2.4 Circular Waveguides: Coaxial Lines

This document illustrates several calculations for a coaxial line. Calculations are available for the line's parameters: characteristic impedance, phase velocity, and attenuation. You can enter values for:

- **f**, the operating frequency,
- the permeability and permittivity of the dielectric,
- **δ(ω)**, the loss tangent,
- the dimensions of the coaxial cable.

References

The values for dissipation factor in this document come from tables of dielectric insulator properties, Government Report No. 345, MIT Center for Insulator Research, *Dielectric Tables Vol. I and II*, Hanscom AFB (1945).

Background

Coaxial cable is perhaps the single most common transmission line in use today. Commercial lines are specified by their characteristic impedance, which can be calculated from the dimensions of the line and properties of the dielectric. It is also possible to determine loss characteristics for the coaxial cable by examining the complex permittivity of the dielectric used to separate the two conductors.

The complex permittivity

$$\varepsilon = \varepsilon_{Re} - j \cdot \varepsilon_{Loss}$$

represents the frequency response of a material. It will enter into the propagation characteristics in a waveguide through the dispersion equation,

$$k^2 = \omega^2 \cdot \mu \cdot \varepsilon \qquad\qquad E = E_0 \cdot e^{-jkz}$$

If part of the exponent is imaginary, then there will be a decaying portion of the wave in the waveguide. It is possible to quantify the loss if we know the loss tangent **d(w)** [or dissipation factor, **tan(d)**] at a particular frequency. The loss tangent is defined as the ratio of the imaginary (loss) part of the permittivity to the real part, and is dependent on frequency, conductivity, and other power-dissipating factors.

Mathcad Implementation

The following equations model a coaxial line operating in TEM mode. Enter parameters below.

Constants

$\varepsilon 0_0 := 8.854 \cdot 10^{-12}\ \frac{F}{m}$ $\qquad$ $\mu 0_0 := 4 \cdot \pi \cdot 10^{-7}\ \frac{H}{m}$

System Specifications

First, specify the operating frequency for the coax, and several material parameters for the dielectric:

$f := 25 \cdot MHz$ $\qquad$ $\omega := 2 \cdot \pi \cdot f$

Permeability of dielectric: $\mu := \mu 0_0$

Permittivity of the dielectric (polyethylene): $\varepsilon := 2.5 \cdot \varepsilon 0_0$

Loss tangent at frequency **w** (polyethylene, good to 10 GHz): $\delta := 2 \cdot 10^{-4}$

Now specify the **dimensions** of the coaxial line:

OD of inner conductor: $OD := 0.25\ cm$

ID of outer conductor: $ID := 1.60\ cm$

Corresponding radii: $a := \frac{OD}{2}$ $\qquad$ $b := \frac{ID}{2}$

Cable Loss and Impedance

Now it is possible to calculate complex quantities which will affect the loss in the cable, and the impedance.

Complex permittivity: $\varepsilon_c := \varepsilon \cdot (1 - 1\mathrm{j} \cdot \operatorname{atan}(\delta))$

Intrinsic impedance of dielectric: $\eta := \sqrt{\frac{\mu}{\varepsilon}}$

Propagation constant: $k(f) := 2 \cdot \pi \cdot f \cdot \sqrt{\mu \cdot \varepsilon_c}$

Phase velocity as a fraction of the speed of light: $v(f) := \left(\frac{1}{\operatorname{Re}\left(\sqrt{\mu \cdot \varepsilon_c}\right)}\right) \cdot \frac{1}{c}$

Characteristic impedance of line: $Z_0(a, b) := \frac{\eta}{2 \cdot \pi} \cdot \ln\left(\frac{b}{a}\right)$

For the radii **a** and **b**, calculated above, the characteristic impedance is

$$\left|Z_0(a, b)\right| = 70.394\ \Omega$$

The phase velocity as a fraction of **c** is

$$v(f) = 0.632$$

The attenuation in **dB per 100 meters** is related to the imaginary part of the propagation constant, which represents loss in the dielectric.

$$A(f) := -20 \cdot \log(\exp(100\ m \cdot \operatorname{Im}(k(f))))$$

$$A(f) = 0.072$$

The voltage decays to half its original value over a distance of

$$d(f) := \frac{-\ln(2)}{\operatorname{Im}(k(f))} \qquad d(f) = 8.367\ km$$