Section 1.2 Voltage Drop Calculations

Section 1.2.1 Introduction

Voltage drops appear across a transmission line as a result of the inductive and resistive impedance of the line. The detrimental effects of voltage drop include: undesirably low voltage at the customer side, excessive power loss in the transmission lines and reduction of system-feeder and bus-power capacity. Loads with large inductive component pose a particular problem, since they increase the line current and, therefore, the voltage drop across the line.

Application of shunt capacitors can significantly improve the voltage profile and power losses of the system. The current of the capacitors reduces the inductive component of the line current and, consequently, the associated voltage drop and power losses. The advantages gained by capacitive compensation include: release of distribution substation capacity, improvement of system voltage profile and reduction of copper losses. The reduction of copper losses can result in substantial savings in both power and operating expense.

Determination of line voltage profile and power losses requires the following information: line impedance, magnitude and power factor of load current, location of the load on the line (load distribution), supply voltage and the manner in which the line is supplied. A line may be supplied from only one end with the other end terminating at the load (**open line**), or from both ends (**closed line**).

In this section, a method is presented for the calculation of the voltage profile and power losses with application examples for both open and closed lines. This method may be used to improve the performance of a line by investigating the application of shunt capacitors and other schemes for mitigating the adverse effects of voltage drops.

Section 1.2.2 Open Lines

Figure 1.2.1 shows a distribution line of length L supplied at one end by a voltage V0. The line load consists of several customer drops distributed along the line and stemming lateral feeders. The current supplied at the different locations on the line varies according to the density of the load at each location. Figure 1.2.1 shows the current density, in amp/m, on the line as a function, i(y), of the distance, y, from the supply end. The currents, Ick, represent the total current of stemming feeders or relatively large customers. Therefore, the line load can be considered as two components: a distributed component represented by the current density function, i(y), and a concentrated (lumped) component represented by the currents Ick.

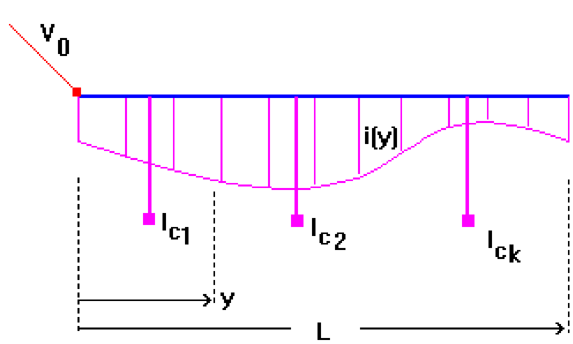

Fig. 1.2.1 Representation of an open line

The distributed component of the line load can be found from information providing the density and size of customers along the line. Typically, a uniform or a linearly increasing load distribution is assumed in voltage drop calculations. However, Mathcad allows the implementation of any desired load distribution for more accurate calculation of the line voltage profile.

To calculate voltage drop, the total line current must be known at each point of the line. This current is found by properly combining the concentrated and distributed load current. A Mathcad implementation is as follows:

The following array describes the concentrated current of a line load.

$$CC := \begin{bmatrix} 0.2 & 1 & 28 \\ 0.4 & 5 & 35 \\ 0.7 & 6 & 0 \end{bmatrix}$$

In the above matrix, **the first column** represents the percent distance of the concentrated current from the supply, **the second column** the current magnitude, and **the third column** the current power factor angle in degrees.

The distributed load current is described by a suitable function.

$i(y) := 0.02 \ \frac{A}{m}$ for uniformly distributed load

$i(y) := 0.03 \ \frac{A}{m^2} \cdot y$ for a linearly increasing load

$\phi := 15 \ deg$ power factor angle

The user may toggle the second equation on to see the effect of a linearly increasing load.

The active and reactive components of the total line current are calculated as follows:

The line length is $L := 1000 \ m$

The voltage drop in the line can be computed numerically. For this purpose, the line is divided into N sections. The value of N determines the accuracy of the solution. However, larger values of N require longer solution times. For this discussion:

$$N := 50$$

The length of each section is: $\Delta y := \frac{L}{N}$

Discretizing the line lengths, we obtain

$$l := 0 .. N$$

$$y_l := l \cdot \Delta y$$ discrete distance from supply

The concentrated component of the current is described as

$$M := \text{rows}(CC)$$

$$k := 0 .. M - 1$$

$$J_k := \text{floor}\left(\frac{CC_{k,0} \cdot L}{\Delta y}\right)$$ indexed current location

