Section 1 Field Patterns of a Uniform Linear Antenna Array

This document calculates the far-field radiation pattern of a uniformly spaced linear antenna array as a function of azimuth angle, and displays the field pattern as both an azimuth distribution and a power pattern. You set the following physical parameters:

- **N**, the number of array elements
- **d**, the interelement spacing
- **f**, the radiating frequency
- **a**, the progressive phase shift from element to element

You can also set **df**, which controls the resolution of the plots.

The document then calculates and plots the radiated far-field pattern in two dimensions for an elevation of 0 degrees, as well as the power pattern as a function of azimuth angle.

References

1. John D. Kraus and Keith R. Carver, *Electromagnetics*, McGraw-Hill Book Company (New York, 1978).
2. Jin Au Kong, *Electromagnetic Wave Theory,* John Wiley & Sons (New York, 1986).

Background

Far-field patterns for an array of uniformly spaced, omnidirectional radiators can be described using the principle of superposition. The energy pattern produced by such an array will be a phase-dependent summation of the individual field patterns. The group behavior of the array is governed by a term known as the array factor, **F(f),** given by**:**

$$F(\phi) = \frac{1}{N} \cdot \sum_{array} e^{-1j \cdot (k \cdot d \cdot cos(\phi) - \alpha)}$$

where **a** is the progressive phase shift from element to element in the array, and **k** = 2**p/l** is the wavenumber. The variable **f** is the azimuth angle, defined as the angle in the x-y plane perpendicular to the antennas.

The array factor is a scaling term which multiplies the radiation field from a single antenna to give the group far-field pattern. This model works only in the plane of zero elevation (perpendicular to the direction of the antennas.) Additional terms are necessary to describe different cross sections through the radiation field.

Mathcad Implementation

First, input the following parameters to define the antenna array and the operating conditions. You can do this in your main worksheet window.

Array Parameters

$N := 6$	number of elements
$d := 50\ \boldsymbol{m}$	interelement spacing
$f := 3\ \boldsymbol{MHz}$	frequency
$\alpha := 0\ \boldsymbol{deg}$	progressive phase shift

Now it is possible to define an expression for the normalized array factor. First, create a range variable for the number of antennas in the array, then the expression for the array factor in terms of the parameters above:

$$n := 0 .. N-1$$

$$F(\phi) := \frac{1}{N} \cdot \left(\sum_{n} e^{-\left(1j \cdot n \cdot d \cdot \frac{2 \cdot \pi \cdot f}{c} \cdot \cos(\phi) + 1j \cdot n \cdot \alpha\right)} \right)$$

The array factor can be considered a scaling term for the radiated power, and can be expressed in dB, as follows:

$$P(\phi) := 20 \cdot \log\left(\frac{|F(\phi)|}{N}\right)$$

To plot this expression as a function of azimuth angle, define a range variable, **f**, and the step size, **df**, to be used in the plot. Note that decreasing the step size will increase the resolution of the plot but will take longer to calculate.

$\delta\phi := 5\ deg$ step size for azimuth angle

$\phi := -180\ deg, -180\ deg + \delta\phi .. 180\ deg$

To see the field pattern, edit the definitions of the four array parameters. Then choose **Calculate Worksheet** from the **Math** menu to plot the power pattern as a function of azimuth.

For the simple case of an antenna array with six elements, spaced **l**/2 apart, the array factor appears as follows:

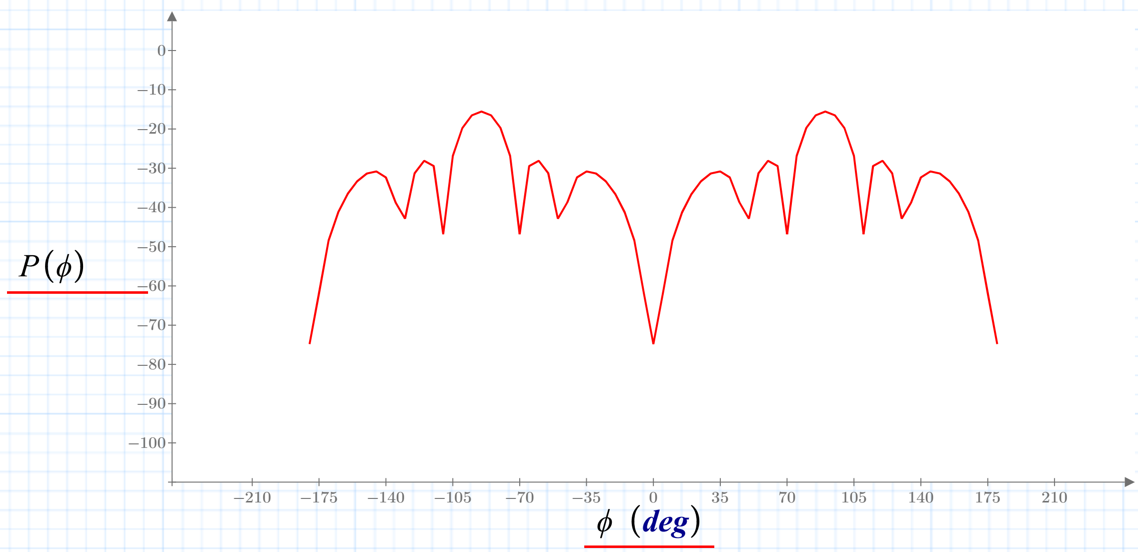


Fig. 1.1 Power pattern with respect to angle

The more familiar way to view the power pattern is in two dimensions, which is simply a conversion to polar coordinates from the azimuth angle. This is equivalent to looking at a radiation contour curve in the x-y plane. Define the conversion as follows:

$$x(r,\phi) := r \cdot \sin(\phi) \qquad\qquad y(r,\phi) := r \cdot \cos(\phi)$$

are the **x** and **y** coordinates, and

$$A(\phi) := |F(\phi)|$$

is the value for the radius. The plot appears below:

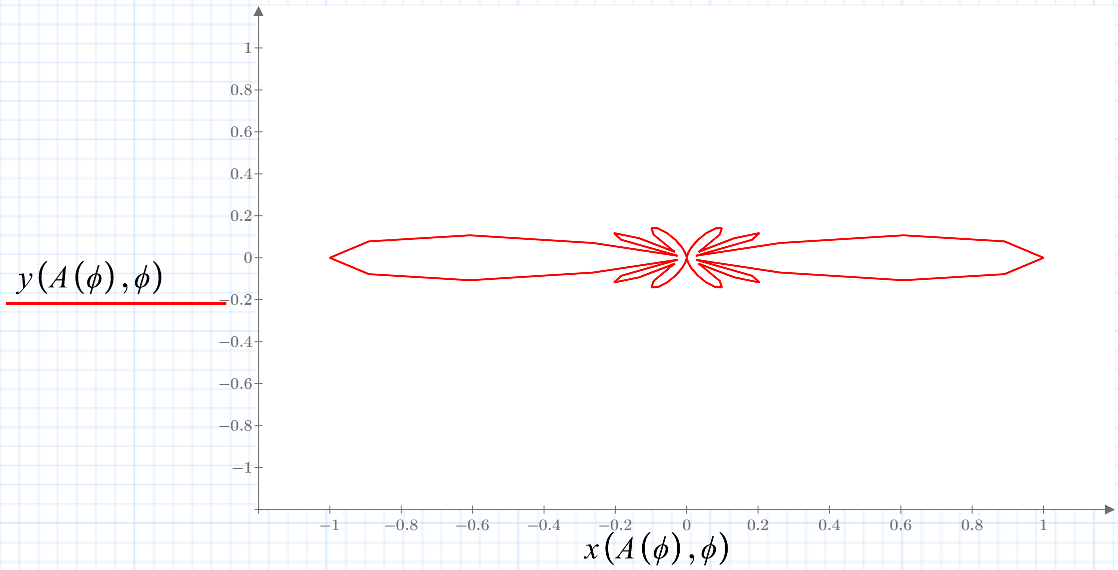
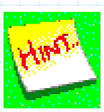


Fig. 1.2 2-D radiation pattern at zero elevation



It may also be useful to define additional variables that depend on the array parameters, such as the wavenumber, **k**, defined as follows:

$$k := \frac{2 \cdot \pi \cdot f}{c} \qquad\qquad k = 0.063 \ \frac{1}{m}$$

Or in terms of additional variables...

$$\omega := 2 \cdot \pi \cdot f \qquad k := \frac{\omega}{c} \qquad \text{or} \qquad \lambda := \frac{c}{f} \qquad k := \frac{2 \cdot \pi}{\lambda} \qquad d := \frac{3 \cdot \lambda}{2}$$

Mathcad will display additional variables in any units desired, and new units can be defined if necessary. In this way, constraints for the antenna array can be specified to emphasize particular relationships; special cases can be explored (such as the case of **d = nl**/2, where **n** is an integer, or **a = kd**).

Another way to enhance your understanding of the system might be to use global variable definitions, so that the parameters for the problem appear next to the graph. To do this, use the global assignment operator. The global assignment operator is located on the **Math** tab/**Operators** panel/**Definition and Evaluation** category. It is represented by a triple equal sign. You may also consider moving the definitions for **N**, **d**, **f**, and **a** next to the graphs.