Section 1.3b Load Flow Calculations

Section 1.3.4 Application

Start all arrays from 1: **ORIGIN** $\equiv 1$

Consider the five-bus power system of Figure 1.3.2. For distribution networks, the generators may represent distribution substations and the load (shown by arrows on buses) may represent network feeders. The system operating conditions are defined from the generated (or imported) and load power at each bus as well as the voltage magnitude at generating buses (or distribution substations). In addition, **the impedances must be known for the network lines**.

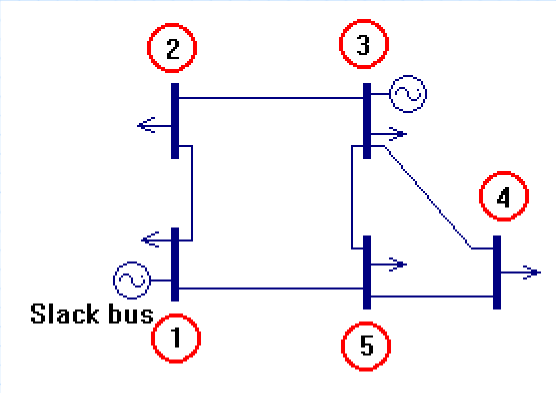

Fig. 1.3.2 Example power system.

System data can be entered most compactly in Mathcad as a matrix.

Transmission Lines and Transformer Data

These data are entered in the array **Series**. Each transformer and line occupies a row in that array with the following meaning associated with each column:

Sending End bus # = column 1
Receiving End bus # = column 2
Series Impedance in pu = column 3
Shunt Admittance in pu = column 4

Long transmission lines are represented by **Pi equivalents**. Column 4 of the array **Series** contains the admittance of each of the Pi shunt elements. For the case of transformers, this column may represent the admittance of the magnetizing branch of the transformer or may be set to zero if this branch is to be omitted. For this example we have the following network description:

$$Series := \begin{bmatrix} 1 & 2 & 0.042 + 1j \cdot 0.168 & 1j \cdot 0.020 \\ 1 & 5 & 0.031 + 1j \cdot 0.126 & 1j \cdot 0.014 \\ 2 & 3 & 0.031 + 1j \cdot 0.126 & 1j \cdot 0.014 \\ 3 & 4 & 0.084 + 1j \cdot 0.336 & 1j \cdot 0.04 \\ 3 & 5 & 0.053 + 1j \cdot 0.210 & 1j \cdot 0.024 \\ 4 & 5 & 0.063 + 1j \cdot 0.252 & 1j \cdot 0.06 \end{bmatrix}$$

Shunt Data

The data of the shunt equipment are entered in the array **Shunt**. The columns of this array correspond to

Bus # = column 1
shunt admittance in pu = column 2

If no shunt equipment is available, enter any bus number in the first column and 0 in the second column of the shunt array.

$$Shunt := \begin{bmatrix} 1 & 0 \end{bmatrix}$$

Shunt elements may represent reactive power compensation at certain system buses. In this case, the reactance of the corresponding compensation device (capacitor or inductor) is calculated and its admittance is entered in the above array. If lines are compensated by series capacitors, the reactance of the capacitor is subtracted from the line series reactance and the net reactance is included in the array **Series**.

Bus Data

These data are entered in the array **Bus**, one row per bus with the following meaning of columns:

	Bus #	P_G	P_L	Q_L	V	δ	Type
Column#	1	2	3	4	5	6	7

where

PG is the pu generated real power on the bus.
PL is the pu real load power on the bus.
QL is the pu reactive load power on the bus.
V is the pu magnitude of the bus voltage. This value remains unchanged during the solution process for the slack bus and PV-buses and is the initial guess for PQ-buses.
δ is the bus voltage phase angle in radians. It is normally zero for the slack bus. The value of this column is taken as the initial guess of the bus voltage phase angles for the other bus types.
Type is the bus type. Enter 0 for the slack bus, 1 for each PV-bus and 2 for each PQ-bus.