The active concentrated current is given by

$$Icr_k := CC_{k,1} \cdot \cos\left(CC_{k,2}\ \textbf{deg}\right)\ \textbf{A}$$

and the reactive concentrated current by

$$Icx_k := CC_{k,1} \cdot \sin\left(CC_{k,2}\ \textbf{deg}\right)\ \textbf{A}$$

Therefore, the total line current at the location yl is found as

Active component

$$Ir_l := \int_{y_l}^{L} i(y) \cdot \cos(\phi)\, \mathrm{d}y + \sum_k \left(Icr_k \cdot \left(1 - \Phi\left(l - J_k\right)\right)\right)$$

Reactive component

$$Ix_l := \int_{y_l}^{L} i(y) \cdot \sin(\phi)\, \mathrm{d}y + \sum_k \left(Icx_k \cdot \left(1 - \Phi\left(l - J_k\right)\right)\right)$$

The two current components are shown in the graph below.

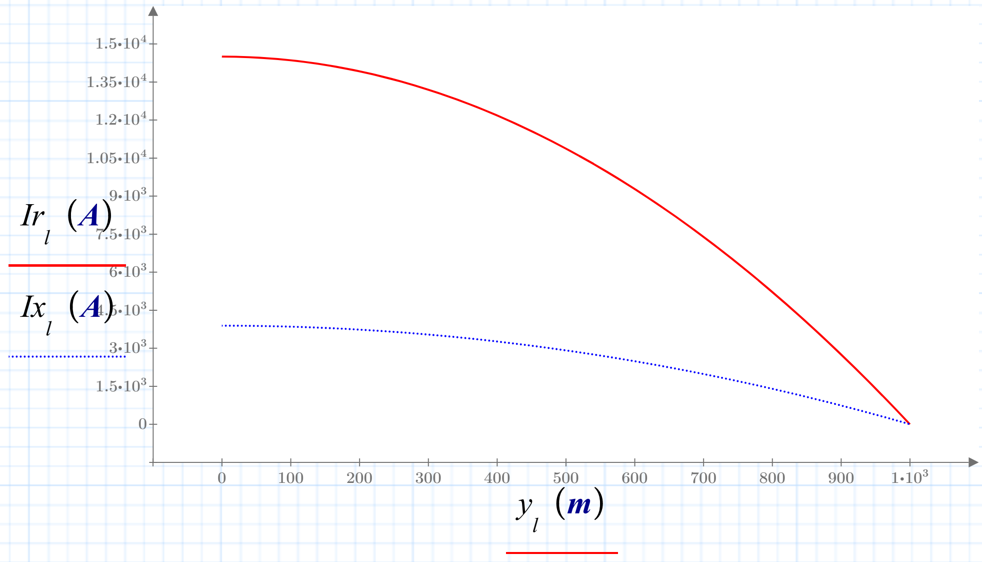

Fig. 1.2.2 Line current as function of distance from supply end

With a known line current, the voltage at any distance from the supply end can be calculated. Let the voltage at the supply end be

$$V_0 := 2400\ \mathbf{V}$$

Let the line resistance and series inductive reactance per unit length be respectively

$$r := 0.001\ \frac{\Omega}{m} \qquad x := 0.003\ \frac{\Omega}{m}$$

The voltage drop caused at location y due to the infinitesimal line impedance is

$$dV = (r + 1j \cdot x) \cdot (Ir(y) + 1j \cdot Ix(y)) \qquad \textbf{(1.2.1)}$$

Equation (1.2.1) can be approximated as

$$dV = (r \cdot Ir(y) + x \cdot Ix(y)) \qquad \textbf{(1.2.2)}$$

Therefore, the line voltage at the location y is given as

$$V(y) = V_0 - \int_0^y dV\,\mathrm{d}y \qquad \textbf{(1.2.3)}$$

Equation (1.2.3) is discretized to obtain the line voltage at the discrete locations yl.

