] \right] \ldots \\ + \left[t - (k+1) \cdot T_9 \right] \cdot \Phi\left[t - (k+1) \cdot T_9 \right] \end{array} \right] \right] - V_{pp}$$

$V_{pp} = 5 \cdot volt$ $\qquad v_{tri0}\left(\dfrac{T_9}{2}, T_9, V_{pp}\right) = 5 \cdot V$

$$t := -1 \cdot T_9, -1 \cdot T_9 + \frac{20 \cdot T_9 + 1 \cdot T_9}{5000} .. 20 \cdot T_9$$



Adimensional function: $\qquad Vi3(t, T_9, V_{pp}) := \dfrac{v_{tri0}(t, T_9, V_{pp})}{V}$

Periodic Signals

16 Periodic Voltage Pulse Train

Signal amplitude: $V_{pp} = 5 \cdot V$

Period: $T_{0ts} = 1 \times 10^{3} \cdot \mu s$

Pulse width: $k2 := 4$ $\qquad \frac{T_{0ts}}{k2} = 250 \cdot \mu s$

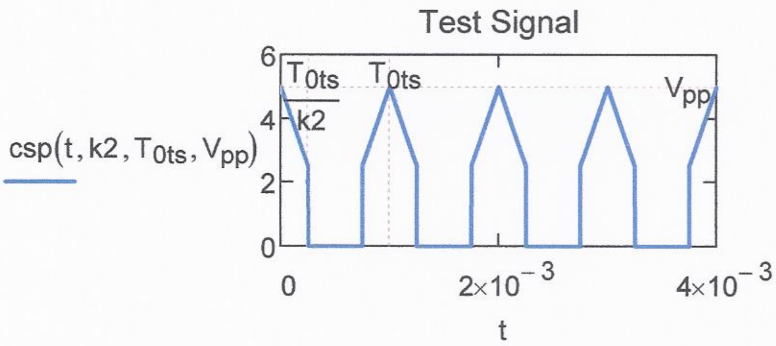
Frequence $\quad f_{10} := \frac{1}{T_{0ts}} \qquad \omega_{10} := 2 \cdot \pi \cdot f_{10}$

$N = 50$

$$csp(t,k2,T_{0ts},V_{pp}) := V_{pp} \cdot \left[\sum_{k=-N}^{N} \left[\left(2 \cdot \frac{t + k \cdot T_{0ts}}{T_{0ts}} + 1\right) \cdot \left(\Phi\left(t + k \cdot T_{0ts} + \frac{T_{0ts}}{k2}\right) - \Phi(t + k \cdot T_{0ts})\right) \ldots \atop + \left(-2 \cdot \frac{t + k \cdot T_{0ts}}{T_{0ts}} + 1\right) \cdot \left(\Phi(t + k \cdot T_{0ts}) - \Phi\left(t + k \cdot T_{0ts} - \frac{T_{0ts}}{k2}\right)\right) \right] \right]$$

$$csp(t,k2,T_{0ts},V_{pp}) := V_{pp} \cdot \left[\sum_{k=-N}^{N} \left[\left(2 \cdot \frac{t + k \cdot T_{0ts}}{T_{0ts}} + 1\right) \cdot rect1\left[t, -\left(k \cdot T_{0ts} + \frac{T_{0ts}}{k2}\right), \frac{T_{0ts}}{k2}\right] \ldots \atop + \left(-2 \cdot \frac{t + k \cdot T_{0ts}}{T_{0ts}} + 1\right) \cdot rect1\left(t, -k \cdot T_{0ts}, \frac{T_{0ts}}{k2}\right) \right] \right]$$

$$t := -1 \cdot T_{0ts}, -1 \cdot T_{0ts} + \frac{4 \cdot T_{0ts} + 1 \cdot T_{0ts}}{5000} \,..\, 4 \cdot T_{0ts}$$



$N1 = 50$

Adimensional function: $\quad fc5(t,k2,T_{0ts},V_{pp}) := \frac{csp(t,k2,T_{0ts},V_{pp})}{V}$

Periodic Signals

17 Bipolar Sawtooth with positive slope Pulse Train

Amplitude: $V_{sawth} = 50 \cdot V$

Sawtooth length: $\delta_{sawth} = 1 \cdot \mu s$

Slope: $p_{sawth} = 50 \cdot \frac{V}{\mu s}$

Period: $T_{sawth} = 1 \cdot \mu s$

$f_{sawth} = 1 \times 10^{6} s^{-1}$

$$f_{sw}(t, T_{sawth}, V_{sawth}) := p_{sawth} \cdot t \cdot \text{rect1}(t, 0.0 \cdot T_{sawth}, T_{sawth})$$

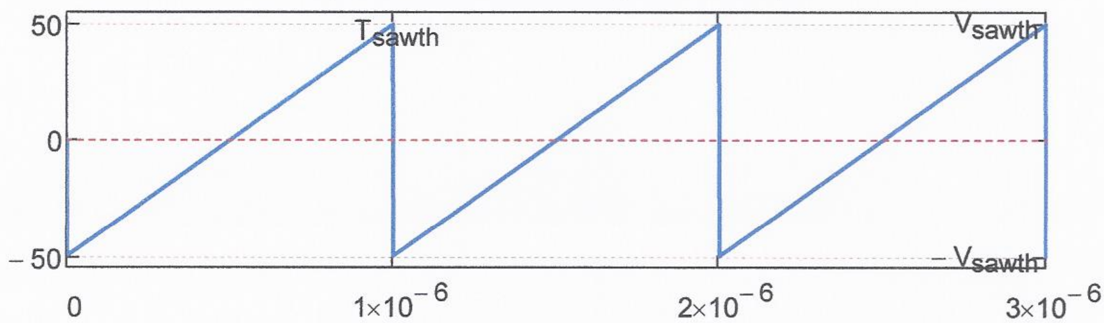
Defined in -11 Sawtooth Voltage Pulse with positive slope

$$\alpha := \text{atan}\left(p_{sawth} \cdot \frac{sec}{volt}\right) \qquad \alpha = 1.571$$

$$t := -T_{sawth} \cdot 0, T_{sawth} \cdot 0 + \frac{5 \cdot T_{sawth} + T_{sawth} \cdot 0}{5000} .. 5 \cdot T_{sawth}$$

$$N = 50 \qquad v1_{sw}(t, T_{sawth}, V_{sawth}) := \sum_{k=-N}^{N} \left(f_{sw}(t - k \cdot T_{sawth}, T_{sawth}, 2 \cdot V_{sawth})\right) - V_{sawth}$$

Bipolar Sawtooth with positive slope



$$\text{Adimensional function: } v1sw(t, T_{sawth}, V_{sawth}) := \frac{v1_{sw}(t, T_{sawth}, V_{sawth})}{V}$$

Periodic Signals

18 Bipolar Sawtooth with negative slope Pulse Train

Amplitude: $V_{sawth} = 50 \cdot V$

Sawtooth length: $\delta_{sawth} = 1 \cdot \mu s$

Slope: $p_{sawth} = 50 \cdot \frac{V}{\mu s}$

Period: $T_{sawth} = 1 \cdot \mu s$

$f_{sawth} = 1 \times 10^{6} s^{-1}$

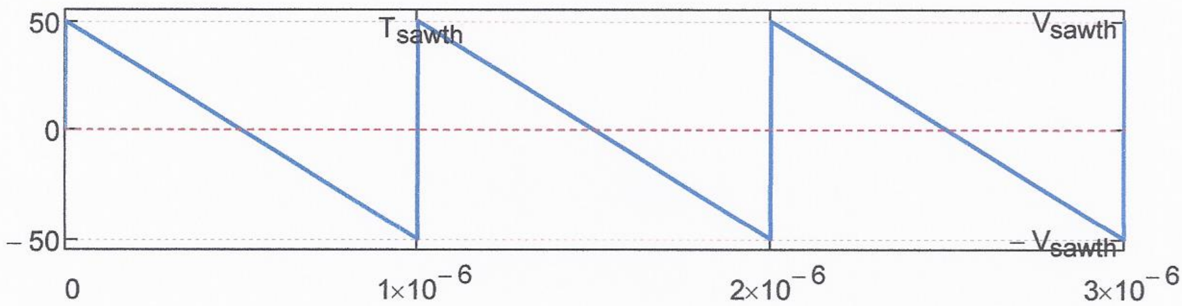
$$f(t, T_{sawth}, V_{sawth}) := V_{sawth} \cdot \left(\frac{-t}{T_{sawth}} + 1 \right) \cdot \left(\Phi(t) - \Phi(t - T_{sawth}) \right)$$

$$f_{12} := \frac{1}{\delta_{sawth}}$$

Defined in -12 Sawtooth Voltage Pulse with negative slope

$$v2_{sw}(t, \delta_{sawth}, V_{sawth}) := \sum_{k=-N}^{N} f(t - k \cdot \delta_{sawth}, \delta_{sawth}, 2 \cdot V_{sawth}) - V_{sawth}$$

Sawtooth



Adimensional function: $fc7(t, T_{sawth}, V_{sawth}) := \frac{v2_{sw}(t, T_{sawth}, V_{sawth})}{V}$

Periodic Signals

19 Raised-Cosine (RC) Pulse Train

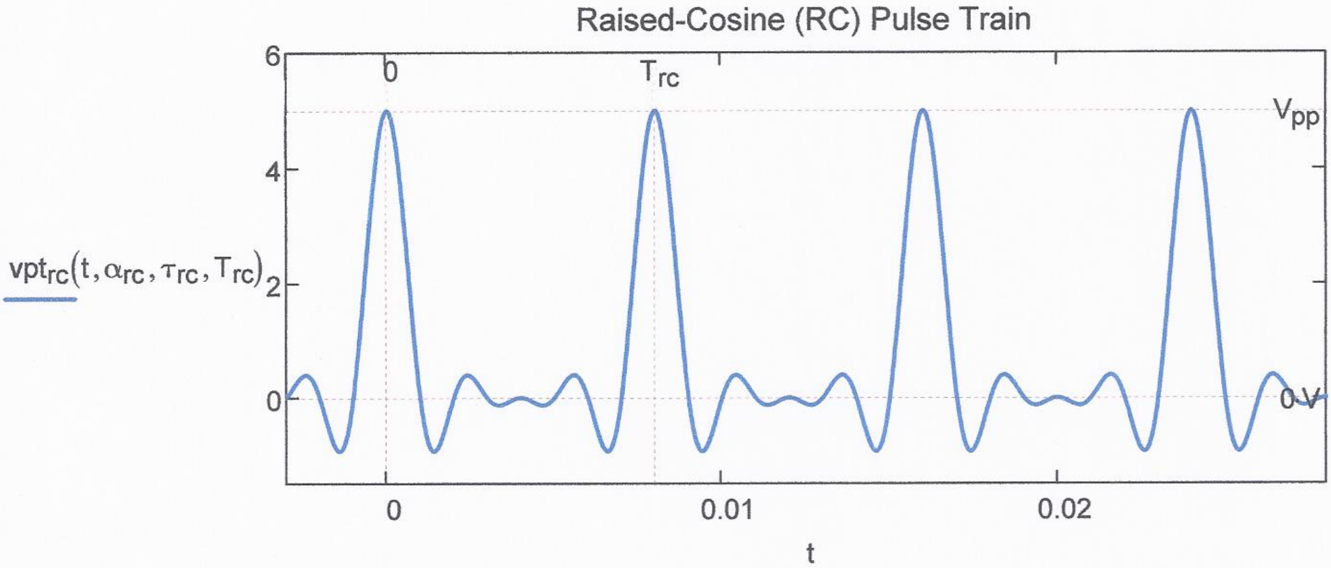
α_{rc} is the excess bandwidth factor since the bandwidth of this pulse is $Bw_{rc} = \dfrac{1+\alpha_{rc}}{2 \cdot T_{rc}}$

$$T_{rc} := 8 \cdot T_{0ts} \qquad \tau_{rc} := T_{0ts}$$

$$vpt_{rc}(t, \alpha_{rc}, \tau_{rc}, T_{rc}) := \sum_{k=-N}^{N} v_{rc}(t - k \cdot T_{rc}, \alpha_{rc}, \tau_{rc})$$

Example $\quad \alpha_{rc} := 0.3 \quad T_{0ts} = 1 \times 10^{3} \cdot \mu s \quad Bw_{rc} = 6.5 \times 10^{-4} \cdot MHz$

$$t := -T_{rc}, -T_{rc} + \frac{20 \cdot T_{rc} + T_{rc}}{5000} .. 20 \cdot T_{rc}$$



Adimensional function: $Vpt_{rc}(t, \alpha_{rc}, \tau_{rc}, T_{rc}) := \dfrac{vpt_{rc}(t, \alpha_{rc}, \tau_{rc}, T_{rc})}{V}$

$$Vpt_{rc}(T_{rc}, \alpha_{rc}, \tau_{rc}, T_{rc}) = 5$$

Periodic Signals

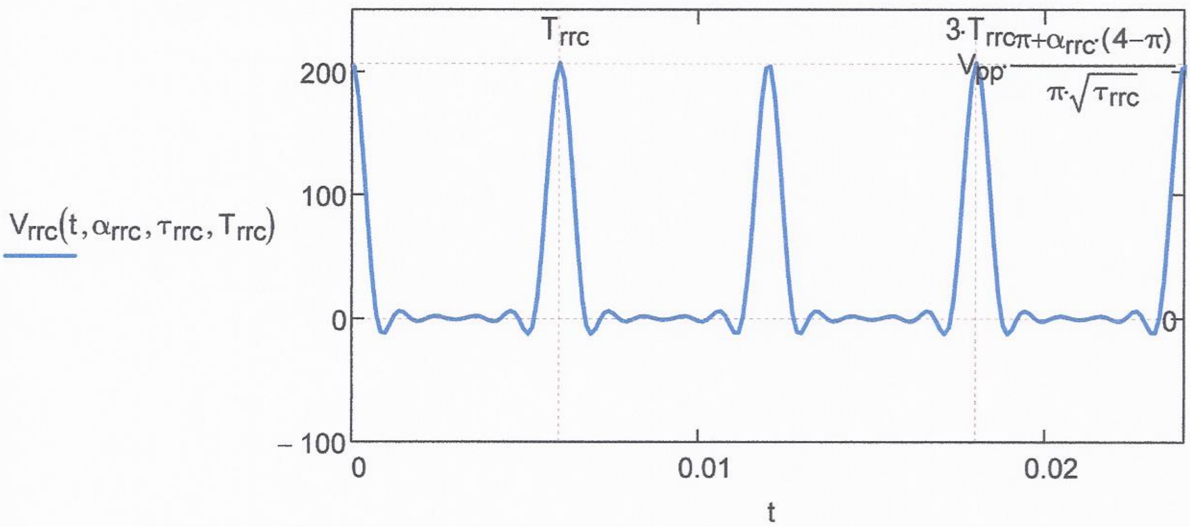
20 Root Raised-Cosine (RC) Pulse Train

$$V_{rrc}(t,\alpha_{rrc},\tau_{rrc},T_{rrc}) := \sum_{k=-N}^{N} Vrrc(t-k\cdot T_{rrc},\alpha_{rrc},\tau_{rrc})$$

Example $\alpha_{rrc} := 1.10$ $\tau_{rrc} = 1\times 10^{3}\cdot \mu s$

$$t := -T_{rrc}\cdot 3, -T_{rrc}\cdot 3 + \frac{40\cdot T_{rrc} + T_{rrc}\cdot 3}{2000} \,..\, 10\cdot T_{rrc}$$

Root Raised-Cosine (RRC) Pulse Train



The pulse is ortogonal while: $\int_{-\infty}^{\infty} v_{rrc}(t,\alpha,T)\cdot v_{rrc}(t-n\cdot T,\alpha,T)\,dt = 0$