ENTER the slack bus first in the array, **Bus**.

$$Bus := \begin{bmatrix} 1 & 0.0 & 0.65 & 0.30 & 1.04 & 0.0 & 0 \\ 2 & 0.0 & 1.15 & 0.6 & 0.961 & 0.0 & 2 \\ 3 & 1.80 & 0.70 & 0.40 & 1.02 & 0.0 & 1 \\ 4 & 0.0 & 0.70 & 0.3 & 0.92 & 0.0 & 2 \\ 5 & 0.0 & 0.85 & 0.40 & 0.968 & 0.0 & 2 \end{bmatrix}$$

The solution follows these steps: initialization of system arrays, functions and indices, construction of the system bus admittance matrix, construction and inversion of the system Jacobian matrix and iterative solution of the problem.

Initialization

$N\_bus := \text{rows}(Bus)$ number of buses

$N\_ser := \text{rows}(Series)$ number of series elements

$N\_sh := \text{rows}(Shunt)$ number of shunt elements

Define auxiliary arrays holding the terminals of series and shunt equipment and the number and type of each bus.

$Is := \text{Re}\left(Series^{\langle 1\rangle}\right)$ $Js := \text{Re}\left(Series^{\langle 2\rangle}\right)$

$Ish := \text{Re}\left(Shunt^{\langle 1\rangle}\right)$ $It := \text{Re}\left(Bus^{\langle 7\rangle}\right)$

Calculate the net power injections at each bus.

$i := 1 .. N\_bus$

$Pb_i := Bus_{i,2} - Bus_{i,3}$ real power injection

$Qb_i := -Bus_{i,4}$ reactive power injection

Initialize the voltage and angle vectors.

$V := \left(Bus^{\langle 5\rangle}\right)^{T}$ $V := V^{T}$

$\delta := \left(Bus^{\langle 6\rangle}\right)^{T}$ $\delta := \delta^{T}$

Initialize the Ybus matrix.

$j := 1 .. N\_bus$ $Y_{bus_{i,j}} := 0.0$

Form the Ybus Matrix

The Ybus matrix is formed as follows:

$$m := 1 .. N\_ser$$

Add series elements.

$$Y_{bus_{Is_m, Is_m}} := Y_{bus_{Is_m, Is_m}} + Series_{m,4} + \frac{1}{Series_{m,3}}$$

$$Y_{bus_{Is_m, Js_m}} := Y_{bus_{Is_m, Js_m}} - \frac{1}{Series_{m,3}}$$

$$Y_{bus_{Js_m, Is_m}} := Y_{bus_{Js_m, Is_m}} - \frac{1}{Series_{m,3}}$$

$$Y_{bus_{Js_m, Js_m}} := Y_{bus_{Js_m, Js_m}} + Series_{m,4} + \frac{1}{Series_{m,3}}$$

$$m := 1 .. N\_sh$$

Add shunt elements.

$$Y_{bus_{Is_m, Is_m}} := Y_{bus_{Is_m, Is_m}} + Shunt_{m,2}$$

$$Y_{bus_{Js_m, Js_m}} := Y_{bus_{Js_m, Js_m}} + Shunt_{m,2}$$

Define functions of the net real and reactive power injections using Equations (1.3.2).

$$f_p(k, x, y) := \sum_i \left(x_k \cdot x_i \cdot \left| Y_{bus_{k,i}} \right| \cdot \cos\left(\mathbf{if}\left(Y_{bus_{k,i}} \neq 0, \arg\left(Y_{bus_{k,i}} \right), 0 \right) + y_i - y_k \right) \right)$$

$$f_q(k, x, y) := -\left(\sum_i \left(x_k \cdot x_i \cdot \left| Y_{bus_{k,i}} \right| \cdot \sin\left(\mathbf{if}\left(Y_{bus_{k,i}} \neq 0, \arg\left(Y_{bus_{k,i}} \right), 0 \right) + y_i - y_k \right) \right) \right)$$