$$V_{l+1} := V_l - \left(r \cdot Ir_l + x \cdot Ix_l\right) \cdot \Delta y$$

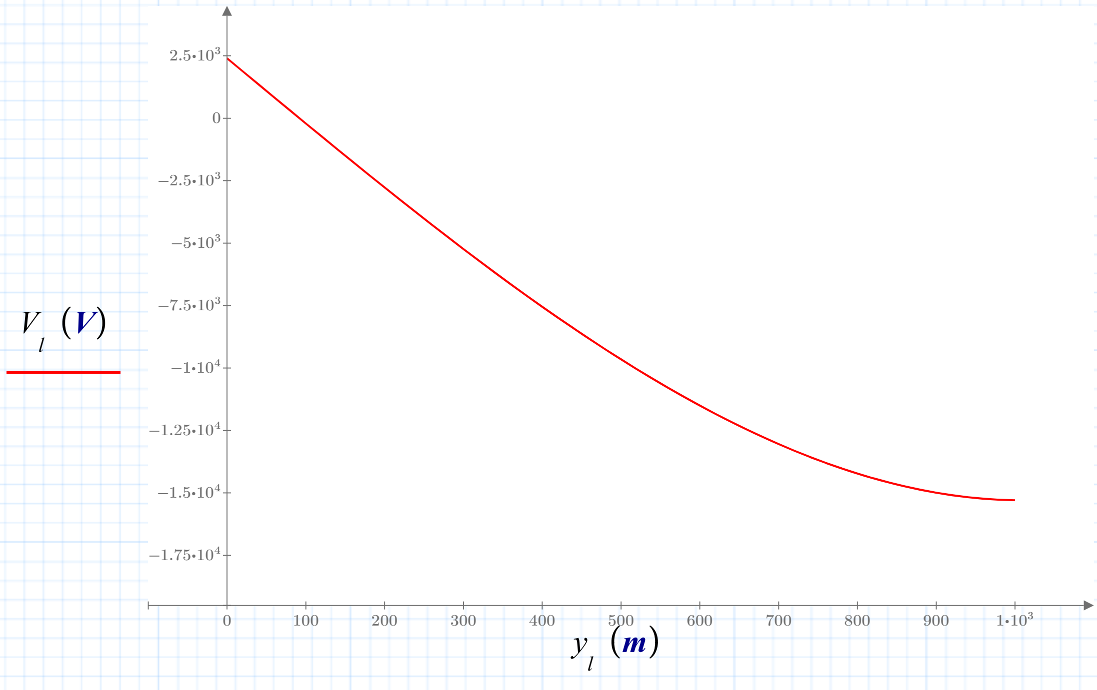

Fig. 1.2.3 Voltage profile of the open line

From the above calculation, the minimum line voltage is

$$Vmin := min(V)$$

$$Vmin = -1.529 \cdot 10^4 \ V$$

The user may investigate the effect of capacitive compensation on the line voltage profile by including the proper capacitor current in the array CC. The user may vary both the capacitive current magnitude and its location on the line to see the improvements.

Section 1.2.3 Closed Lines

For a line segment supplied at both ends, shown in Figure 1.2.2, the current at any location y from end 0 consists of two components: current due to the line load from y to end L, and currents from the source. The current viewed from end 0 is expressed as

$$I(y) = I0 - \int_0^y (i(y) + Ic(y))\, dy \qquad \textbf{(1.2.4)}$$

When the current is viewed from end L, it takes an equivalent form

$$I(y) = \int_y^L (i(y) + Ic(y))\, dy + IL \qquad \textbf{(1.2.5)}$$

The terms in Equations (1.2.4) and (1.2.5) are

i(y) is the distributed load current,
Ic(y) is the concentrated load current,
I0 is the current into the source at the 0 end,
and
IL is the current out of the source L end.

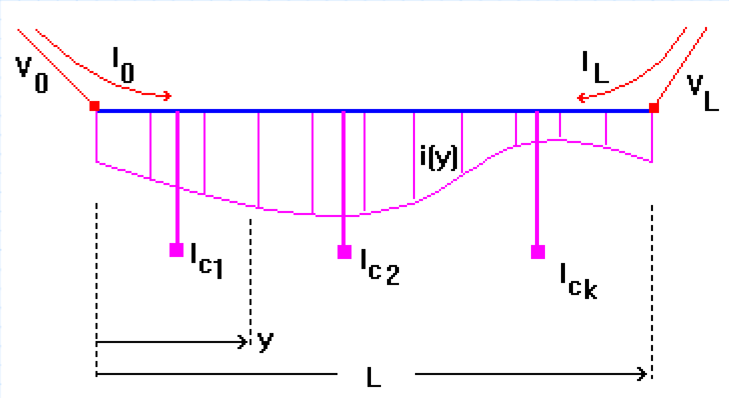

Fig. 1.2.4 Closed line representation

Substituting Equation (1.2.4) into Equation (1.2.3) and calculating the voltage with reference to end 0, we obtain

$$V(y) = V_0 + r \cdot \int_0^y Ir(y)\,dy + x \cdot \int_0^y Ix(y)\,dy + (r \cdot I0r + x \cdot I0x) \cdot y \qquad \textbf{(1.2.6a)}$$

Defining C as the voltage drop per unit length due to supply current, we have

$$C = r \cdot I0r + x \cdot I0x$$

$$V(y) = V_0 + r \cdot \int_0^y Ir(z)\,dz + x \cdot \int_0^y Ix(z)\,dz + C \cdot y \qquad \textbf{(1.2.6b)}$$

where the r's denote the resistive component, and the x's denote the reactive component.