Adimensional function $Vrrc(t,\alpha_{rrc},T_{rrc}) := v_{rrc}(t,\alpha_{rrc},T_{rrc})\cdot\frac{sec^{0.5}}{V}$

$$Vrrc(3\cdot T_{rrc},\alpha_{rrc},T_{rrc}) = -0.275$$

Periodic Signals

21 AM test signal (single tone)

Carrier Amplitude: $A1 := 10 \cdot \text{volt}$

Modulating signal amplitude: $B1 := 5.5 \cdot \text{volt}$

$$\omega 1_c := \frac{\omega_{0ts}}{2} \qquad T1_c := \frac{2 \cdot \pi}{\omega 1_c} \qquad \omega 1_m := \frac{\omega_{0ts}}{10} \qquad T1_m := \frac{2 \cdot \pi}{\omega 1_m} \qquad f1_m := \frac{\omega 1_m}{2 \cdot \pi} \qquad f_{15} := \frac{\omega 1_c}{2 \cdot \pi}$$

$$v_{ammax} := A1 + B1 \qquad v_{ammin} := A1 - B1 \qquad A1 = v_{ammax} + v_{ammin} \qquad B1 = v_{ammax} - v_{ammin}$$

$$v_{ammax} = 15.5 \cdot \text{volt}$$

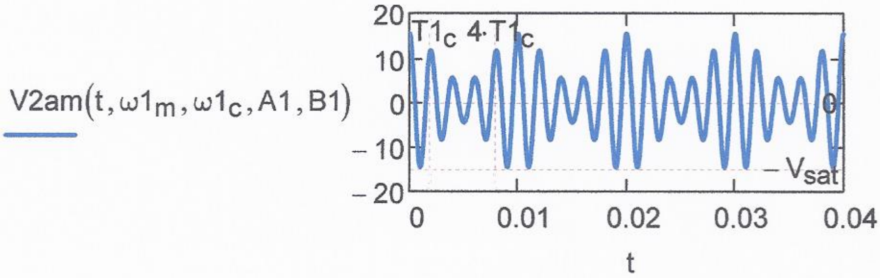
$$v_{ammin} = 4.5 \cdot \text{volt} \qquad m_{am} := \frac{v_{ammax} - v_{ammin}}{v_{ammax} + v_{ammin}} \qquad m_{am} = 55 \cdot \%$$

$$v2_i(t, \omega 1_m, \omega 1_c, A1, B1) := A1 \cdot \cos(\omega 1_c \cdot t) \ldots$$
$$+ \frac{B1}{2} \cdot \cos[(\omega 1_c + \omega 1_m) \cdot t] \ldots$$
$$+ \frac{B1}{2} \cdot \cos[(\omega 1_c - \omega 1_m) \cdot t]$$

$$A1 = 10 \cdot \text{volt} \qquad T1_c = 2 \times 10^3 \cdot \mu s \qquad \omega 1_m = 6.283 \times 10^{-7} \cdot \frac{\text{Grads}}{\text{sec}} \qquad \omega 1_c = 3.142 \times 10^{-6} \cdot \frac{\text{Grads}}{\text{sec}}$$

Adimensional function: $V2am(t, \omega 1_m, \omega 1_c, A1, B1) := \dfrac{v2_i(t, \omega 1_m, \omega 1_c, A1, B1)}{V}$

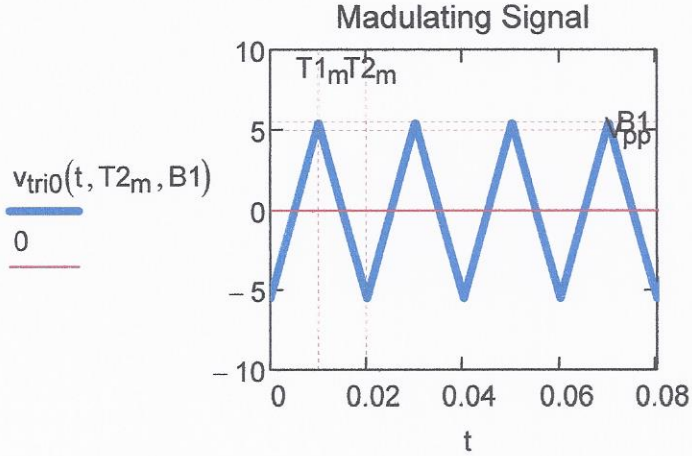
$$t := -T_{0ts} \cdot 3, -T_{0ts} \cdot 3 + \frac{20 \cdot T1_c + T_{0ts} \cdot 3}{5000} .. 20 \cdot T1_c$$



Periodic Signals

22 AM test signal (triangular wave)

$$\omega_{2m} := \frac{\omega 1_c}{10} \qquad T2_m := \frac{2 \cdot \pi}{\omega_{2m}} \qquad t := 0 \cdot sec, 40 \cdot \frac{T2_m}{5000} .. 40 \cdot T2_m$$



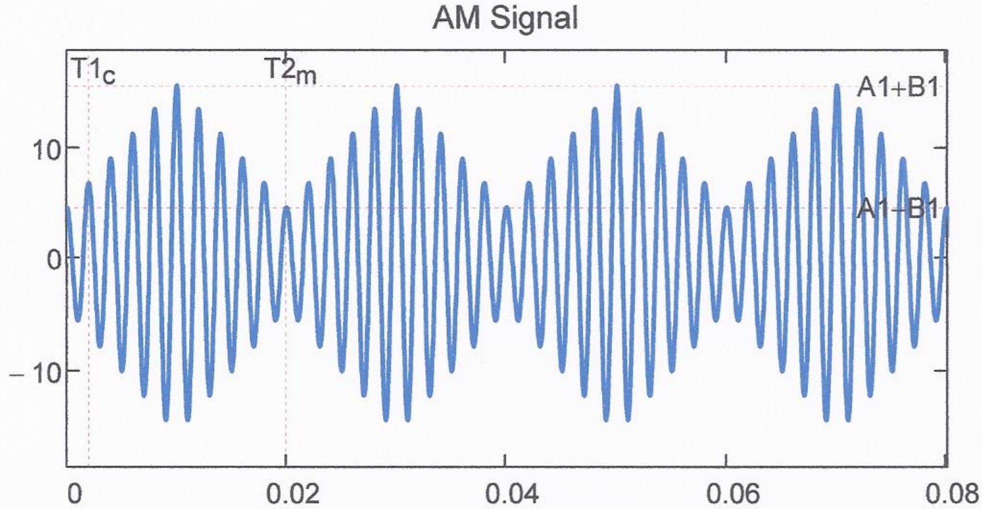
$$f_{16} := \frac{\omega 1_c}{2 \cdot \pi}$$

$$\omega 1_c = 3.142 \times 10^{-3} \cdot \frac{Mrads}{sec} \qquad \omega 1_m = 0.628 \cdot \frac{krads}{sec} \qquad m_{am} = 55 \cdot \%$$

$$v_{am}(t, \omega_{2m}, \omega 1_c, A1, B1) := A1 \cdot \left[\left(1 + \frac{m_{am}}{B1} \cdot v_{tri0}(t, T2_m, B1)\right) \cdot \cos(\omega 1_c \cdot t)\right]$$

Adimensional function: $V3am(t, \omega 1_m, \omega 1_c, A1, B1) := \dfrac{v_{am}(t, \omega 1_m, \omega 1_c, A1, B1)}{V}$

$$t := -T_{0ts} \cdot 3, -T_{0ts} \cdot 3 + \frac{8 \cdot T2_m + T_{0ts} \cdot 3}{5000} .. 8 \cdot T2_m$$



Periodic Signals

23 AM DSBSC test signal (single tone)

$$\omega 2_m := \frac{\omega 1_c}{10} \qquad T2_m := \frac{2\cdot\pi}{\omega 2_m} \qquad \omega 2_m = \frac{2\cdot\pi}{T2_m} \qquad \frac{A1\cdot B1}{2} = 27.5\cdot volt^2$$

$$\omega 1_c = 3.142\times 10^{-3}\cdot\frac{Mrads}{sec} \qquad \omega 2_m = 3.142\times 10^{-4}\cdot\frac{Mrads}{sec} \qquad f2_m := \frac{1}{T2_m} \qquad f_{1c} := \frac{\omega 1_c}{2\cdot\pi}$$

$$T_{1c} := \frac{1}{f_{1c}} \qquad V_{dsbsc}(t,\omega 1_c,\omega 2_m) = A1\cdot\cos(\omega 1_c\cdot t)\cdot v_m(t)$$

$$V_m(t) = B1\cdot\cos(\omega 2_m\cdot t)$$

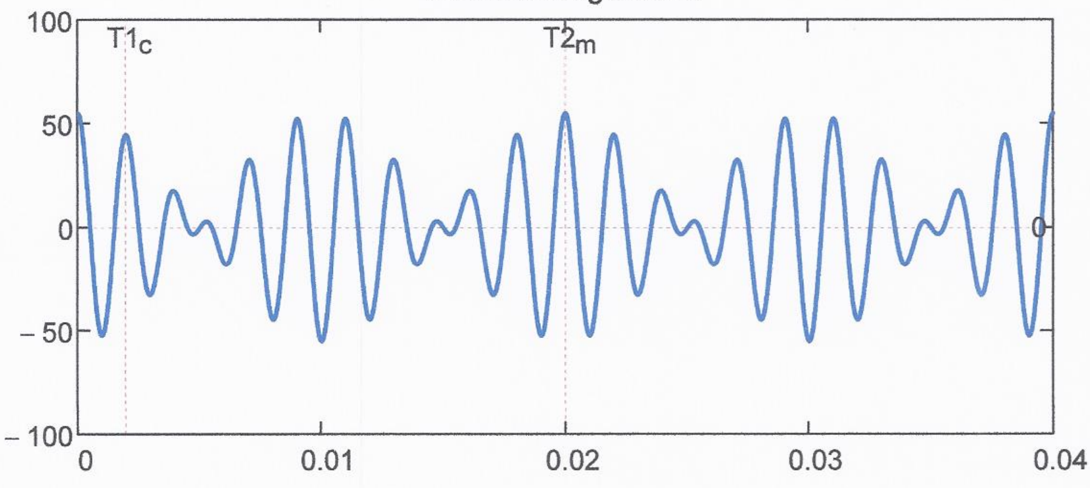
$$V_{dsbsc}(t,\omega 1_c,\omega 2_m,A1,B1) = A1\cdot\cos(\omega 1_c\cdot t)\cdot B1\cdot\cos(\omega 2_m\cdot t)$$

$$V_{dsbsc}(t,\omega 1_c,\omega 2_m,A1,B1) := \frac{A1\cdot B1}{2}\cdot\cos[(\omega 1_c+\omega 2_m)\cdot t] + \frac{A1\cdot B1}{2}\cdot\cos[(\omega 1_c-\omega 2_m)\cdot t]$$

Adimensional function: $$V4dsbsc(t,\omega 1_c,\omega 2_m,A1,B1) := \frac{V_{dsbsc}(t,\omega 1_c,\omega 2_m,A1,B1)}{V^2}$$

$$\nu := 20 \qquad t := 0\cdot sec,\nu\cdot\frac{T1_c}{5000}\,..\,\nu\cdot T1_c$$

DSBSC single tone



Periodic Signals

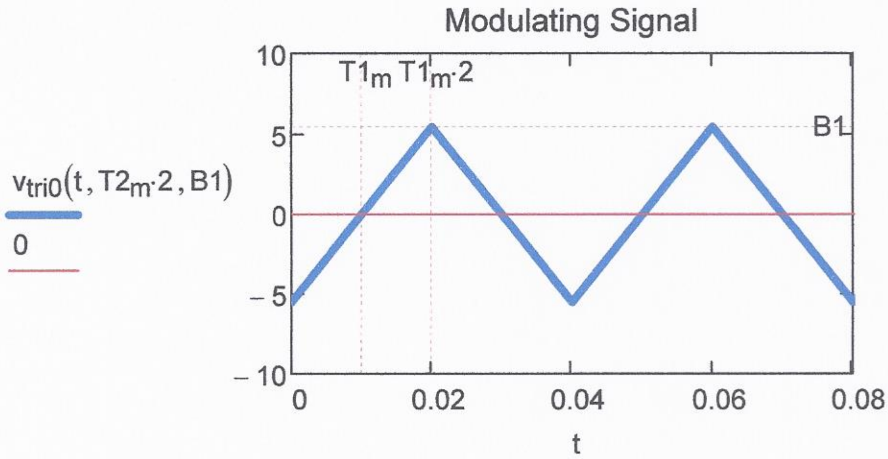
24 AM DSBSC test signal (triangular wave)

$T_{18} := T2_m \qquad V3_{dsbsc}(t, \omega 1_c, \omega 2_m, A1, B1) := A1 \cdot \cos(\omega 1_c \cdot t) \cdot v_{tri0}(t, T2_m \cdot 2, B1)$

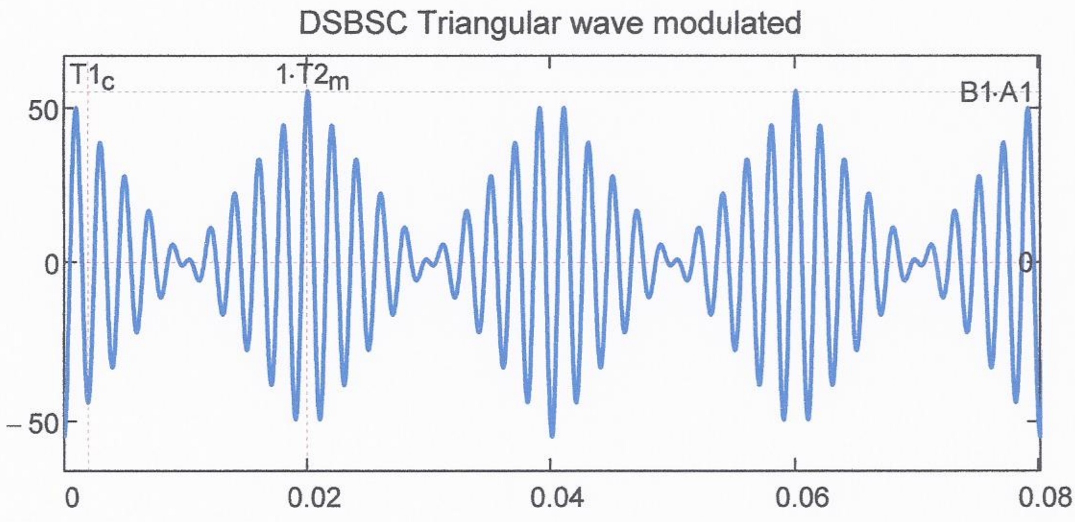
Adimensional function: $V5_{dsbsc}(t, \omega 1_c, \omega 2_m, A1, B1) := \dfrac{V3_{dsbsc}(t, \omega 1_c, \omega 2_m, A1, B1)}{volt^2}$

$f_{18} := \dfrac{1}{T_{18}} \qquad A1 = 10 \cdot V \qquad \omega 1_c = 3.142 \times 10^{-6} \cdot \dfrac{Grads}{sec} \qquad \omega 2_m = 3.142 \times 10^{-4} \cdot \dfrac{Mrads}{sec}$

$$t := -T_{18} \cdot 3, -T_{18} \cdot 3 + \frac{8 \cdot T_{18} + T_{18} \cdot 3}{5000} .. 8 \cdot T_{18}$$



$$v_{tri0}\left(\frac{T2_m \cdot 2}{2}, T2_m \cdot 2, B1\right) = 5.5 \cdot V$$



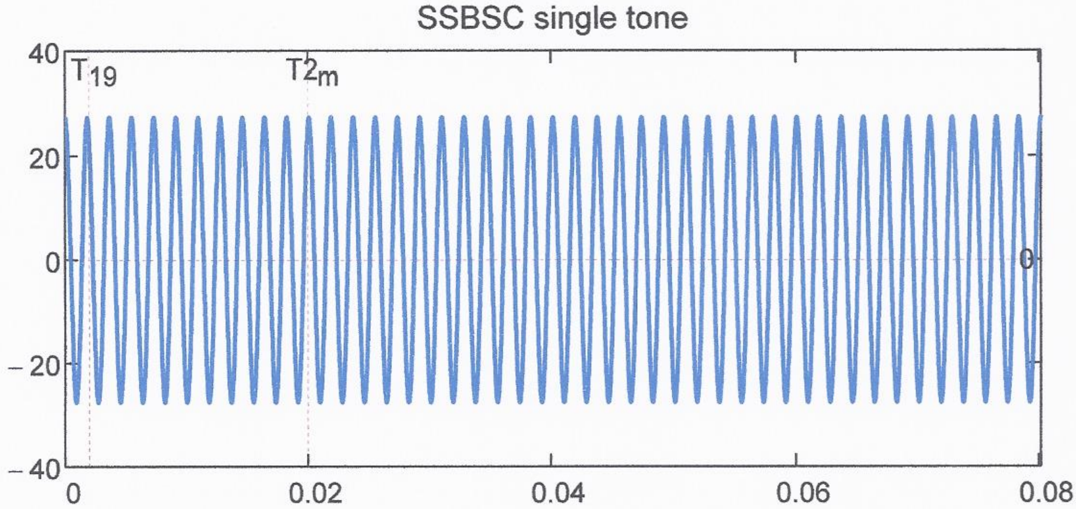
Periodic Signals

25 AM SSBSC test signal (single tone)

$$\frac{A1 \cdot B1}{2} = 27.5 \cdot V^2 \quad V_{ssbsc}(t, \omega 1_c, \omega 2_m, A1, B1) := \frac{A1 \cdot B1}{2} \cdot \cos\left[\left(\omega 1_c + \omega 2_m\right) \cdot t\right]$$

$$f_{19} := \frac{\omega 1_c}{2 \cdot \pi} \qquad T_{19} := \frac{1}{f_{19}} \qquad \omega 2_m = 3.142 \times 10^{-4} \cdot \frac{Mrads}{sec} \qquad \omega 1_c = 3.142 \times 10^{-3} \cdot \frac{Mrads}{sec}$$

$$A1 = 10 \cdot V \qquad t := 0 \cdot sec, \frac{4 \cdot T2_m}{5000} .. 4 \cdot T2_m \qquad B1 = 5.5 \cdot V$$



