



APPLICATIONS IN CIRCUIT ANALYSIS

Section 7 American Wire Gauge Table

This document calculates the resistance per unit length for copper wire as a function of the AWG number, the temperature, and the frequency. Resistance is calculated for two regions of operation: DC operation at a particular temperature, and high frequency current operation. High frequency calculations account for the change in resistance due to the skin effect. The AWG tables are generated from formulas that approximately reproduce the dimension and DC resistance values. These calculated values agree within 1%. You supply:

- **n**, the American Wire Gauge (AWG) number,
- **T**, the operating temperature,
- **f**, the operating frequency.

References

Siemens Components XIX (1984), No. 5: The Skin Effect

Donald G. Fink and H. Wayne Beaty, *Standard Handbook for Electrical Engineers*, McGraw-Hill Book Company (New York, 1987).

Handbook of Mathematical and Scientific, and Engineering Formulas, Research and Education Association (New York, 1984).

Marcel Daoust suggested this application and supplied helpful references.

Background

Resistance in a good conductor will depend on the cross section (per unit length) of the conductor, the average temperature of the conductor and the frequency of the energy propagating in the conductor.

DC Resistance Corrections: Temperature

It is reasonable to expect that a hot conductor will conduct electricity less well than a cool one, because as the electrons in the material move around faster, it will be harder to transfer energy from one to the next in an orderly fashion. Hence, the hotter the wire, the greater the resistance. It is difficult to quantify this relationship, however, so in the data table below we have included several measured values, and will extrapolate along the curve between them.

AC Resistance Corrections: Skin Depth

At higher frequencies, bulk conductors will react like an inductance (open circuit at high frequency), i.e. the resistance will increase substantially. Most of the energy will be transported in the outer layer of the conductor, which means it has a narrower cross section than for DC current. The thickness of the conducting layer is a characteristic penetration depth, or skin depth, that is associated with the conductivity of the wire.

Mathcad Implementation

The following formulas and extrapolation tables will be used to calculate the various properties of wire, according to the American Wire Gauge Table. The calculations are shown on the last page of this document.

DC Formulas

Approximate formula for diameter in terms of AWG number:

$$C(n) := \left(10.2059 - n \cdot 8.269 \cdot 10^{-3}\right) \cdot \frac{2.54}{\pi} \cdot 10^{-(.05 \cdot n)}$$

$$B(n) := \text{if}(C(n) > 1, .0001 \cdot \text{floor}(1000 \cdot C(n) + .5), .00001 \cdot \text{floor}(10000 \cdot C(n) + .95))$$

Resistivity as a function of temperature:

$$A := \begin{bmatrix} 273.15 & 1.543 \\ 293 & 1.678 \\ 300 & 1.725 \\ 350 & 2.063 \end{bmatrix} \quad vs := \text{cspline}(A^{(0)}, A^{(1)})$$

$$\rho(T) := \text{interp}(vs, A^{(0)}, A^{(1)}, T + 273.15) \cdot 1.03 \cdot 10^{-6} \text{ } \Omega \cdot \text{cm}$$

$$DIAM(n) := B(n) \text{ cm}$$

$$AREA(n) := .25 \cdot \pi \cdot DIAM(n) \cdot DIAM(n)$$

$$CURR(n) := 450 \cdot AREA(n) \frac{A}{\text{cm}^2} \quad RES(n, T) := \frac{\rho(T)}{AREA(n)}$$

AC Formulas

Equivalent conducting layer (skin depth) for copper conductor as a function of frequency and temperature:

$$\delta(f, T) := \sqrt{\frac{\rho(T)}{\pi \cdot f \cdot \mu_0}}$$

The skin effect correction for resistance per unit length at high frequency is calculated from formulas in the first reference, with an interpolated transition to DC behavior given by **g**. Here, **x** is the skin effect ratio in the limit **d** << **DIAM**.

$$x(n, T, f) := \frac{DIAM(n)}{4 \cdot \delta(f, T)}$$

$$g(x) := \text{if}(x < .8, 1, \text{if}(x < 1.2, 1 + 1.25 \cdot (x - .8)^2, x))$$

Approximate value for **SRES**, the AC resistance per unit length:

$$SRES(n, T, f) := RES(n, T) \cdot g(x(n, T, f))$$

Alternative formula, following Fink and Beaty:

$$y(n, T, f) := .063598 \cdot \sqrt{\frac{f}{RES(n, T)} \frac{\Omega}{mi}}$$

$$h(y) := \text{if}(y < 1, 1, \text{if}(y < 3, 1 + .034 \cdot (y - 1), .356 \cdot y))$$

$$SRES(n, T, f) := RES(n, T) \cdot h\left(y\left(n, T, \frac{f}{Hz}\right)\right)$$

This gives values about 1% lower than **SRES** calculated from first formula.

To use this formula, use the **Save As** button in the toolbar above and resave the file in your worksheet window. (**Note:** The version of the files in the **Web Library** can *not* be resaved. You may download the book through the MathSoft Web Store.)

AMERICAN WIRE GAUGE TABLE, COPPER WIRE, RESISTANCE PER UNIT LENGTH AS A FUNCTION OF AWG NUMBER, TEMPERATURE, AND FREQUENCY

To use the table, enter values for the AWG number (from 0 to 50) and the temperature in Celsius. Mathcad displays the wire diameter and cross section. It then calculates the current corresponding to 450 amp/cm² and the resistance per unit length for DC operation. You can edit the length units given for **DIAM**, **AREA**, and **RES**, using mm, cm, meter, km, in, ft, and mi as convenient.

Enter AWG number **n** (from 0 to 50) $n := 10$

The diameter is: $DIAM(n) = 0.2588 \text{ cm}$

The cross-section area is: $AREA(n) = 0.052604 \text{ cm}^2$

Current producing 450 amp/cm squared is: $CURR(n) = 23.672 \text{ A}$

Enter temperature **T** in Celsius (from 0 to 100) $T := 20$

DC Resistance

The resistance is:

$$RES(n, T) = 3.29 \frac{\Omega}{km}$$

High Frequency Resistance

Enter frequency: $f := 10^5 \text{ Hz}$

Skin depth at this frequency: $\delta(f, T) = 0.209 \text{ mm}$

Diameter for chosen AWG number: $DIAM(n) = 2.588 \text{ mm}$

Resistance per unit length at frequency **f**: $SRES(n, T, f) = (1.02 \cdot 10) \frac{\Omega}{km}$