Expressing the voltage with reference to end L, we obtain

$$V(y) = V_N - r \cdot \int_y^L Ir(y)\,dy - x \cdot \int_y^L Ix(y)\,dy - C \cdot (L - y) \qquad \textbf{(1.2.7)}$$

Comparing Equations (1.2.6b) and (1.2.7), we calculate C.

Define $Cr = \int_0^y Ir(y)\,dy$ and $Cx = \int_0^y Ix(y)\,dy$

in a discrete form

$$Cr := \sum_l \left(Ir_l \cdot \Delta y\right) \qquad Cx := \sum_l \left(Ix_l \cdot \Delta y\right)$$

For the system data as in Section 1.2.2, we have

$$Cr = \left(9.809 \cdot 10^6\right)\ \boldsymbol{A \cdot m} \qquad Cx = \left(2.628 \cdot 10^6\right)\ \boldsymbol{A \cdot m}$$

Assume the voltage of end 0 to be $V_0 := 2400\ \boldsymbol{V}$

and the voltage of the L end to be $V_N := 2400\ \boldsymbol{V}$

Again using a line resistance and series inductive reactance per unit length of

$$r = 0.001\ \frac{\Omega}{m} \qquad x = 0.003\ \frac{\Omega}{m}$$

we find that $C := \dfrac{V_N - V_0 + r \cdot Cr + x \cdot Cx}{L} = 17.693\ \dfrac{\boldsymbol{V}}{\boldsymbol{m}}$

Calculation of I0

Calculation of the constant C, above, provides one of the equations needed for the calculation of the active and reactive components of I0, the current into the source. The other equation is derived by comparing the imaginary parts of Equations (1.2.6b) and (1.2.7). Thus, we obtain

$$C2 = x \cdot I0r - r \cdot I0x = \frac{x \cdot Cr - r \cdot Cx}{L}$$

Using the values of the example of the closed line in Section 1.2.3, we have

$$C2 := \frac{x \cdot Cr - r \cdot Cx}{L} = 26.8\ \frac{\boldsymbol{V}}{\boldsymbol{m}}$$

Therefore, $I0r := \dfrac{r \cdot C + x \cdot C2}{r^2 + x^2} = \left(9.809 \cdot 10^3\right)\ \boldsymbol{A}$

and $I0x := \dfrac{x \cdot C - r \cdot C2}{r^2 + x^2} = \left(2.628 \cdot 10^3\right)\ \boldsymbol{A}$

The line currents are defined according to Equation (1.2.4). Therefore, in a discrete form we have