Adimensional function: $V6_{ssbsc}(t, \omega 1_c, \omega 2_m, A1, B1) := \frac{V_{ssbsc}(t, \omega 1_c, \omega 2_m, A1, B1)}{V^2}$

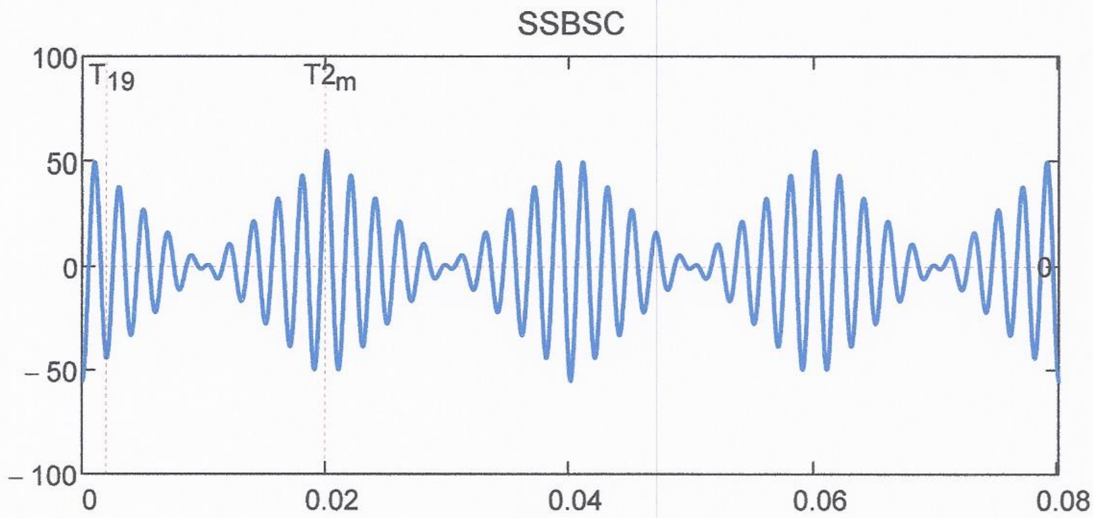
Periodic Signals

26 AM SSBSC test signal (triangular wave)

$$f_{20} := \frac{1}{T1_c} \qquad T_{20} := T1_c \qquad v_{tri0}\left(\frac{2\cdot\pi}{\omega2_m}, \frac{2\cdot\pi}{\omega2_m}\cdot 2, B1\right) = 5.5\cdot V$$

$$V4_{ssbsc}(t, \omega1_c, \omega2_m, A1, B1) := A1\cdot\cos(\omega1_c\cdot t)\cdot v_{tri0}\left(t, 2\cdot\frac{2\cdot\pi}{\omega2_m}, B1\right)$$

Adimensional function: $V7_{ssbsc}(t, \omega1_c, \omega2_m, A1, B1) := \dfrac{V4_{ssbsc}(t, \omega1_c, \omega2_m, A1, B1)}{V^2}$



Periodic Signals

27 FM test signal (single tone) (change data in FM data.xmcd)

Scroll the slider, to change the carrier frequency:

$j_p :=$

Carrier Frequency...: $f_c := j_p \cdot f_{0ts}$ $\quad f_c = 1 \times 10^4 \cdot \mathrm{MHz}$

Carrier period...: $T_c := \dfrac{1.0}{f_c}$

Angular frequency of the carrier...................................: $\omega_c := 2.0 \cdot \pi \cdot f_c$ $\quad \omega_c = 62.832 \cdot \dfrac{\mathrm{Grads}}{\mathrm{sec}}$

Amplitude of the single tone modulating signal............: $B_{fmm} := 50 \cdot \mathrm{volt}$ $\quad B_{fmm} = 15 \cdot \mathrm{V}$

Period of the modulating signal....................: $T_{fmm} := T_c \cdot 20$ $\quad f_{fmm} := \dfrac{1}{T_{fmm}}$

Frequency of the single tone modulating signal.............: $f_{fmm} = 737.557 \cdot \mathrm{MHz}$

Angular frequency of the single tone modulating signal: $\omega_{fmm} := 2.0 \cdot \pi \cdot f_{fmm}$

$$\frac{T_{fmm}}{T_c} = 13.558 \qquad \omega_{fmm} = 4.634 \cdot \frac{\mathrm{Grads}}{\mathrm{sec}} \qquad \frac{\omega_c}{\omega_{fmm}} = 13.558$$

Scroll the slider to change the modulation index m_f:

Frequency modulation index: $m_{fm} :=$

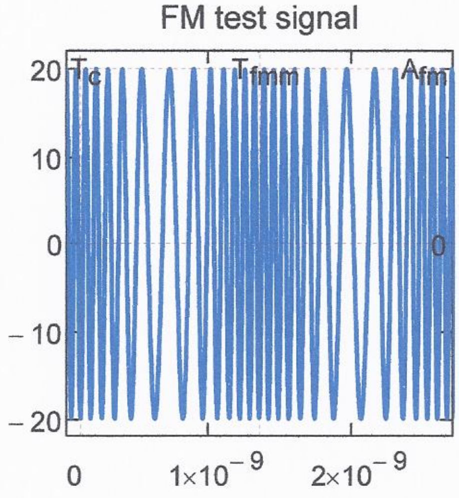
$$m_{fm} = 7 \qquad Kst_{fm} := \frac{m_{fm} \cdot \omega_{fmm}}{2 \cdot \pi \cdot B_{fmm}} \qquad T_{fmm} = 1.356 \cdot \mathrm{ns}$$

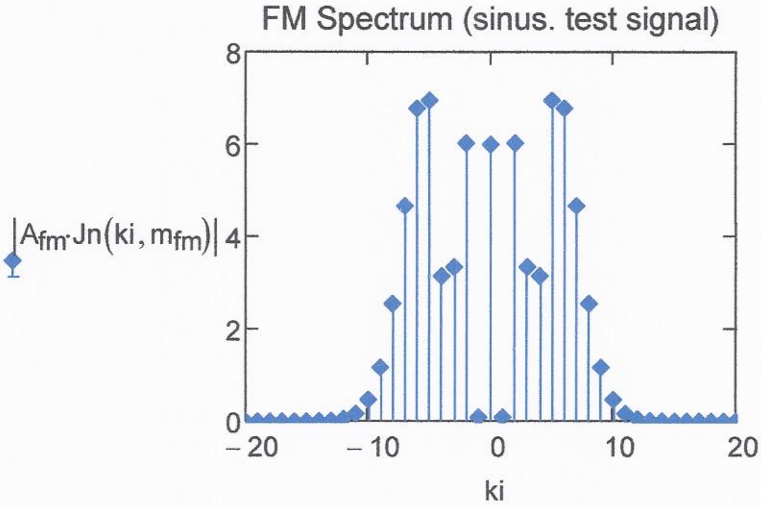
$$\omega_c = 62.832 \cdot \frac{\mathrm{Grads}}{\mathrm{sec}} \qquad \omega_{fmm} = 4.634 \cdot \frac{\mathrm{Grads}}{\mathrm{sec}} \qquad T_c = 0.1 \cdot \mathrm{ns}$$

$$t_{fm} := T_{fmm} \cdot 0 , T_{fmm} \cdot 0 + \frac{10 \cdot T_{fmm} - T_{fmm} \cdot 0}{5000} \;..\; 10 \cdot T_{fmm} \qquad \frac{T_{fmm}}{T_c} = 13.558$$

$N = 50$ $\quad v_{fmts}(t, \omega_c, \omega_{fmm}, A_{fm}, m_{fm}) := \mathrm{Re}\left[A_{fm} \cdot e^{j \cdot \omega_c \cdot t} \cdot \sum_{k=-N}^{N} \left(Jn(k, m_{fm}) \cdot e^{j \cdot k \cdot \omega_{fmm} \cdot t}\right)\right]$

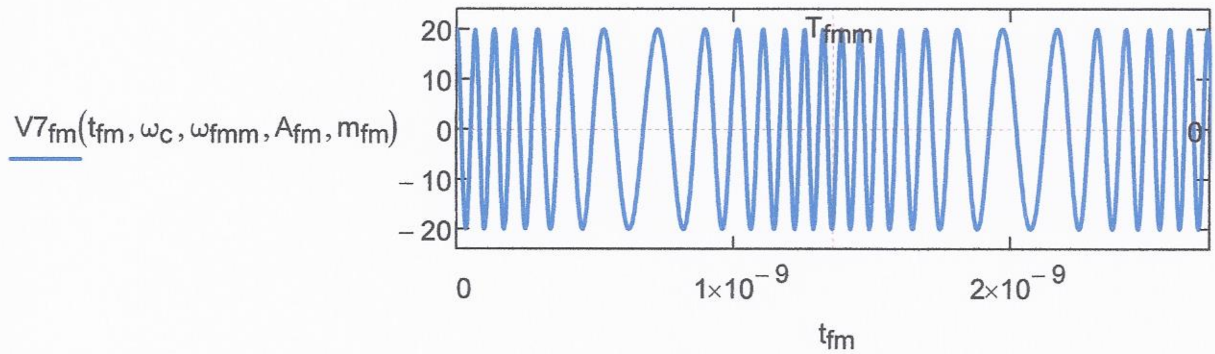
$m_{fm} = 7$ $\qquad ki := -30 .. 30$





Carrier Amplitude: $\quad A_{fm} = 20 \cdot \mathrm{volt}$ $\qquad B_{fmm} = 15 \cdot \mathrm{volt}$ $\qquad Cars = 55.61 \cdot \frac{\mathrm{Grads}}{\mathrm{sec}}$

Adimensional function: $\boxed{V7_{fm}(t, \omega_c, \omega_{fmm}, A_{fm}, m_{fm}) := \frac{v_{fmts}(t, \omega_c, \omega_{fmm}, A_{fm}, m_{fm})}{V}}$



Periodic Signals

28 FM test signal (triangular wave)

$V_{pp} := 30 \cdot V$

Modulating triangular voltage wave: $vtri_m(t) := v_{trio}(t, T_{fmm}, B_{fmm})$ $\quad Kst_{fm} = 3.442 \times 10^8 \cdot \frac{1}{volt \cdot sec}$

Generic FM signal: $v_{fmts}(t) = A \cdot \cos(\varphi(t))$ $\quad B_{fmm} = 15 \cdot V$

where: $\varphi(t) = \omega_c \cdot t + Kst_{fm} \cdot \int vtri_m(t)\, dt$

results:

$$\int v_{trio}(t)\, dt = \frac{4 \cdot B_{fmm}}{T_{fmm}} \cdot \sum_{k=0}^{N1} \left[\frac{\Phi(t - T_{fmm} \cdot k) \cdot (t - T_{fmm} \cdot k)^2}{2} \ldots + (-1) \cdot \frac{\Phi\left[t - T_{fmm} \cdot \left(k + \frac{1}{2}\right)\right] \cdot \left[2 \cdot t - 2 \cdot T_{fmm} \cdot \left(k + \frac{1}{2}\right)\right]^2}{4} \ldots + \frac{\Phi[t - T_{fmm} \cdot (k+1)] \cdot [t - T_{fmm} \cdot (k+1)]^2}{2} \right] - B_{fmm} \cdot t$$

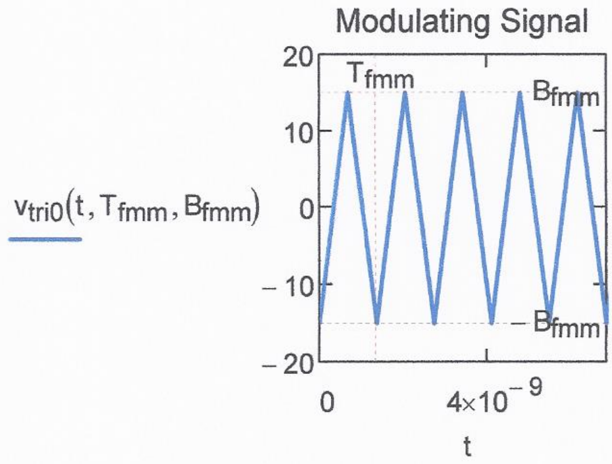
or written as a function of t.

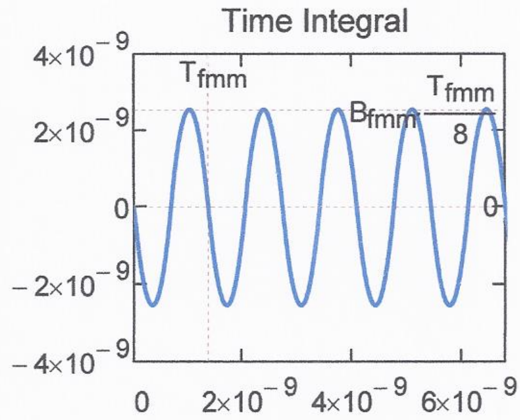
$N1 = 50$

$$I_{vtri}(t, T_{fmm}, B_{fmm}) := \frac{4 \cdot B_{fmm}}{T_{fmm}} \cdot \sum_{k=0}^{N1} \left[\frac{\Phi(t - T_{fmm} \cdot k) \cdot (t - T_{fmm} \cdot k)^2}{2} \ldots + (-1) \cdot \Phi\left[t - T_{fmm} \cdot \left(k + \frac{1}{2}\right)\right] \cdot \left[t - T_{fmm} \cdot \left(k + \frac{1}{2}\right)\right]^2 \ldots + \frac{\Phi[t - T_{fmm} \cdot (k+1)] \cdot [t - T_{fmm} \cdot (k+1)]^2}{2} \right] - B_{fmm} \cdot t$$

$I_{vtrimax} := B_{fmm} \cdot \frac{T_{fmm}}{8}$ $\quad I_{vtrimax} = 2.542 \cdot V \cdot ns$

$T_{fmm} = 1.356 \times 10^{-3} \cdot \mu s$ $\quad t := 0 \cdot T_{fmm}, \frac{10 \cdot T_{fmm} - 0 \cdot T_{fmm}}{5000} .. 10 \cdot T_{fmm}$ $\quad T_c = 0.1 \cdot ns$ $\quad \frac{T_{fmm}}{T_c} = 13.558$

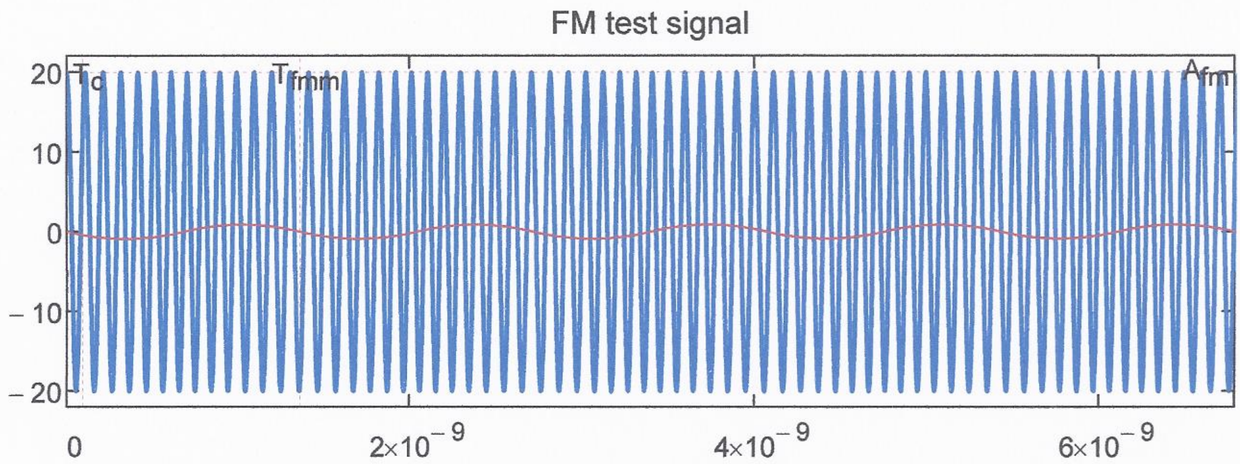




FM signal: $v_{fm3}(t_{fm}, \omega_c, \omega_{fmm}, A_{fm}, B_{fmm}, m_{fm}) := A_{fm} \cdot \cos(\omega_c \cdot t_{fm} + Kst_{fm} \cdot I_{vtri}(t_{fm}, T_{fmm}, B_{fmm}))$

