CLEANING UP LOGSPACE(min,max,npts) RESULTS

Using significant figures for fun and profit

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INTRODUCTION

One new and interesting Mathcad Prime 3.0 function is $logspace$. The purpose of $logspace$ is to create a vector of logarithmically spaced points. The resulting vector is useful for creating log or log-log plots and for manipulating data that spans multiple decades. I learned about the new $logspace$ function in Brent Maxfield's excellent new book **Essential PTC Mathcad Prime 3.0**. [1]

The first application for $logspace$ that came to mind is the Moody Diagram, which is used in fluid mechanics. [2] In the turbulent flow regime, the Moody diagram plots various curves that represent solutions to the Colebrook-White Equation for different values of relative roughnesses. The Colebrook-White Equation relates the Reynolds Number of the flow, the relative roughness of the pipe, and the friction factor of the pipe. The friction factor is then used in the Darcy Equation, usually to calculate head loss. The Colebrook-White Equation is empirical and requires iteration for solve for the friction factor, which is why—in the pre-computer days—Moody created his diagram.

Years ago, I manually created a quasi-$logspace$ function in Excel specifically to generate the Reynolds Numbers for calculating solutions to the Colebrook-White Equation so that I could create my own Moody Diagram. Also years ago, I created a very nice direct solution approximation for the the "standard equations" for the smooth pipe curve {1} and I used quasi-logarithmically spaced points for making statistical comparisons between my equation and the "standard equations" and other published approximations. {2} However, I have not formally published my equation, so it remains proprietary for now.

Back to the $logspace$ function...As an engineer, I don't revel in unlimited decimal places and "messy" numbers except when it comes to fun things like π and e and $\sqrt{2}$. I am just old enough to have been an active part of the transition from slide rules (very limited precision) to the HP-35 and it's successors and to have had "significant figures" drummed into my head by my father (who was my high school chemistry teacher) and the late Mr. Lipston (my high school physics teacher, who was, unfortunately, afraid of RPN). The "sigfig" baton was then handed off to my engineering professors.

So, rather than use the initial $logspace$ results, I would prefer to use values with a limited number of sigfigs, even though the number of sigfigs that is appropriate here is somewhat arbitrary. Unfortunately, rounding off to a fixed number of sigfigs in not built into Mathcad. Fortunately, it can be created in Mathcad.

The purpose of this worksheet, then, is to round off $logspace$ results to a fixed number of sigfigs. This technique can be applied to other functions, but $logspace$ saw it first.

PROCEDURE

Step 1 — Create a *logspace* vector:
(Note all the "messy" values.)

$$x := \text{logspace}(0.001, 10000, 9) = \begin{bmatrix} 0.001000 \\ 0.007499 \\ 0.056234 \\ 0.421697 \\ 3.162278 \\ 23.713737 \\ 177.827941 \\ 1333.521432 \\ 10000.000000 \end{bmatrix}$$

Step 2 — Define the number of sigfigs to round to:
(For this example, I created three options so that the results can be compared.)

$SigFig_a := 1$ $SigFig_b := 2$ $SigFig_c := 3$

Step 3 — Create a function that generates a vector, whose elements are the numbers of places to round off each member of x per the sigfig specification:
(This step can be skipped if the essence of the RND function is incorporated into Step 4. However, I included it here so that the intermediate step in the process can be examined.)

$$\text{RND}(Vector, SigFigs) := (SigFigs - 1) - \text{floor}(\log(Vector))$$

Step 4 — Create a function that generates the final results, with each member of x rounded off to the same number of significant figures as specified in Step 2.
(Q.E.D.)

$$\text{RND\_VEC}(Vector, Rounding) := \text{round}(Vector, Rounding)$$

Step 5 — Examine the results

$$RND_a := \text{RND}(x, SigFig_a) = \begin{bmatrix} 3 \\ 3 \\ 2 \\ 1 \\ 0 \\ -1 \\ -2 \\ -3 \\ -4 \end{bmatrix}$$

$$RND\_VEC_a := \text{RND\_VEC}(x, RND_a) = \begin{bmatrix} 1.000 \cdot 10^{-3} \\ 7.000 \cdot 10^{-3} \\ 6.000 \cdot 10^{-2} \\ 4.000 \cdot 10^{-1} \\ 3.000 \\ 2.000 \cdot 10 \\ 2.000 \cdot 10^{2} \\ 1.000 \cdot 10^{3} \\ 1.000 \cdot 10^{4} \end{bmatrix}$$

$$RND_b := \mathrm{RND}\left(x, SigFig_b\right) = \begin{bmatrix} 4 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \\ -1 \\ -2 \\ -3 \end{bmatrix}$$

$$RND\_VEC_b := \mathrm{RND\_VEC}\left(x, RND_b\right) = \begin{bmatrix} 1.000 \cdot 10^{-3} \\ 7.500 \cdot 10^{-3} \\ 5.600 \cdot 10^{-2} \\ 4.200 \cdot 10^{-1} \\ 3.200 \\ 2.400 \cdot 10 \\ 1.800 \cdot 10^{2} \\ 1.300 \cdot 10^{3} \\ 1.000 \cdot 10^{4} \end{bmatrix}$$

$$RND_c := \mathrm{RND}\left(x, SigFig_c\right) = \begin{bmatrix} 5 \\ 5 \\ 4 \\ 3 \\ 2 \\ 1 \\ 0 \\ -1 \\ -2 \end{bmatrix}$$

$$RND\_VEC_c := \mathrm{RND\_VEC}\left(x, RND_c\right) = \begin{bmatrix} 1.000 \cdot 10^{-3} \\ 7.500 \cdot 10^{-3} \\ 5.620 \cdot 10^{-2} \\ 4.220 \cdot 10^{-1} \\ 3.160 \\ 2.370 \cdot 10 \\ 1.780 \cdot 10^{2} \\ 1.330 \cdot 10^{3} \\ 1.000 \cdot 10^{4} \end{bmatrix}$$

NOTES

{1} The smooth pipe curve on the Moody Diagram can be found by using either the Colebrook-White Equation with $\varepsilon / D = 0$ or the Von Karman-Nikuradse Equation...they're virtually identical. My primary application for the smooth pipe curve is natural gas flow.
{2} Creating direct solution equations for the Colebrook-White and Von Karman-Nikuradse Equations is somewhat of a cottage industry and many such equations can be found in the literature. I was introducted to this pastime by Dr. Walter Rowland, one of my favorite college professors.

REFERENCES

[1] Maxfield, Brent, P.E., Essential Mathcad Prime 3.0, A Guide for New and Current Users, Academic Press, Elsevier, Inc., Waltham, MA (2014...but released in November 2013), p. 236.
[2] Daugherty, Robert L., and Franzini, Joseph B., *Fluid Mechanics, With Engineering Applications*, McGraw-Hill Book Company, New York (1977), p. 217–217.