2.2 2-D Waveguides

This document calculates various quantities for rectangular waveguides. For a 2-D waveguide, cutoff frequencies and guide wavelengths are calculated. Values for the cutoff frequency are calculated for TEmn and TMmn modes over a range of values for **m** and **n**, and a corresponding **kz-k** diagram is shown for the 2-D guide. You provide the following values:

- **a** and **b**, the waveguide dimensions,
- **m** and **n**, mode numbers,
- **f**, the operating frequency.

Background

Refer to the following diagram for dimensions and orientation.

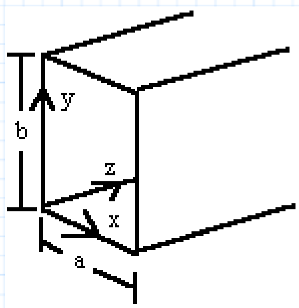

Fig. 2.2.1 A 2-D rectangular waveguide

Cutoff Frequencies

Any waveguide system will have cutoff frequencies: frequencies below which no waves will propagate in the guide. These determine operating bandwidth for a particular mode, or alternatively the number of simultaneous modes for a propagating frequency.

Note: For more information on cutoff frequencies, see Background for **2.1 1-D Waveguides: Striplines**.

Valid Modes

When calculating cutoff frequencies, it's important to constrain the mode integers **m** and **n** appropriately. In some cases setting these integers to zero means that there is no electric field.

TMmn: $E_z = E_0 \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right) \cdot e^{-j \cdot k_z \cdot z}$

$$E_x = \frac{-j \cdot k_z \cdot k_x \cdot E_0}{\omega^2 \cdot \mu \cdot \varepsilon - k_z^2} \cdot \cos\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot \sin\left(\frac{n \cdot \pi \cdot y}{b}\right) \cdot e^{-j \cdot k_z \cdot z}$$

etc...

TEmn: $E_y = E_0 \cdot \sin\left(\frac{m \cdot \pi \cdot x}{a}\right) \cdot e^{-j \cdot k_z \cdot z}$

For TEmn modes, **n** can be zero. But the y-dependence of the TMmn modes means that **m** and **n** must both be greater than or equal to one for there to be a propagating wave in the guide.

Mathcad Implementation

The equations that follow define propagation constants and cutoff frequencies for a rectangular waveguide.

$\varepsilon_0 := 8.854 \cdot 10^{-12} \ \frac{F}{m}$ $\qquad \mu_0 := 4 \cdot \pi \cdot 10^{-7} \ \frac{H}{m}$

First, enter waveguide dimensions along the x and y axes:

$a := 3 \ cm$ $\qquad b := 1.5 \ cm$

For the air-filled rectangular waveguide shown in Fig. 2.2.1, the propagation constant in the z-direction for a given propagating frequency **ω** is

$$k_z(\omega, m, n) := \sqrt{\omega^2 \cdot (\mu_0 \cdot \varepsilon_0) - \left(\frac{\pi \cdot m}{a}\right)^2 - \left(\frac{\pi \cdot n}{b}\right)^2}$$

The cutoff frequency for the same guide is

$$f_c(m, n) := \frac{1}{2 \cdot \sqrt{\mu_0 \cdot \varepsilon_0}} \cdot \sqrt{\left(\frac{m}{a}\right)^2 + \left(\frac{n}{b}\right)^2}$$

Now we can enter the mode numbers, **m** and **n,** as array indices, and calculate a matrix of values for the cutoff frequency and the propagation constant, yielding design numbers for waveguides.

Enter the range of modes for **m** and **n**.

$m := 0 .. 4$ $\qquad n := 0 .. 4$

The table below gives cutoff frequencies in GHz. The rows correspond to **n** from 0 to 4, and the columns to **m** from 0 to 4. The first row and column apply only to the TEmn modes, and **m** = **n** = 0 is not valid for any mode.

$$F_{n,m} := f_c(m,n)$$

$\longleftarrow$ **m** $\longrightarrow$

$$F = \begin{bmatrix} 0 & 5 & 9.99 & 14.99 & 19.99 \\ 9.99 & 11.17 & 14.13 & 18.02 & 22.35 \\ 19.99 & 20.6 & 22.35 & 24.98 & 28.27 \\ 29.98 & 30.39 & 31.6 & 33.52 & 36.03 \\ 39.97 & 40.28 & 41.2 & 42.69 & 44.69 \end{bmatrix} GHz$$

$\updownarrow$ **n**

Note: Remember that **m, n** = 0 is not valid for TM modes.

Using the expression for **kz** and the dispersion relation, it is possible to plot the **kz-k** diagram, defining bandwidths for particular modes.