Adimensional function:

$m_{fm} = 7$ $\boxed{V8_{fm}(t_{fm}, \omega_c, \omega_{fmm}, A_{fm}, B_{fmm}, m_{fm}) := \frac{v_{fm3}(t_{fm}, \omega_c, \omega_{fmm}, A_{fm}, B_{fmm}, m_{fm})}{V}}$ $A_{fm} = 20 \cdot V$



$$\frac{\omega_c}{\omega_{fmm}} = 13.558$$

TEST SIGNALS

Periodic Signals

29 PM test signal (single tone)

Carrier Amplitude:..: $\boxed{A_{pm} := 20 \cdot volt}$, $A_{pm} = 20 \cdot V$

Scroll the slider to change the carrier frequency in the range (1MHz - 10GHz):

$j_p :=$

$j_p = 1 \times 10^7 \quad f_{0ts} = 1 \times 10^{-6} \cdot GHz$

Carrier Frequency..: $f_c := j_p \cdot f_{0ts}$, $\quad \boxed{f_c = 1 \times 10^4 \cdot MHz}$,

Carrier period...: $T_c := \dfrac{1.0}{f_c}$, $\quad T_c = 0.1 \cdot ns$,

Angular frequency of the carrier............................: $\omega_c := 2.0 \cdot \pi \cdot f_c$, $\quad \omega_c = 62.832 \cdot \dfrac{Grads}{sec}$,

Amplitude of the modulating signal.......................: $\boxed{B_{pm} := 8 \cdot volt}$

Modulating signal period..: $T_{pmm} := T_c \cdot 10$, $\quad f_{pmm} := \dfrac{1}{T_{pmm}}$,

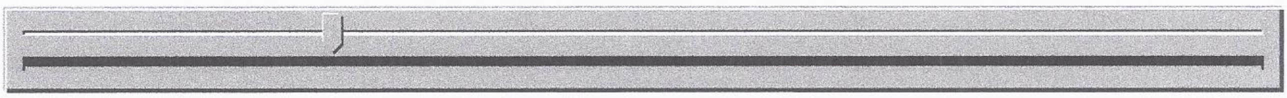
$T_{pmm} = 1 \times$

Frequency of the harmonic modulating signal.....: $\boxed{f_{pmm} = \blacksquare \cdot MHz}$, $\quad \dfrac{T_{pmm}}{T_c} = 10$,

Angular frequency of the modulating signal:.......: $\omega_{pmm} := 2.0 \cdot \pi \cdot f_{pmm}$, $\quad \omega_{pmm} = 6.283 \times 10^3 \cdot \dfrac{Mrad}{sec}$

Scroll the slider to change the modulation index m_{pm}:

$j_{pm} :=$



$m_{pm} := \dfrac{j_{pm}}{100} \qquad k_{pm} := \dfrac{m_{pm}}{B_{pm}}$

Phase modulation index.....................................: $m_{pm} = 5.02 \cdot rad$

Phase-sensitivity factor.......................................: $k_{pm} = 0.627 \cdot \dfrac{rad}{V}$

$m_{pm} = 5.02 \qquad A_{pm} = 20 \cdot V$

$\omega_c = 6.283 \times 10^4 \cdot \dfrac{Mrads}{sec} \qquad B_{pm} = 8 \cdot V \qquad \omega_{pmm} = 6.283 \times 10^3 \cdot \dfrac{Mrads}{sec}$

For any modulating signal $v_m(t)$, results: $v_{pm}(t) := A_{pm} \cdot \cos\left(\omega_c \cdot t + k_{pm} \cdot v_m(t)\right)$

$$v_{pm}(t, \omega_c, \omega_{pmm}, A_{pm}, m_{pm}) := A_{pm} \cdot \sum_{k=-N}^{N} \left[Jn(k, m_{pm}) \cdot \cos\left[\left(\omega_c + k \cdot \omega_{pmm}\right) \cdot t - k \cdot f_m(t)\right]\right]$$

while for a cosinusoidal test signal, the modulated carrier is:

$$v_{pm}(t) := A_{pm} \cdot \cos(\omega_c \cdot t + m_{pm} \cdot \cos(\omega_{pmm} \cdot t))$$

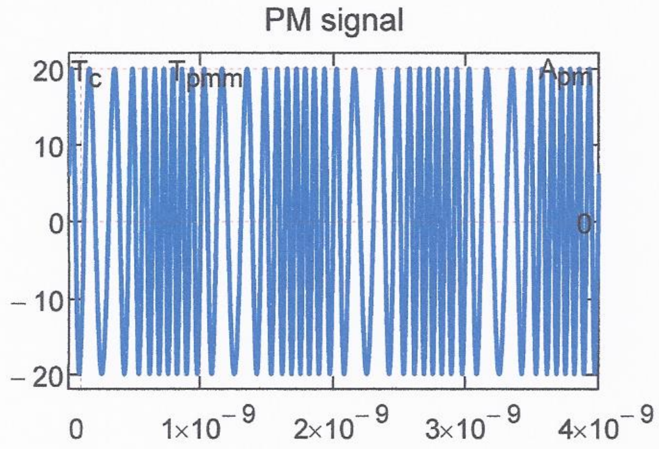
$$v_{pm}(t, \omega_c, \omega_{pmm}, A_{pm}, m_{pm}) := A_{pm} \cdot \cos(\omega_c \cdot t + m_{pm} \cdot \cos(\omega_{pmm} \cdot t))$$

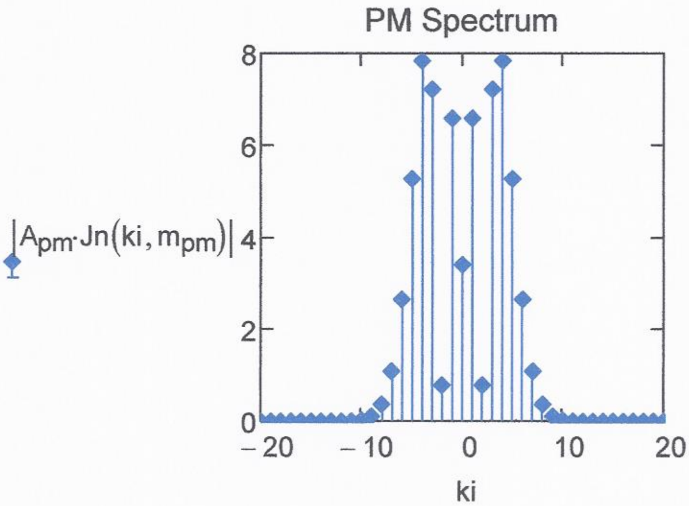
$$v_{pm}(t, \omega_c, \omega_{pmm}, A_{pm}, m_{pm}) := \mathrm{Re}\left[A_{pm} \cdot e^{j \cdot \omega_c \cdot t} \cdot \sum_{k=-N}^{N} \left(e^{j \cdot \frac{k \cdot \pi}{2}} \cdot Jn(k, m_{pm}) \cdot \cos(k \cdot \omega_{pmm} \cdot t)\right)\right]$$

Adimensional function: $V9_{pm}(t, \omega_c, \omega_{pmm}, A_{pm}, m_{pm}) := \dfrac{v_{pm}(t, \omega_c, \omega_{pmm}, A_{pm}, m_{pm})}{V}$

$A_{pm} = 20 \cdot volt$

$B_{pm} = 8 \cdot volt$ $\quad t_{pm} := T_c \cdot 0, T_c \cdot 0 + \dfrac{40 \cdot T_c - 0 \cdot T_c}{5000} .. 40 \cdot T_c$ $\quad \dfrac{T_{pmm}}{T_c} = 10$





TEST SIGNALS

Periodic Signals

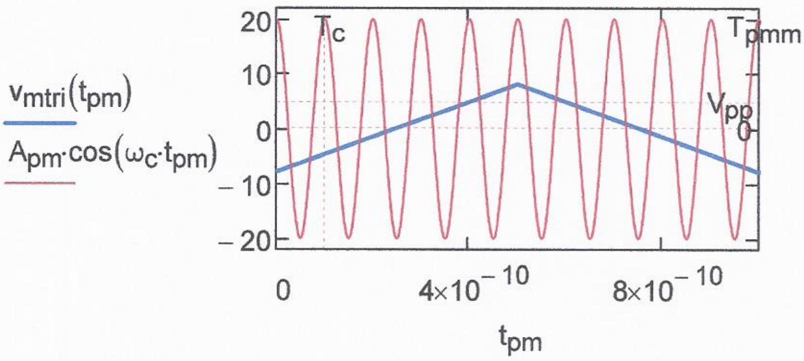
30 PM test signal (triangular wave)

$V_{pp} = 5 \cdot V$ $\quad V_{pp} := 15V$ $\quad k_{pm} := \dfrac{m_{pm}}{B_{pm}}$ $\quad k_{pm} = 0.627 \cdot \dfrac{rad}{volt}$

$\omega_{pmm} = 6.283 \times 10^3 \cdot \dfrac{Mrads}{sec}$ $\quad v_{mtri}(t) := v_{trio}(t, T_{pmm}, B_{pm})$ $\quad \dfrac{T_{pmm}}{T_c} = 10$ $\quad \dfrac{A_{pm}}{B_{pm}} = 2.5$

$B_{pm} = 8 \cdot V$ $\quad v_{pm}(t) := A_{pm} \cdot \cos(\omega_c \cdot t + k_{pm} \cdot v_{mtri}(t))$

$A_{pm} = 20 \cdot V$ $\quad t_{pm} := T_{pmm} \cdot 0, T_{pmm} \cdot 0 + \dfrac{5 \cdot T_{pmm} - 0 \cdot T_{pmm}}{5000} .. 5 \cdot T_{pmm}$



$$\omega_c = 62.832 \cdot \frac{\text{Grads}}{\text{sec}}$$

$$\omega_{pmm} = 6.283 \times 10^3 \cdot \frac{\text{Mrads}}{\text{sec}}$$

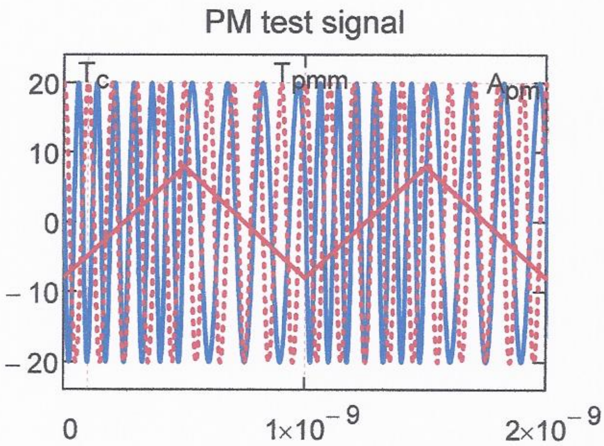
$B_{pm} = 8 \cdot V$ $\qquad m_{pm} = 5.02$

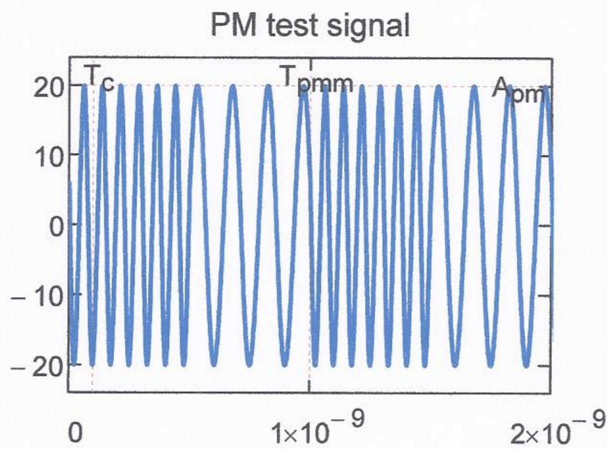
$$v_{pmtri}(t, \omega_c, \omega_{pmm}, A_{pm}, m_{pm}) := A_{pm} \cdot \cos\left(\omega_c \cdot t + \frac{m_{pm}}{B_{pm}} \cdot v_{tri0}\left(t, \frac{2 \cdot \pi}{\omega_{pmm}}, V_{pp}\right)\right)$$

Adimensional function:

$$V10_{pm}(t, \omega_c, \omega_{pmm}, A_{pm}, B_{pm}, m_{pm}) := \frac{A_{pm} \cdot \cos\left(\omega_c \cdot t + \frac{m_{pm}}{B_{pm}} \cdot v_{tri0}\left(t, \frac{2 \cdot \pi}{\omega_{pmm}}, B_{pm}\right)\right)}{V}$$

$\omega_c = 62.832 \cdot \frac{\text{Grads}}{\text{sec}}$ $\qquad \omega_{pmm} = 6.283 \times 10^3 \cdot \frac{\text{Mrads}}{\text{sec}}$ $\quad A_{pm} = 20 \cdot V$ $\quad B_{pm} = 8 \cdot V$ $\qquad k_{pm} = 0.627 \cdot \frac{\text{rad}}{\text{volt}}$





$\omega_c = 62.832 \cdot \frac{\text{Grads}}{\text{sec}}$ $\qquad \omega_{fmm} = 4.634 \times 10^3 \cdot \frac{\text{Mrads}}{\text{sec}}$ $A_{fm} = 20 \cdot V$ $\qquad B_{fmm} = 15 \cdot V$

Periodic Signals

31 Staircase based test signal .

$$T_T := 2 \cdot T_{2stpl\_} \cdot (3 \cdot m2_{steps} + 4) + 5 \cdot T_{2stpl\_}$$

$$T_{2stpl\_} := \frac{T_T}{2 \cdot (3 \cdot m2_{steps} + 5)} \qquad T_{2stpl\_} = 4.207 \cdot \mu s$$

$$v_{HH}(t, T_T, V_{stc}, m2_{steps}) := \sum_{k=0}^{10} v_H\left[t - k \cdot T_T, \frac{T_T}{2 \cdot (3 \cdot m2_{steps} + 5)}, V_{stc}, m2_{steps}\right]$$

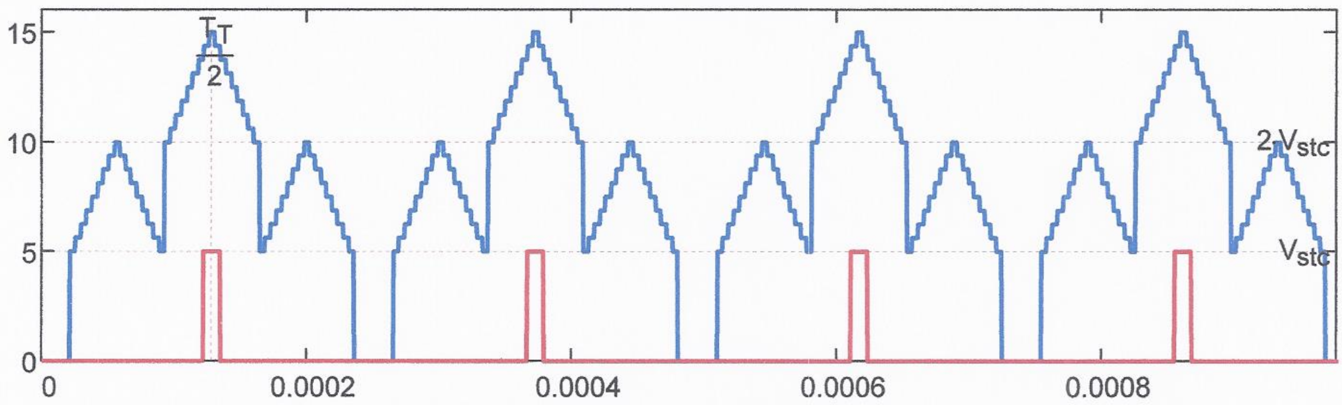
V

$$V_H(t, T_T, V_{stc}, m3_{steps}) := \frac{v_{HH}(t, T_T, V_{stc}, m2_{steps})}{V} \qquad V_{stc} = 5 \cdot V$$

$$v_{HD}(t, T_T, V_{stc}, m2_{steps}) := \sum_{k=0}^{10} v_{HDoor}\left(t - k \cdot T_T - \frac{3 \cdot T_{2stpl\_}}{2}, T_{2stpl\_}, V_{stc}, m2_{steps}\right)$$

$$t := 0 \cdot T_H, 0 \cdot T_H + \frac{10 \cdot T_H}{5000} .. 10 \cdot T_H$$

Staircase based test signal...