$$Ir_l := I0r - Ir_l \qquad Ix_l := I0x - Ix_l$$

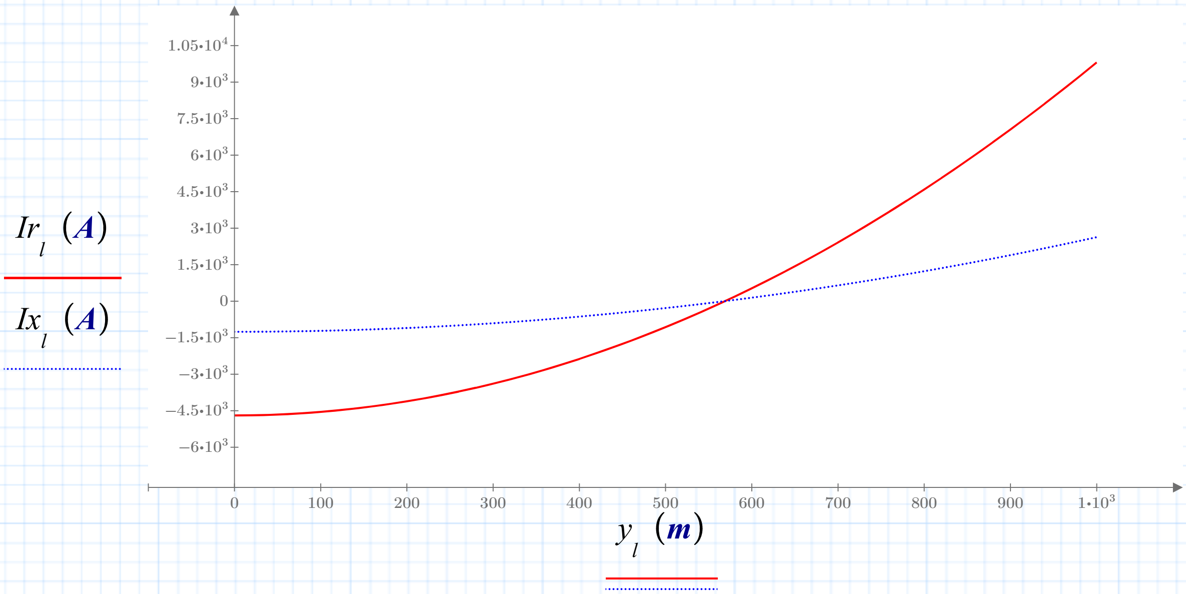

Fig. 1.2.5 Current profile of closed line

Voltage Profile Calculation

Using the constant C, the line voltage profile can be calculated from the following discretized Equation (1.2.6b).

$$V_{l+1} := V_l + r \cdot \left(Ir_l - I0r\right) \cdot \Delta y + x \cdot \left(Ix_l - I0x\right) \cdot \Delta y + C \cdot \Delta y$$

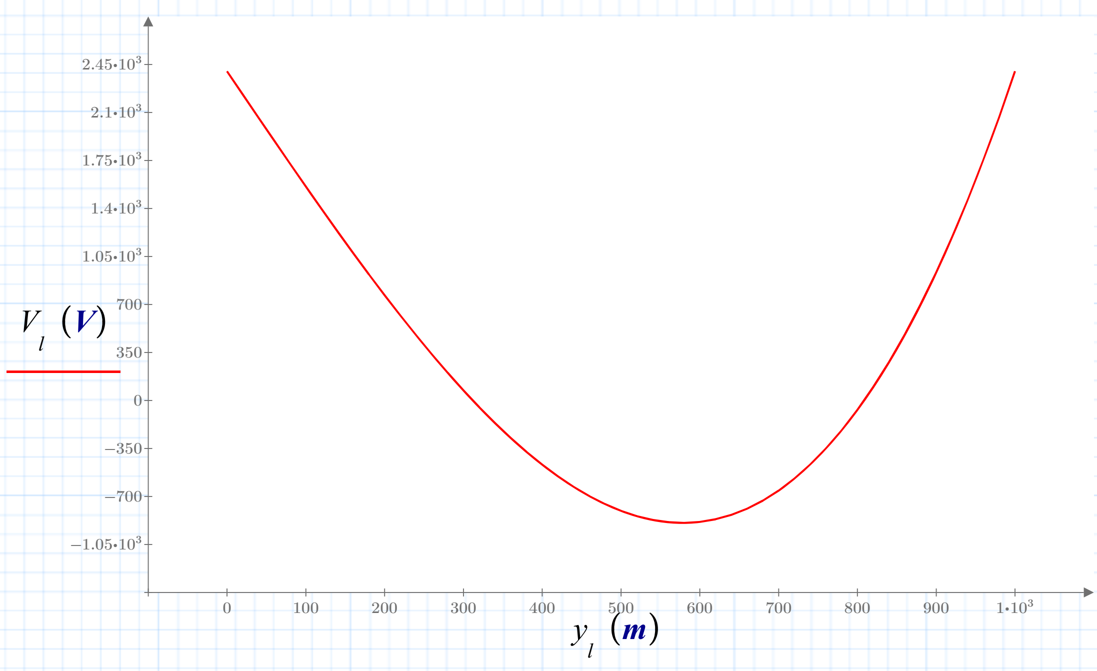

Fig. 1.2.6 Closed line Voltage profile

The minimum line voltage is $Vmin := min(V) = -892.843$ V

Section 1.2.4 Power Losses

The power losses in the line segment can be calculated from the line current and the line resistance. The incremental power losses due to the current at location y are

$$dP = r \cdot \left(Ir(y)^2 + Ix(y)^2 \right)$$

Therefore, the total power losses are

$$P = \int_0^L dP \, \mathrm{d}y \qquad \textbf{(1.2.8)}$$

This integral can be expressed as a recursion equation. For the case of closed lines presented above, we have

$$P_0 := 0 \; W$$

$$P_{l+1} := P_l + r \cdot \sum_{i=1}^{\mathrm{length}(Ir)} \left(\left(\left(Ir_l \right)^2 + \left(Ix_l \right)^2 \right) \cdot \Delta y \right)$$

$$P_{N+1} = \left(1.089 \cdot 10^9 \right) \; W$$