Formation of the Jacobian Matrix

A Jacobian matrix, formed from the partial derivatives of the real and reactive power matrices, is required for the Newton-Raphson algorithm. The elements of the Jacobian matrix are formed as follows:

$k := 2 .. N\_bus$ $\qquad\qquad$ $n := 2 .. N\_bus$

$$\frac{d}{dV_n} f_{p_k}$$

$$Jac_{k-1,(n+N\_bus)-2} := V_k \cdot \left(\left|Ybus_{k,n}\right|\right) \cdot \cos\left(\mathbf{if}\left(Ybus_{k,n} \neq 0, \arg\left(Ybus_{k,n}\right), 0\right) + \delta_n - \delta_k\right)$$

$$\frac{d}{dV_k} f_{p_k}$$

$$Jac_{k-1,(k+N\_bus)-2} := \sum_i \left(V_i \cdot \left(\left|Ybus_{k,i}\right|\right) \cdot \cos\left(\mathbf{if}\left(Ybus_{k,i} \neq 0, \arg\left(Ybus_{k,i}\right), 0\right) + \delta_i - \delta_k\right)\right) + V_k \cdot \left(\left|Ybus_{k,k}\right|\right) \cdot \cos\left(\arg\left(Ybus_{k,k}\right)\right.$$

$$\frac{d}{dV_n} f_{q_k}$$

$$Jac_{(k+N\_bus)-2,(n+N\_bus)-2} := \left(-V_k\right) \cdot \left(\left|Ybus_{k,n}\right|\right) \cdot \sin\left(\mathbf{if}\left(Ybus_{k,n} \neq 0, \arg\left(Ybus_{k,n}\right), 0\right) + \delta_n - \delta_k\right)$$

$$\frac{d}{dV_k} f_{q_k}$$

$$Jac_{(k+N\_bus)-2,(k+N\_bus)-2} := \sum_i \left(\left(-V_i\right) \cdot \left(\left|Ybus_{k,i}\right|\right) \cdot \sin\left(\mathbf{if}\left(Ybus_{k,i} \neq 0, \arg\left(Ybus_{k,i}\right), 0\right) + \delta_i - \delta_k\right)\right) + -\left(V_k \cdot \left(\left|Ybus_{k,k}\right|\right) \cdot \sin\right.$$

$$\frac{d}{d\delta_n} f_{p_k}$$

$$Jac_{k-1,n-1} := -\left(V_k \cdot V_n \cdot \left(\left|Ybus_{k,n}\right|\right)\right) \cdot \sin\left(\mathbf{if}\left(Ybus_{k,n} \neq 0, \arg\left(Ybus_{k,n}\right), 0\right) + \delta_n - \delta_k\right)$$

$$\frac{d}{d\delta_k} f_{p_k}$$

$$Jac_{k-1,k-1} := \sum_i \left(V_k \cdot V_i \cdot \left(\left|Ybus_{k,i}\right|\right) \cdot \sin\left(\mathbf{if}\left(Ybus_{k,i} \neq 0, \arg\left(Ybus_{k,i}\right), 0\right) + \delta_i - \delta_k\right)\right) + -\left(\left(V_k\right)^2 \cdot \left|Ybus_{k,k}\right| \cdot \sin\left(\arg\left(Ybus_{k,k}\right)\right.\right.$$

$$\frac{d}{d\delta_n} f_{q_k}$$

$$Jac_{(k+N\_bus)-2,\,n-1} := \left(-V_k\right)\cdot V_n \cdot \left(\left|Y_{bus_{k,n}}\right|\right)\cdot \cos\left(\mathbf{if}\left(Y_{bus_{k,n}} \neq 0, \arg\left(Y_{bus_{k,n}}\right), 0\right) + \delta_n - \delta_k\right)$$