$$V_{HD}(t, T_T, V_{stc}, m2_{steps}) := \sum_{k=0}^{10} v_{HDoor}\left[t - k \cdot T_T - \frac{3}{2} \cdot \frac{T_T}{2 \cdot (3 \cdot m2_{steps} + 5)}, \frac{T_T}{2 \cdot (3 \cdot m2_{steps} + 5)}, V_{stc}, m2_{steps}\right]$$

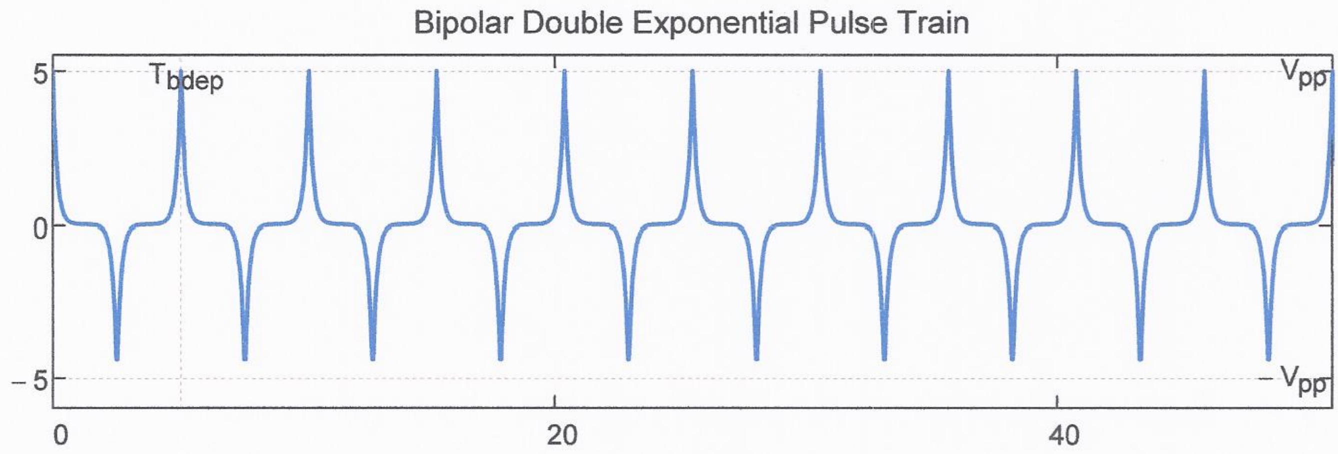
Periodic Signals

32 Bipolar Double Exponential Pulse Train

$$T_{bdep} := 32 \cdot \tau_{ts}$$

$$V_{bdept}(t, \tau_{ts}, T, V_{pp}) := \sum_{k=0}^{N1} V_{bdep}(t - k \cdot T, \tau_{ts}, V_{pp})$$

$$t := -20 \cdot T_{bdep}, -20 \cdot T_{bdep} + \frac{20 \cdot T_{bdep} + 20 \cdot T_{bdep}}{5000} .. 20 \cdot T_{bdep}$$



$$V_{bdepta}(t, \tau_{ts}, T, V_{pp}) := \frac{V_{bdept}(t, \tau_{ts}, T, V_{pp})}{V}$$

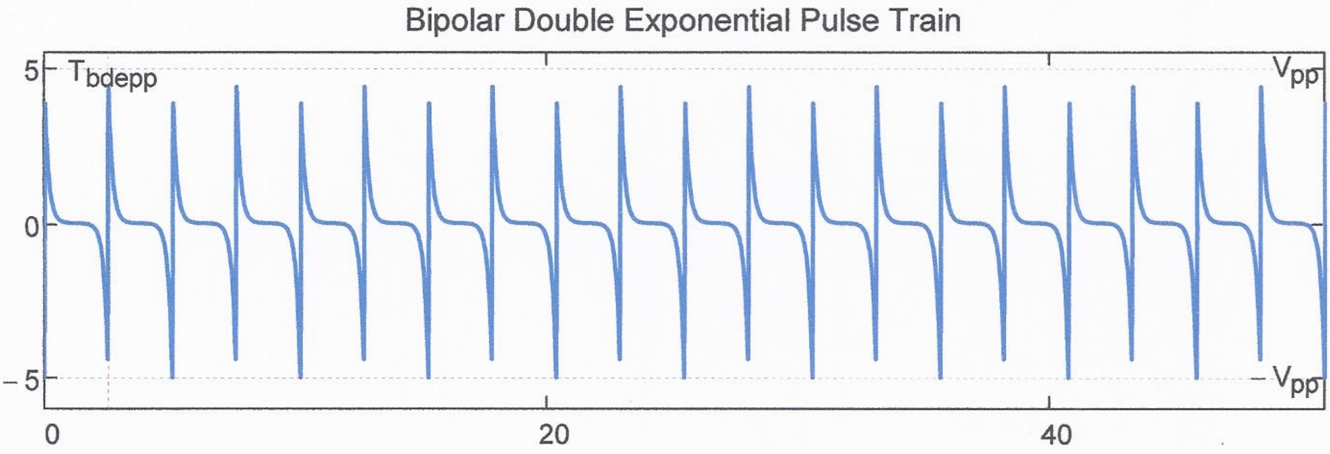
TEST SIGNALS

Periodic Signals

33 Bipolar Double Exponential Odd symmetrc Pulse Train

$$T_{bdepp} := 16 \cdot \tau_{ts}$$

$$V_{bdeospp}(t, \tau_{ts}, T, V_{pp}) := \sum_{k=0}^{N1} V_{bdeosp}(t - k \cdot T, \tau_{ts}, V_{pp})$$



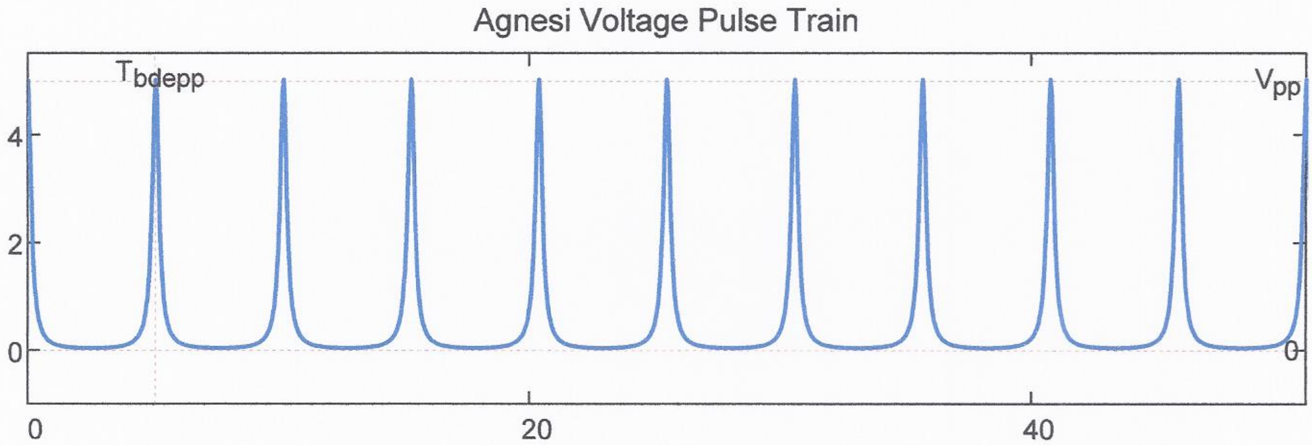
TEST SIGNALS

Periodic Signals

34 Agnesi Profile Voltage Pulse Train

$$T_{bdepp} := 32 \cdot \tau_{ts}$$

$$V_{agnp}(t, \tau_{ts}, T, V_{pp}) := \sum_{k=0}^{N1} V_{agn}(t - k \cdot T, \tau_{ts}, V_{pp})$$



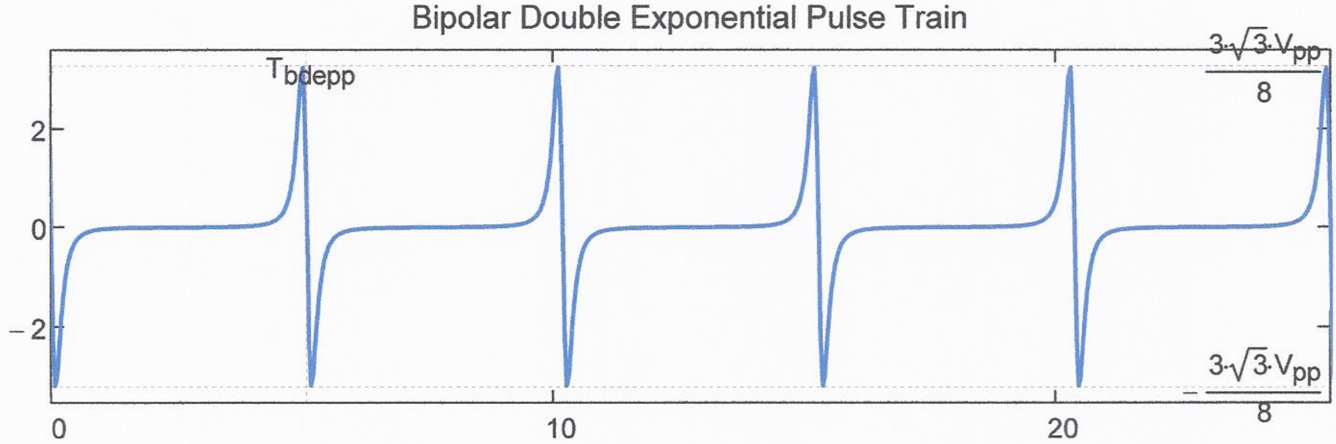
TEST SIGNALS

Periodic Signals

35 Agnesi Derivative Profile Voltage Pulse Train

$$T_{bdepp} := 32 \cdot \tau_{ts}$$

$$V_{Dagnp}(t, \tau_{ts}, T, V_{pp}) := \sum_{k=0}^{N1} V_{Dagn}(t - k \cdot T, \tau_{ts}, V_{pp})$$



TEST SIGNALS

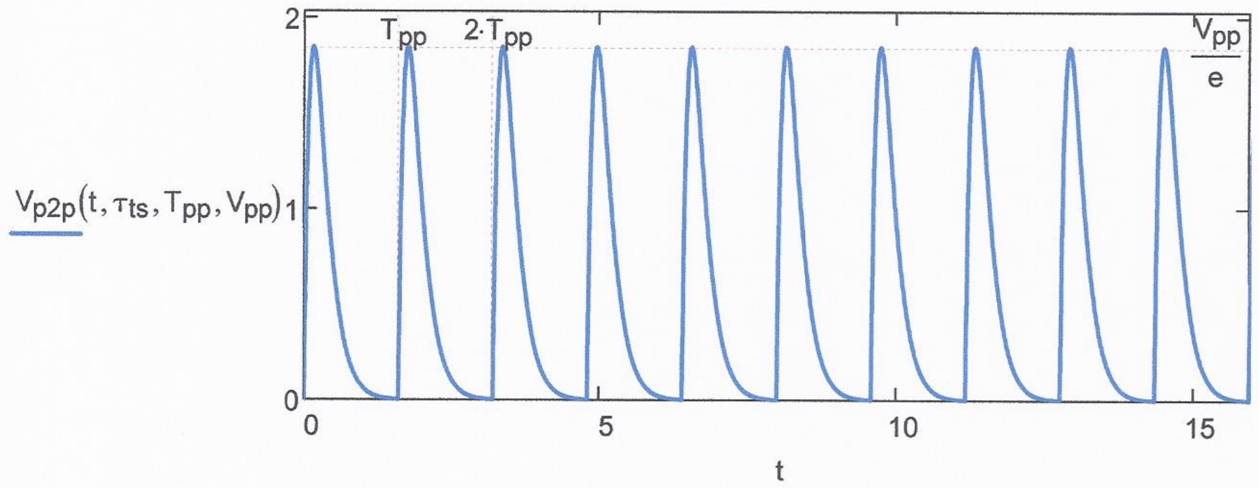
Periodic Signals

36 Poisson Profile Voltage Pulse Train

$$t := 0 \cdot \tau_{ts}, 0 \cdot \tau_{ts} + \frac{200 \cdot \tau_{ts}}{5000} .. 200 \cdot \tau_{ts}$$

$N1 = 50 \qquad \tau_{ts} = 0.159\,s \qquad T_{pp} := 10 \cdot \tau_{ts}$

$$V_{p2p}(t, \tau_{ts}, T, V_{pp}) := \sum_{k=0}^{N1} \left(V_p(t - k \cdot T, \tau_{ts}, V_{pp}) \cdot \mathrm{rect1}(t - k \cdot T, 0 \cdot T, T)\right)$$

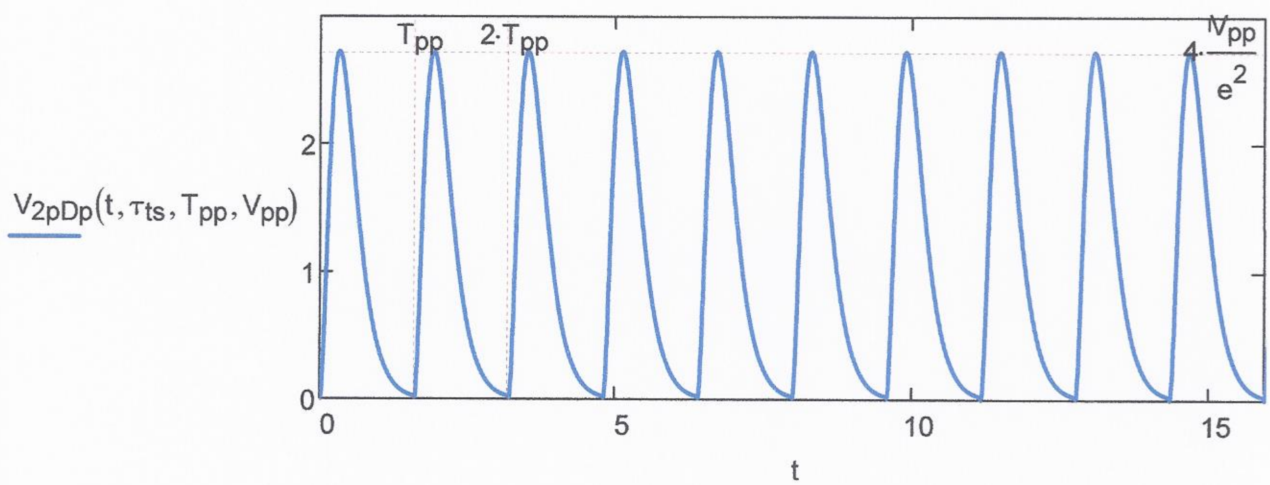


TEST SIGNALS

Periodic Signals

37 Poisson Derivative Profile Voltage Pulse Train

$$V_{2pDp}(t, \tau_{ts}, T, V_{pp}) := \sum_{k=0}^{N1} \left(V_{2p}(t - k \cdot T, \tau_{ts}, V_{pp}) \cdot \text{rect1}(t - k \cdot T, 0 \cdot T, T)\right)$$

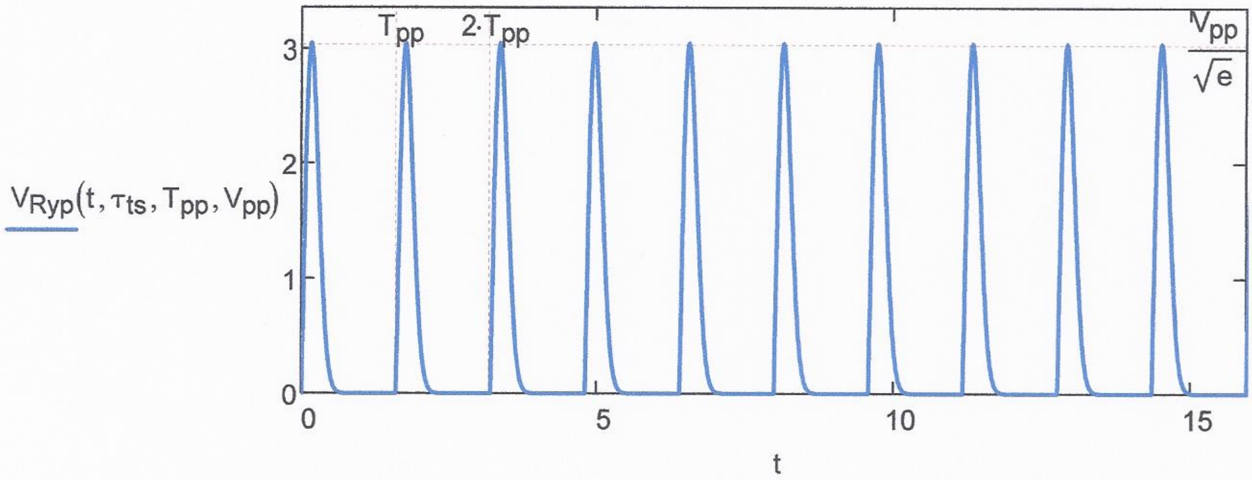


TEST SIGNALS

Periodic Signals

38 Rayleigh Profile Voltage Pulse Train

$$V_{Ryp}(t,\tau_{ts},T,V_{pp}) := \sum_{k=0}^{N1} \left(V_{Ry}(t-k\cdot T,\tau_{ts},V_{pp})\cdot \mathrm{rect1}(t-k\cdot T,0\cdot T,T)\right)$$



Worksheet "global data.xmcd"

Definitions and necessary constants.