Begin range for frequency at the lowest cutoff frequency:

$$\omega_0 := 2 \cdot \pi \cdot f_c(1,0) \qquad \omega_0 = 31.39\ GHz$$

$$k(\omega) := \omega \cdot \sqrt{\mu_0 \cdot \varepsilon_0} \qquad \omega := \omega_0, \omega_0 + 1\ GHz .. 120\ GHz$$

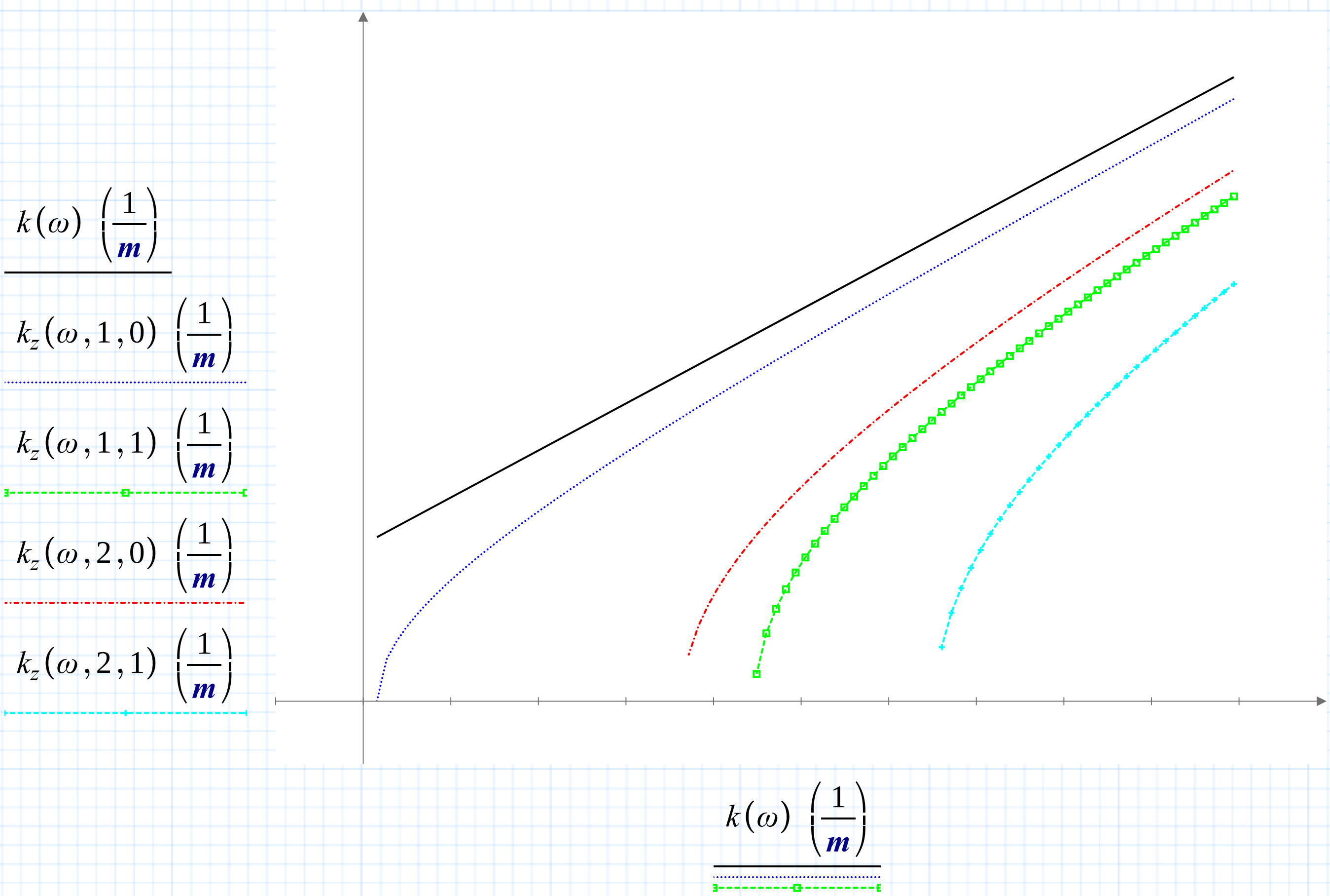

Fig. 2.2.2 Design curve: graph of k vs. kz.

The x-intercepts of this diagram are the cutoff wavenumbers, and are given by the expression

$$k_c(m,n) := \sqrt{\left(2 \cdot \pi \cdot f_c(m,n)\right)^2 \cdot \left(\mu_0 \cdot \varepsilon_0\right)}$$

For the **a** and **b** given above, the cutoff wavenumbers are as follows:

$$k_c(1,0) = 104.72\ \frac{1}{m}$$

$$k_c(2,1) = 296.19\ \frac{1}{m}$$

Additionally, the guide wavelength λ is defined as a function of **kz**. The example given calculates the guide wavelength for a particular choice of dimensions, mode, and frequency.

The guide wavelength for a given frequency **ω** is

$$\lambda_{guide}(\omega,m,n) := \frac{2 \cdot \pi}{k_z(\omega,m,n)}$$

For example, for the **TE10** mode propagating at three times the cutoff angular frequency, the guide wavelength is

$$\lambda_{guide}\left(3 \cdot \omega_0, 1, 0\right) = 2.12\ cm$$