$krads \equiv 10^3 \cdot rad$, $Mrads \equiv 10^6 \cdot rad$, $Grads \equiv 10^3 \cdot Mrads$, $THz \equiv 10^3 \cdot GHz$.,

$Trads \equiv 10^3 \cdot Grads$

$dB \equiv 1$, $nH \equiv 10^{-3} \cdot \mu H$, $rt \equiv 1.0\%$

Op. Amp. saturation voltage: $V_{sat} \equiv 15 \cdot volt$,

$\mu mho \equiv 10^{-6} \cdot mho$ $nmho \equiv 10^{-9} \cdot mho$ $m\Omega \equiv 10^{-3} \cdot \Omega$ $mmho \equiv 10^{-3} \cdot mho$

Generic Amplitude.........: $V_{pp} \equiv 5.0 \cdot V$

Signal frequency.............: $f_{0ts} \equiv 1.0 \cdot kHz$

Signal period...................: $T_{0ts} \equiv \frac{1}{f_{0ts}}$, $T_{0ts} = \blacksquare \cdot \mu s$

Signal angular frequency: $\omega_{0ts} \equiv 2 \cdot \pi \cdot f_{0ts}$, $\omega_{0ts} = \blacksquare \cdot \frac{Mrads}{sec}$,

Arbitrary time constant....: $\tau_{ts} \equiv \frac{1}{\omega_{0ts}} \cdot 1000$, $\tau_{ts} = \blacksquare \cdot \mu s$,

Step length.......................: $T_{stpl\_} \equiv \frac{T_{0ts}}{100}$,

Number of samples for the FFT: $N0 \equiv 2^8$,

Number of elements of a series: $N \equiv 50$

$k := 0 .. N0 - 1$,

The Bode diagrams will have an extension defined by a multiple $U \equiv 100$, of ω_s, freely chosen.

Worksheet "RF data.xmcd"

RF Pulse Data

Reference:C:\new folder\global data.xmcd

Reference:C:\new folder\Pulse Train Data.xmcd

Reference:C:\new folder\Fourier Series.xmcd

Reference:C:\new folder\Dirac Pulse - formulas.xmcd

Step amplitude: $V_{rf} := B_{ptd}$, $V_{rf} = 5 \cdot V$

Signal frequency: $f_{rf} \equiv 70 \cdot f_{ptd}$,

Signal period: $T_{rf} := \dfrac{1}{f_{rf}}$

Signal angular frequency: $\omega_{rfpt} \equiv 2 \cdot \pi \cdot f_{rf}$, $\omega_{rfpt} = 4.398 \times 10^{-4} \cdot \dfrac{Grads}{sec}$,

$$\omega_{rfpt}^{2} = 1.934 \times 10^{-7} \cdot \left(\frac{Grads}{sec}\right)^{2}$$

time constant: $\tau_{rf} \equiv \dfrac{10}{\omega_{rfpt}}$,

Pulse Cadence: $f_{ptd} = 1 \times 10^{-3} \cdot MHz$ $T_{ptd} = 1 \times 10^{3} \cdot \mu s$

Pulse width: $\tau_{ptd} = 250 \cdot \mu s$

Frequency: $f_{ptd} = 1 \times 10^{-3} \cdot MHz$ $f_{ptd} = \dfrac{\delta_{ptd}}{\tau_{ptd}}$

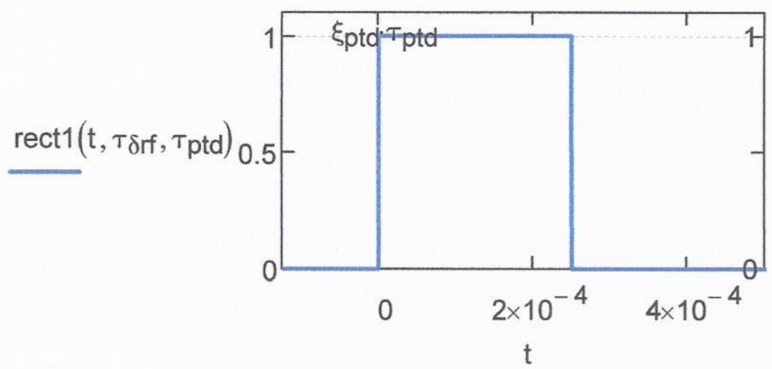
Duty Cycle: $\delta_{ptd} = 25 \cdot \%$ $\delta_{rfpt} := \dfrac{\tau_{ptd}}{T_{ptd}}$ $\delta_{rfpt} = 25 \cdot \%$

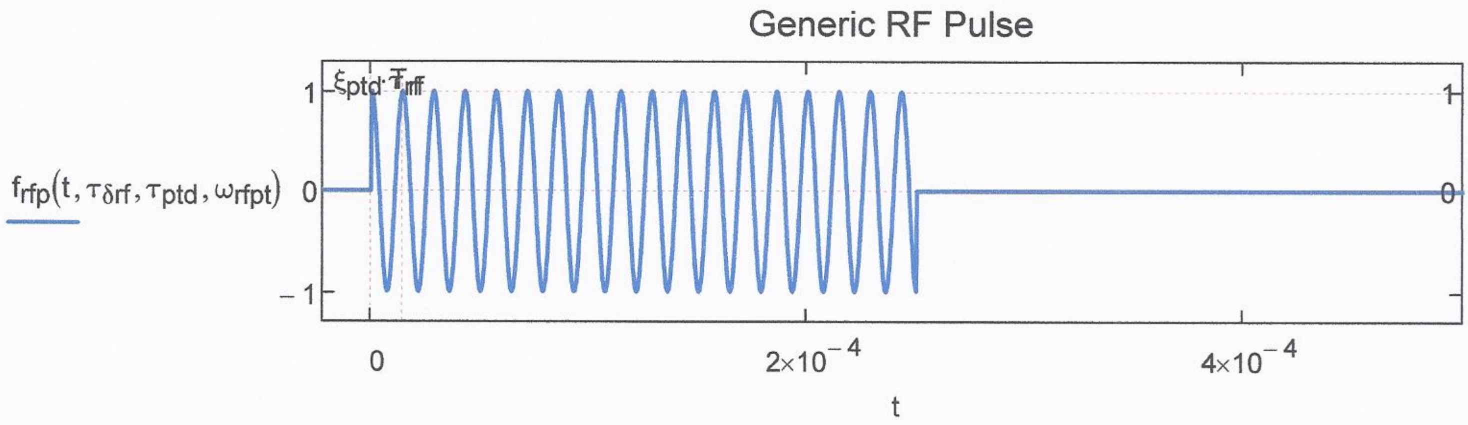
$\tau_{\delta rf} := 0 \cdot sec$

Generic pulse definition: $rect1(t, risingedge, width) := rect1(t, \tau_{\delta rf}, \tau_{pwts})$

$$f_{rfp}(t, \tau_{\delta rf}, \tau_{ptd}, \omega_{rfpt}) := rect1(t, \tau_{\delta rf}, \tau_{ptd}) \cdot \cos(\omega_{rfpt} \cdot t)$$

$$t := -1 \cdot T_{ptd}, -1 \cdot T_{ptd} + \frac{4 \cdot T_{ptd} + T_{ptd}}{20000} .. 4 \cdot T_{ptd}$$





Generic RF Pulse
$f_{rfp}(t, \tau_{\delta rf}, \tau_{ptd}, \omega_{rfpt})$
1
0
−1
0
2×10^{-4}
4×10^{-4}
t

Worksheet "sawtooth pulse data.xmcd

Reference:C:\new folder\global data.xmcd

Amplitude:	$V_{sawth} := 50 \cdot V$	$V_{sawth} = 50 \cdot V$
Sawtooth length:	$\delta_{sawth} := 1 \cdot \mu s$	$\delta_{sawth} = 1 \times 10^{3} \cdot ns$
Slope:	$p_{sawth} := \dfrac{V_{sawth}}{\delta_{sawth}}$	$p_{sawth} = 50 \cdot \dfrac{V}{\mu s}$
Period:	$T_{sawth} := 1 \cdot \delta_{sawth}$	
Frequence:	$f_{sawth} := \dfrac{1}{T_{sawth}}$	$f_{sawth} = 1 \cdot MHz$

Worksheet "staircase pulse data.xmcd"

Reference:C:\new folder\global data.xmcd

$$k_{stplength} := 300$$

Step Amplitude (arbitrary choise):	$V_{stcstp0} := \dfrac{V_{pp}}{5000}$	$V_{stcstp0} = 1 \cdot mV$
Number of steps:	$m1_{steps} := 2^3$	$m1_{steps} = 8$
Signal amplitude:	$V_{stcs} := V_{stcstp0} \cdot m1_{steps}$	$V_{stcs} = 8 \cdot mV$
Step length:	$T_{1stpl\_} := \dfrac{T_{0ts}}{k_{stplength} \cdot (m1_{steps} + 1)}$	$T_{1stpl\_} = 0.37 \cdot \mu s$
Period:	$T_{stcpt} := (m1_{steps} + 1) \cdot T_{1stpl\_} \cdot 2$	$T_{stcpt} = 6.667 \cdot \mu s$

Worksheet "staircase 2 pulse data.xmcd"

➔ Reference:C:\new folder\global data.xmcd

➔ Reference:C:\new folder\Pulse Train Data.xmcd

Signal amplitude:	$V_{stc} := V_{pp}$	$V_{stc} = 5 \cdot \mathrm{volt}$
Step length (arbitrary):	$T_{2stpl\_} := 4 \cdot \frac{T_{ptd}}{1000}$	$T_{2stpl\_} = 4 \cdot \mu s$
Number of steps:	$m2_{steps} := 2^3$	$m2_{steps} = 8$
Time constant:	$\tau 2\_ := \frac{T_{ptd}}{5}$	
Step amplitude:	$V_{stcstp} := \frac{V_{stc}}{m2_{steps}}$	$V_{stcstp} = 0.625 \cdot V$
Period:	$T2_{stp\_} := \left(m2_{steps} + 1\right) \cdot T_{2stpl\_} \cdot 2$	$T2_{stp\_} = 72 \cdot \mu s$

Worksheet "staircase 3 pulse data.xmcd"

Reference:C:\new folder\global data.xmcd

Reference:C:\new folder\Pulse Train Data.xmcd

Signal amplitude: $V_{stc3} := \frac{V_{pp}}{1000}$ $V_{pp} = \blacksquare \cdot V$ $V_{stc3} = \blacksquare \cdot mV$

Step length (Arbitrary): $T3_{stpl\_} := 4 \cdot \frac{T_{ptd}}{1000}$ $T3_{stpl\_} = \blacksquare \cdot \mu s$

Number of steps: $m3_{steps} := 2^2$

Time constant: $\tau 3\_ := \frac{T3_{stpl\_}}{5}$

Period: $T3 := 2 \cdot m3_{steps} \cdot T3_{stpl\_}$ $T3 = \blacksquare \cdot \mu s$

Frequence: $f33 := \frac{1}{T3}$ $f33 = \blacksquare \cdot MHz$

Worksheet "staircase 4 pulse data.xmcd"

Reference:C:\new folder\global data.xmcd

Reference:C:\new folder\Pulse Train Data.xmcd

Amplitude: $V_{stc4} := 15 \cdot V$

Signal amplitude: $V_{stc4} = 15 \cdot \text{volt}$

Step length: $T_{4stpl\_} := 4 \cdot T_{ptd}$ $\quad T_{4stpl\_} = 4 \cdot \text{ms}$

Number of steps: $m4_{steps} := 2^3$

Time constant: $\tau4\_ := \dfrac{T_{ptd}}{5}$

Period: $T_4 := 2 \cdot (m4_{steps} + 1) \cdot T_{0ts}$ $\quad T_4 = 18 \cdot \text{ms}$

Frequency: $f_{44} := \dfrac{1}{T_4}$ $\quad f_{44} = 5.556 \times 10^{-5} \cdot \text{MHz}$

$\omega_{44} := 2 \cdot \pi \cdot f_{44}$ $\quad \omega_{44} = 3.491 \times 10^{-4} \cdot \dfrac{\text{Mrads}}{\text{sec}}$

Worksheet "FM data.xmcd" (& theory)

When saving or printing, disable Automatic Calculation.

Reference:C:\new folder\global data.xmcd
Reference:C:\new folder\Fourier Series.xmcd
Reference:C:\new folder\Dirac Pulse - formulas.xmcd

Carrier Amplitude:..: $\boxed{A_{fm} := 20 \cdot volt}$

Scroll the slider, to change the carrier frequency in the range 100kHz-10GHz:

$j_p :=$

$j_p = 100 \qquad f_{0ts} = 1 \cdot kHz$

Carrier Frequency..: $f_c := j_p \cdot f_{0ts} \qquad \boxed{f_c = 1 \times 10^{-4} \cdot GHz}$

Carrier period.......: $T_c := \dfrac{1.0}{f_c} \qquad T_c = 1 \times 10^{4} \cdot ns$

Angular frequency of the carrier..: $\omega_c := 2.0 \cdot \pi \cdot f_c \; \omega_c = 6.283 \times 10^{-4} \cdot \dfrac{Grads}{sec}$

Amplitude of the modulating signal.....................................: $B_{fmm} := 15 \cdot volt$

Modulating signal period...: $T_{fmm} := T_c \cdot 1\text{(} f_{fmm} := \dfrac{1}{T_{fmm}}$

Frequency of the modulating signal.................................: $f_{fmm} = 0.01 \cdot MHz$

Angular frequency of the modulating signal:...................: $\omega_{fmm} := 2.0 \cdot \pi \cdot f_{fmm}$

Single tone Modulating signal..: $V_m(t)$

Scroll the slider to change the modulation index m_{fm} :

Frequency modulation index: $m_{fm} :=$

$m_{fm} = 5 \cdot rad \quad k_{fm} := \dfrac{m_{fm} \cdot \omega_{fmm}}{2 \cdot \pi \cdot B_{fmm}} \qquad \dfrac{T_{fmm}}{T_c} = 10 \qquad k_{fm} = 3.333 \times 10^{-3} \cdot \dfrac{MHz}{V}$

Frequency modulation index definition:............................ $: m_{fm} := \dfrac{2 \cdot k_{fm} \cdot \pi \cdot B_{fmm}}{\omega_{fmm}}$ (rad)

Frequency sensitivity factor...: $k_{fm} = \frac{m_{fm} \cdot \omega_{fmm}}{2 \cdot \pi \cdot B_{fmm}} \quad \left(\frac{Hz}{V}\right)$

FM Theory (general case).

The Istantaneous voltage is related to the Istantaneous phase θ(t) by the trigonometric relation:

$$v_{fm}(t) = A_{fm} \cdot \cos(\theta(t)) \quad (V)$$

while ***the istantaneous angular frequency*** of the FM signal is defined as $\omega_{fm}(t) = 2 \cdot \pi \cdot (f_c + \Delta f)$,

that is: $\omega_{fm}(t) = 2 \cdot \pi \cdot (f_c + k_{fm} \cdot v_m(t)) = 2 \cdot \pi \cdot f_c + 2 \cdot \pi \cdot k_{fm} \cdot v_m(t) \quad \left(\frac{rad}{sec}\right)$.

The Istantaneous frequency is : $f_{fm}(t) = f_c + k_{fm} \cdot v_m(t)$, namely $f_{fm}(t) = f_c + \Delta f(t)$ (Hz),

hence k_{fm} must have unit $\left(\frac{Hz}{V}\right)$.

The istantaneous frequency deviation is: $\Delta f(t) = k_{fm} \cdot v_m(t)$.

furthermore it is linked to the istantaneous phase by the relation: $\omega_{fm}(t) = \frac{d}{dt}\theta(t), \quad \left(\frac{rad}{sec}\right)$,

The Istantaneous phase is given, then, by: $\theta(t) = \int \omega_{fm}(t)\, dt \quad$ (rad),

$$\theta(t) = 2 \cdot \pi \cdot \int (f_c + k_{fm} \cdot v_m(t))\, dt = \omega_c \cdot t + 2 \cdot \pi \cdot k_{fm} \cdot \int v_m(t)\, dt,$$

Using the phasor representation one gets:

$$\mathbf{V_{fm}} = \mathbf{A_{fm}} \cdot e^{j \cdot \theta(t)},$$

that is:

$$\mathbf{V_{fm}} = \mathbf{A_{fm}} \cdot e^{j \cdot \int \omega(t)\, dt},$$

or:

$$\mathbf{V_{fm}} = \mathbf{A_{fm}} \cdot e^{j \cdot \int (2 \cdot \pi \cdot f_c + 2 \cdot \pi \cdot k_{fm} \cdot v_m(t))\, dt},$$

hence

$$\mathbf{V_{fm}} = \mathbf{A_{fm}} \cdot e^{j \cdot \left(\omega_c \cdot t + 2 \cdot \pi \cdot k_{fm} \cdot \int v_m(t)\, dt\right)},$$

The time domain modulated wave is: $v_{fm}(t) = \mathrm{Re}\left[\mathbf{A_{fm}} \cdot e^{j \cdot \left(\omega_c \cdot t + 2 \cdot \pi \cdot k_{fm} \cdot \int v_m(t)\, dt\right)}\right]$,

or:

$$v_{fm}(t) = A_{fm} \cdot \cos\left(\omega_c \cdot t + 2 \cdot \pi \cdot k_{fm} \cdot \int v_m(t)\, dt\right).$$

The Fourier trasform

$$\mathcal{F}(v_m(t)) = \int_{-\infty}^{\infty} v_m(t) \cdot e^{-j \cdot \omega \cdot t}\, dt$$

of the modulating signal is $F(\omega) = \mathcal{F}(v_m(t))$, so that one can express the modulating signal as an inverse Fourier trasform of $F(\omega)$,

$$v_m(t) = \frac{1}{2 \cdot \pi} \cdot \int_{-\infty}^{\infty} F(\omega) \cdot e^{j \cdot \omega \cdot t}\, d\omega\,,$$

hence the FM signal takes the general form:

$$v_{fm}(t) = A_{fm} \cdot \cos\left(\omega_c \cdot t + 2 \cdot \pi \cdot k_{fm} \cdot \int v_m(t)\, dt\right).$$

Single tone Modulating signal: $v_m(t) = B_{fmm} \cdot \cos(\omega_m \cdot t + \varphi 1)$, where $\varphi 1$ is an arbitrary initial phase for semplicity considered null: $v_m(t) := B_{fmm} \cdot \cos(\omega_{fmm} \cdot t)$.

Phasors:

*Phasors are written in **Bold** face.* $(j = \sqrt{-1}$

modulating signal......................: $\mathbf{V_m} = \mathbf{B_{fmm}} \cdot e^{j \cdot \omega_{fmm} \cdot t}$, $\mathbf{B_{fmm}} = |\mathbf{B_{fmm}}| \cdot e^{j \cdot \varphi 1}$

unmodulated carrier................. : $\mathbf{V_c} = \mathbf{A_{fm}} \cdot e^{j \cdot \omega_c \cdot t}$, : $\mathbf{A_{fm}} = |\mathbf{A_{fm}}| \cdot e^{j \cdot \theta 1}$,

where $\theta 1$ is an arbitrary initial phase, for semplicity considered null.

FM signal.................................. : $\mathbf{V_{fm}} = \mathbf{A_{fm}} \cdot e^{j \cdot \left(\omega_c \cdot t + 2 \cdot \pi \cdot k_{fm} \cdot \int v_m(t)\, dt\right)}$.

Hence $v_{fm}(t) = \mathrm{Re}(\mathbf{V_{fm}})$, $A_{fm} = 20 \cdot \mathrm{volt}$

As seen, ***the istantaneous frequency*** of the FM signal is defined as $f(t) = f_c + \Delta f(t)$,

The frequency deviation is: $\Delta f(t) = k_{fm} \cdot v_m(t) = k_{fm} \cdot B_{fmm} \cdot \cos(\omega_m \cdot t)$.

Therfore the istantaneous frequency is: $f(t) = f_c + k_{fm} \cdot B_{fmm} \cdot \cos(\omega_m \cdot t)$,

while the FM signal is: $v_{fm}(t) = A_{fm} \cdot \cos\left(\omega_c \cdot t + 2 \cdot \pi \cdot k_{fm} \cdot \int v_m(t)\, dt\right)$.

$$\int v_m(t)\, dt = \frac{B_{fmm} \cdot \sin(t \cdot \omega_{fmm})}{\omega_{fmm}}\,,$$

replacing it, one obtains: $v_{fm}(t) = A_{fm} \cdot \cos\left(\omega_c \cdot t + 2 \cdot \pi \cdot \frac{k_{fm} \cdot B_{fmm} \cdot \sin(t \cdot \omega_{fmm})}{\omega_{fmm}}\right)$.

From this relation on define the modulation index as: $m_{fm} = \frac{\Delta f_{max}}{f_{fmm}} = \frac{k_{fm} \cdot B_{fmm}}{\omega_{fmm}} = \frac{k_{fm} \cdot B_{fmm}}{f_{fmm}}$ (rad).

Finally the FM single tone signal is:

$$\boxed{v_{fm}(t) = A_{fm} \cdot \cos(\omega_c \cdot t + m_{fm} \cdot \sin(t \cdot \omega_{fmm}))},$$

or expressed as a phasor:

$$\boxed{v_{fm}(t) = |\mathbf{A_{fm}}| \cdot \mathrm{Re}\left[e^{j \cdot (\omega_c \cdot t + m_{fm} \sin(t \cdot \omega_{fmm}))}\right]}$$

So, it has been found that the modulation index is given by the relation:

$$m_{fm} = \frac{2\cdot\pi\cdot k_{fm}\cdot B_{fmm}}{\omega_{fmm}},$$

and the frequency sensitivity factor by:

$$k_{fm} = \frac{m_{fm}\cdot\omega_{fmm}}{2\cdot\pi\cdot B_{fmm}}.$$

The maximum frequency deviation is given by:

$$\boxed{\Delta f_{max} := k_{fm}\cdot B_{fmm}}.$$

The maximum and minimum frequency values are given by:

$$f_{fmmax} := f_c + \Delta f_{max},\quad f_{fmmin} := f_c - \Delta f_{max},$$

from which, results also: $\Delta f_{max} = \frac{1}{2}\cdot\left(f_{fmmax} - f_{fmmin}\right)$

To determine the spectra of the FM signal one must elaborate the expression of the signal. Well, observing the second term of of the FM signal exprressed as a phasor, one sees that the exponential can be replaced by the second Jacobi-Anger relation, as follows:

$$\mathbf{V_{fm}} = A_{fm}\cdot e^{j\cdot\omega_c\cdot t}\cdot\sum_{k=-\infty}^{\infty}\left(Jn(k,m_f)\cdot e^{j\cdot k\cdot\omega_{fmm}\cdot t}\right),$$

where Jn(k,x) is the k[th] Bessel function of the first kind. The result is a Bessel series of the single tone. Now, this representation of the FM signal (single tone modulated) as an infinite series, allows to distinguish the individual harmonics and therefore to draw the magnitude spectrum.
To do that, define the new function:

$$v_{fm}(t,\omega_c,\omega_{fmm},A_{fm},m_{fm}) := \mathrm{Re}\left[A_{fm}\cdot e^{j\cdot\omega_c\cdot t}\cdot\sum_{k=-N}^{N}\left(Jn(k,m_{fm})\cdot e^{j\cdot k\cdot\omega_{fmm}\cdot t}\right)\right] \text{ where } N = 50,$$

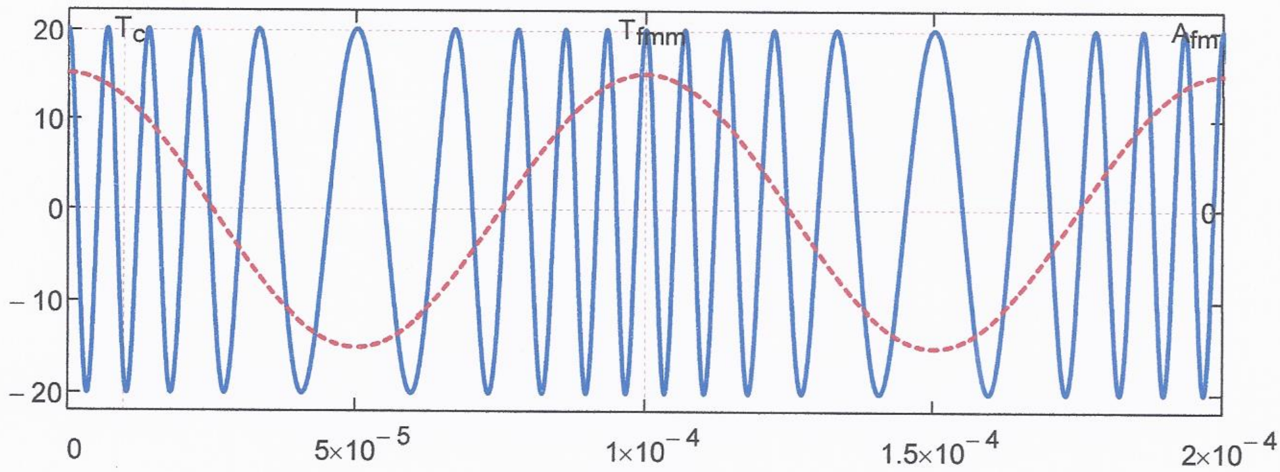
whose graph is here below depicted:

$$f_c = 1\times10^{-4}\cdot GHz\quad \Delta f_{max} = 5\times10^{-5}\cdot GHz$$

$$A_{fm} = 20\cdot volt\qquad B_{fmm} = 15\cdot volt\quad f_{fmmax} = 1.5\times10^{-4}\cdot G\quad f_{fmmin} = 5\times10^{-5}\cdot GHz\quad \frac{T_{fmm}}{T_c} = 10$$

$$t_{fm} := T_c\cdot 0, T_c\cdot 0 + \frac{40\cdot T_c - 0\cdot T_c}{20000} .. 40\cdot T_c$$

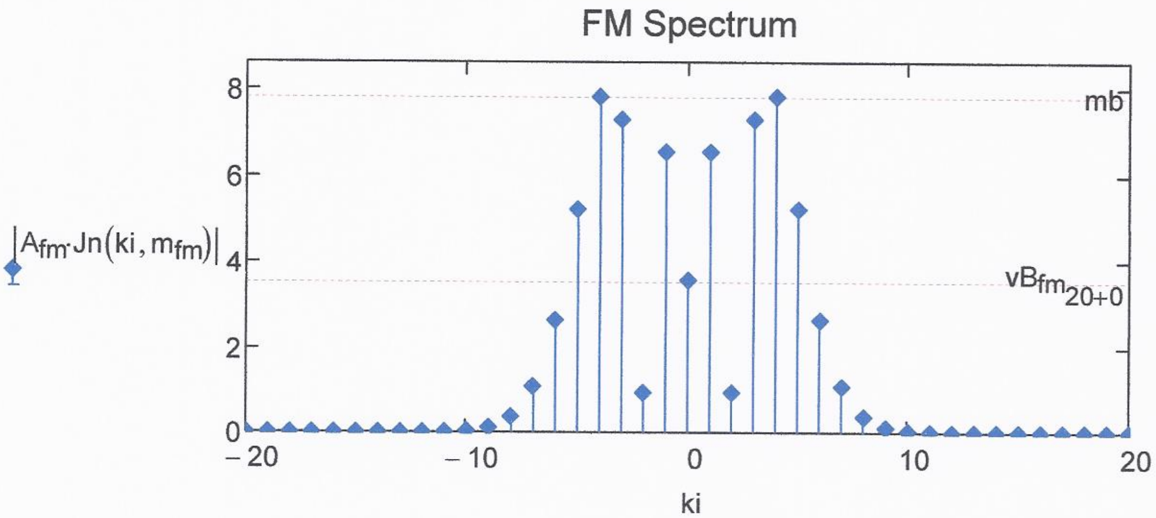
FM signal & Modulating Signal



the magnitude spectrum is:

$ki := -20 .. 20$ $\qquad vB_{fm_{20+ki}} := \left|A_{fm} \cdot Jn\left(ki, m_{fm}\right)\right|$ $\qquad mb := \max\left(vB_{fm}\right)$ $\qquad mb = 7.825 \cdot V$

$vB_{fm_{20+0}} = 3.552 \cdot V$



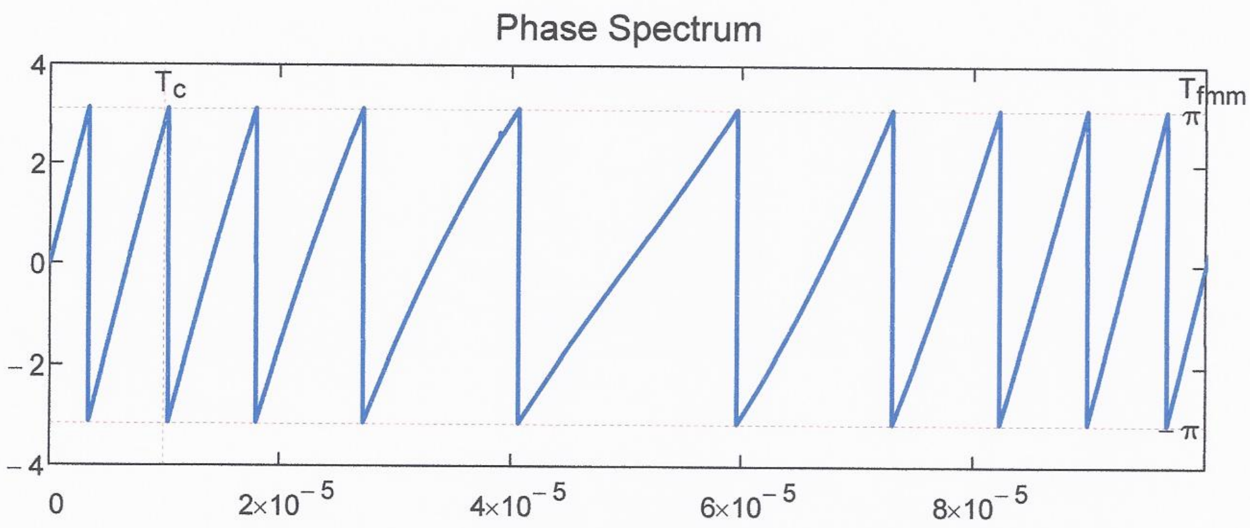
$T_{fmm} = 1 \times 10^{5} \cdot ns$ $\qquad m_{fm} = 5$ $\qquad \omega_{fmm} = 6.283 \times 10^{-5} \cdot \dfrac{Grads}{sec}$ $\qquad \dfrac{T_{fmm}}{T_c} = 10$

$A_{fm} = 20 \cdot volt$ $\qquad B_{fmm} = 15 \cdot volt$

Carson Band: $Cars := 2 \cdot \omega_{fmm} \cdot \left(m_{fm} + 1\right)$

$$Cars = 7.54 \times 10^{-4} \cdot \frac{Grads}{sec}$$

$$\varphi_{fm}(t) := \arg\left[A_{fm} \cdot e^{j \cdot \omega_c \cdot t} \cdot \sum_{k=-N}^{N} \left(Jn\left(k, m_{fm}\right) \cdot e^{j \cdot k \cdot \omega_{fmm} \cdot t}\right)\right]$$



Worksheet "PM data.xmcd"

When saving or printing, disable Automatic Calculation.

PM data (& theory)

Reference:C:\new folder\global data.xmcd
Reference:C:\new folder\Fourier Series.xmcd
Reference:C:\new folder\Dirac Pulse - formulas.xmcd

Carrier Amplitude:...: $\boxed{A_{pm} := 20 \cdot volt}$

Scroll the slider to change the carrier frequency in the range (100kHz - 10GHz):

$j_p :=$

$j_p = 1 \times 10^7 \quad f_{0ts} = 1 \cdot kHz$

Carrier Frequency..: $f_c := j_p \cdot f_{0ts},$ $\qquad \boxed{f_c = 10 \cdot GHz},$

Carrier period...: $T_c := \dfrac{1.0}{f_c},$ $\qquad T_c = 0.1 \cdot ns,$

Angular frequency of the carrier.............................: $\omega_c := 2.0 \cdot \pi \cdot f_c,$ $\qquad \omega_c = 62.832 \cdot \dfrac{Grads}{sec},$

Amplitude of the modulating signal........................: $\boxed{B_{pm} := 8 \cdot volt},$

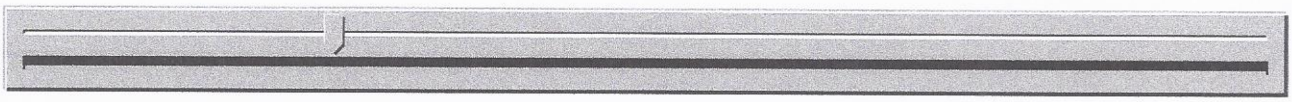
Modulating signal period..: $T_{pmm} := T_c \cdot 10,$ $\qquad f_{pmm} := \dfrac{1}{T_{pmm}},$

Frequency of the harmonic modulating signal.....: $\boxed{f_{pmm} = 1 \times 10^3 \cdot MHz},$ $\qquad \dfrac{T_{pmm}}{T_c} = 10,$

Angular frequency of the modulating signal:........: $\omega_{pmm} := 2.0 \cdot \pi \cdot f_{pmm},$ $\qquad \omega_{pmm} = 6.283 \times 10^3 \cdot \dfrac{Mrads}{sec}.$

Scroll the slider to change the modulation index m_{pm}:

$j_{pm} :=$



$$m_{pm} := \frac{j_{pm}}{100} \qquad k_{pm} := \frac{m_{pm}}{B_{pm}}$$

Phase modulation index....................................: $m_{pm} = 5.02 \cdot rad$

Phase-sensitivity factor.......................................: $k_{pm} = 0.627 \cdot \dfrac{rad}{V}$

Carrier signal:...: $v_c(t) := A_{pm} \cdot \cos(\omega_c \cdot t)$, ***phasor:*** $\mathbf{v_c = A_{pm}} \cdot e^{j \cdot \omega_c \cdot t}$

$$v_c(t) = \mathrm{Re}(\mathbf{v_c}) \qquad A_{pm} = 20 \cdot volt \qquad B_{pm} = 8 \cdot volt$$

PM Theory.

Single tone modulating voltage signal of amplitude B_{pm} and angular frequency $\omega 3_{pm}$ (of arbitrary initial phase $\varphi 1$=const.): $v_m(t) = B_{pm} \cdot \cos(\omega_{pmm} \cdot t + \varphi 1)$,

(represented as a phasor (*written in* ***Bold*** *face*): $\mathbf{V_m} = \mathbf{B_{pm}} \cdot e^{j \cdot (\omega_{pmm} \cdot t + \varphi 1)}$, so that: $v_m(t) = \mathrm{Re}(\mathbf{V_m})$).

The istantaneous voltage of the phase modulated carrier of amplitude A_{pm}, than is:

$$V_{pm}(t) = A_{pm} \cdot \cos(\theta(t)),$$

(represented as a phasor: $\mathbf{V_{pm}} = \mathbf{A_{pm}} \cdot e^{j \cdot \theta(t)}$). where the istantaneous phase is:

$$\theta(t) = \omega_c \cdot t + \theta_1 + k_{pm} \cdot v_m(t) = \omega_c \cdot t + \theta_1 + \Delta\theta(t), \text{ (radians) and } V_{pm}(t) = \mathrm{Re}(\mathbf{V_{pm}}) \text{ (Volt)},$$

where k_{pm} is the ***phase-sensitivity factor*** and the ***istantaneous phase deviation*** is:

$$\Delta\theta(t) = k_{pm} \cdot v_m(t) = k_{pm} \cdot B_{pm} \cdot \cos(\omega_{pmm} \cdot t + \varphi 1),$$

The resulting phase modulated signal is: $\boxed{v_{pm}(t) = A_{pm} \cdot \cos(\omega_c \cdot t + k_{pm} \cdot v_m(t) + \theta_1)}$

namely $\theta(t) = \omega_c \cdot t + \theta_1 + k_{pm} \cdot B_{pm} \cdot \cos(\omega_{pmm} \cdot t + \varphi 1)$, (radians) (arbitrary initial phases: $\varphi 1$=const., θ_1=const.).

The initial phase is: $\theta(0) = \theta_1 + k_{pm} \cdot v_m(0)$,

while the initial modulating voltage is: $v_m(0) = B_{pm} \cdot \cos(\varphi 1)$,

Substituting one gets: $\theta(0) = \theta_1 + k_{pm} \cdot B_{pm} \cdot \cos(\varphi 1)$ (radians),

The product $k_{pm}B_{pm}$ has dimension (radians), while k_{pm} (radians/volt),

The istantaneous angular frequency of the PM signal is defined as $\omega(t) = \frac{d}{dt}\theta(t)$,

while the istantaneous frequency is $f(t) = \frac{1}{2 \cdot \pi} \cdot \frac{d}{dt}\theta(t)$,

therfore

$$\omega(t) = \frac{d}{dt}(\omega_c \cdot t + \theta_1 + k_{pm} \cdot v_m(t)) = \omega_c + k_{pm} \cdot \frac{d}{dt} v_m(t), \text{ (rad/sec)}$$

For a single tone, as previously seen, the istantaneous phase is: $\theta(t) = \omega_c \cdot t + k_{pm} \cdot B_{pm} \cdot \cos(\omega_{pmm} \cdot t + \varphi 1)$ (radians), and the resulting phase modulated signal is:

$$\boxed{v_{pm}(t) = A_{pm} \cdot \cos(\omega_c \cdot t + \theta_1 + k_{pm} \cdot B_{pm} \cdot \cos(\omega_{pmm} \cdot t + \varphi 1))}.$$

The maximum *phase deviation* is: $\Delta\theta_{max} = \pm k_{pm} \cdot B_{pm}$ (radians).

The istantaneous angular frequency is defined as $\omega(t) = \frac{d}{dt}\theta(t) = \omega_c + k_{pm} \cdot \frac{d}{dt} v_m(t)$,

results:

$$\frac{d}{dt} v_m(t) = -B_{pm} \cdot \omega_{pmm} \cdot \sin(\varphi + t \cdot \omega_{pmm}) \text{ (V radians/sec)}$$

hence ***the istantaneous angular frequency is***: $\omega(t) = \omega_c - B_{pm} \cdot k_{pm} \cdot \omega_{pmm} \cdot \sin(t \cdot \omega_{pmm})$ (radians/sec),

The maximum istantaneous angular frequency is ,

$$\omega_{max} = \omega_c + B_{pm} \cdot k_{pm} \cdot \omega_{pmm} = \omega_c + m_{pm} \cdot \omega_{pmm} \quad \text{(radians/sec)}$$

The maximum *angular frequency deviation* is: $\Delta\omega_{max} = \omega_{max} - \omega_c = B_{pm} \cdot k_{pm} \cdot \omega_{pmm}$ (radians/sec)

$$\boxed{\frac{\Delta\omega_{max}}{\omega_{pmm}} = \frac{\Delta f_{max}}{f_{pmm}} = \Delta\theta_{max} = B_{pm}\cdot k_{pm} = m_{pm}}$$ phase modulation index.

Resuming:

$$\Delta f_{max} := m_{pm}\cdot f_{pmm} \;,$$
$$B_{pm} = 8\cdot V$$
$$\Delta f_{max} = 5.02\cdot GHz \;,$$
$$k_{pm} := \frac{m_{pm}}{B_{pm}}$$
$$k_{pm} = 0.627\cdot\frac{rad}{V},$$
$$m_{pm} = 5.02\cdot rad$$

Hence the modulated signal : $v_{pm}(t) = A_{pm}\cdot\cos\left(\omega_c\cdot t + m_{pm}\cdot\cos\left(\omega_{pmm}\cdot t+\varphi\right)+\theta_1\right)$.

To simplify, consider all initial phases null , therfore the modulated signal is:

$$\boxed{v_{pmst}(t, m_{pm}) := A_{pm}\cdot\cos\left(\omega_c\cdot t + m_{pm}\cdot\cos\left(\omega_{pmm}\cdot t\right)\right)},$$

and $\Delta\theta(t) = m_{pm}\cdot\cos\left(\omega_{pmm}\cdot t\right)$.

The corresponding phasor is: $\mathbf{V_{pmst}} = \mathbf{A_{pm}}\cdot e^{j\cdot\left(\omega_c\cdot t + m_{pm}\cdot\cos\left(\omega_{pmm}\cdot t\right)\right)}$.

$$v_{pmst}(t) = \left|\mathbf{A_{pm}}\right|\cdot \mathrm{Re}\left[e^{j\cdot\left(\omega_c\cdot t + m_{pm}\cdot\cos\left(\omega_{pmm}\cdot t\right)\right)}\right]$$

which, using the Jacobi-Anger relation :

$$\boxed{e^{j\cdot z\cdot\cos(\theta)} = \sum_{k=-\infty}^{\infty}\left(j^k\cdot Jn(k,z)\cdot\cos(k\cdot\theta)\right)}$$

where Jn(k,x) is the k^{th} Bessel function of the first kind, and limiting the bounds of the sum to $-n \le k \le n$, with $n := 40$ for example, one gets:

$$\mathbf{V_{pmja}} = \mathbf{A_{pm}}\cdot e^{j\cdot\omega_c\cdot t}\cdot e^{j\cdot m_{pm}\cdot\cos\left(\omega_{pmm}\cdot t\right)} = \mathbf{A_{pm}}\cdot e^{j\cdot\omega_c\cdot t}\cdot\sum_{k=-\infty}^{\infty}\left(e^{j\cdot\frac{k\cdot\pi}{2}}\cdot Jn\left(k, m_{pm}\right)\cdot\cos\left(k\cdot\omega_{pmm}\cdot t\right)\right)$$

$n = 40$ $$\boxed{v_{pmja}(t, m_{pm}) := \mathrm{Re}\left[A_{pm}\cdot e^{j\cdot\omega_c\cdot t}\cdot\sum_{k=-n}^{n}\left(e^{j\cdot\frac{k\cdot\pi}{2}}\cdot Jn\left(k, m_{pm}\right)\cdot\cos\left(k\cdot\omega_{pmm}\cdot t\right)\right)\right]}$$

Example:

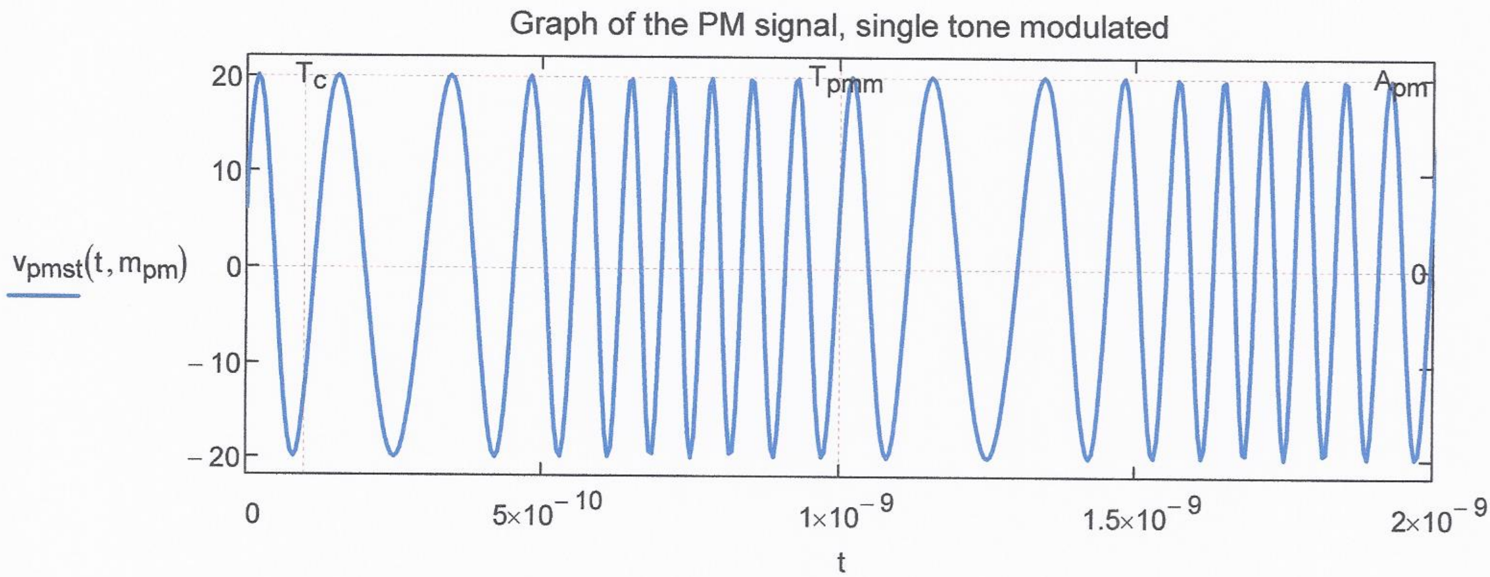
Max amplitude of the Carrier: $A_{pm} = 20\cdot V$,

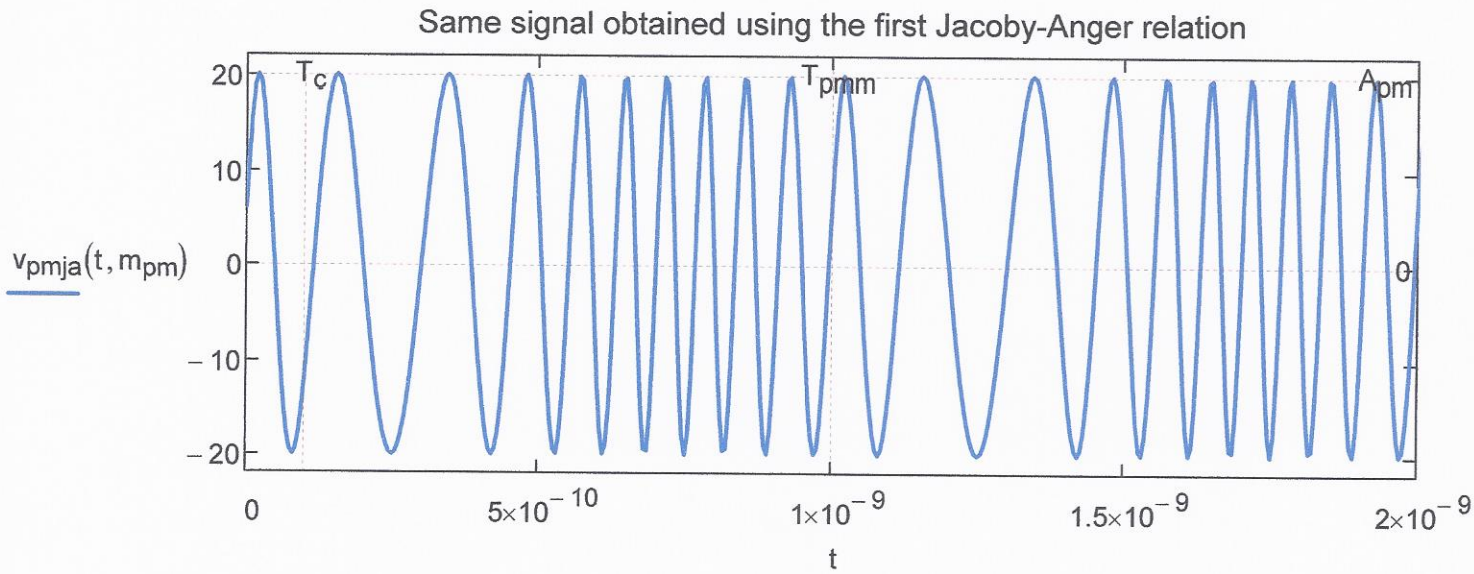
Single tone max amplitude: $B_{pm} = 8\cdot V$,

bounds of the sum: $n = 40$,

phase modulation index: $m_{pm} = 5.02$

$f_c = 10 \cdot GHz$ $\qquad t := T_{pmm} \cdot 0, T_{pmm} \cdot 0 + \frac{40 \cdot T_{pmm} - 0 \cdot T_{pmm}}{8000} .. 40 \cdot T_{pmm}$





$m_{pm} = 5.02$ $\qquad k := -20 .. 20$