$$\frac{d}{d\delta_k} f_{q_k}$$

$$Jac_{k+N\_bus-2,\,k-1} := \sum_i \left(V_k \cdot V_i \cdot \left|Y_{bus_{k,i}}\right| \cdot \cos\left(\mathbf{if}\left(Y_{bus_{k,i}} \neq 0, \arg\left(Y_{bus_{k,i}}\right), 0\right) + \delta_i - \delta_k\right)\right) + -\left(\left(V_k\right)^2 \cdot \left|Y_{bus_{k,k}}\right| \cdot \cos\left(\arg\left(Y_{bu\right.\right.\right.$$

$$Jac_{k-1,\,n+N\_bus-2} := \mathbf{if}\left(It_n = 1, 0, Jac_{k-1,\,n+N\_bus-2}\right)$$

$$Jac_{k+N\_bus-2,\,n-1} := \mathbf{if}\left(It_k = 1, 0, Jac_{k+N\_bus-2,\,n-1}\right)$$

$$Jac_{k+N\_bus-2,\,n+N\_bus-2} := \mathbf{if}\left(It_k = 1, 0, Jac_{k+N\_bus-2,\,n+N\_bus-2}\right)$$

$$Jac_{k+N\_bus-2,\,n+N\_bus-2} := \mathbf{if}\left(It_n = 1, 0, Jac_{k+N\_bus-2,\,n+N\_bus-2}\right)$$

$$Jac_{k+N\_bus-2,\,k+N\_bus-2} := \mathbf{if}\left(It_k = 1, 1, Jac_{k+N\_bus-2,\,k+N\_bus-2}\right)$$

Invert Jacobian matrix.

$$Jinv := Jac^{-1}$$

Solve Load Flow Iteratively

The Iterative solution of the problem is as follows:

Define, using Equation (1.3.5), functions that provide the corrections of voltage and phase angle for the new iteration step.

$$\Delta V(l,V,\delta) := \left(\sum_k \left(Jinv_{l+N\_bus-2,\,k-1} \cdot \left(Pb_k - f_p(k,V,\delta)\right)\right)\right) + \sum_k \left(Jinv_{l+N\_bus-2,\,k+N\_bus-2} \cdot \left(\mathbf{if}\left(It_k = 1, 0, Qb_k - f_q(k,\right.\right.\right.$$

$$\Delta\delta(l,V,\delta) := \sum_k \left(Jinv_{l-1,\,k-1} \cdot \left(Pb_k - f_p(k,V,\delta)\right)\right) + \left(\sum_k \left(Jinv_{l-1,\,k+N\_bus-2} \cdot \left(\mathbf{if}\left(It_k = 1, 0, Qb_k - f_q(k,V,\delta)\right)\right)\right)\right)$$

Iterative Solution

Define the maximum iteration number.

$$Max\_it := 6$$

Define the acceleration coefficient, λ. This coefficient takes values **less than one** and improves the convergence characteristics of the problem. The user may change the value of λ to see its effect on the mismatch at the end of the iterations.

$$\lambda := 1$$

Define the iteration index.

$$Iter := 1 \,..\, Max\_it$$

$$m := 2 \,..\, N\_bus$$

$$Num_1 := 0$$

Iterations:

$$\begin{bmatrix} Num_{Iter+1} \\ V_m \\ \delta_m \end{bmatrix} := \begin{bmatrix} Num_{Iter} + 1 \\ V_m + \Delta V(m,V,\delta) \cdot \lambda \\ \delta_m + \Delta\delta(m,V,\delta) \cdot \lambda \end{bmatrix}$$

The power mismatch is

$$\varepsilon := \left(\sum_m \left(\left(Pb_m - f_p(m, V, \delta)\right)^2 + \textbf{if}\left(It_m = 1, 0, \left(Qb_m - f_q(m, V, \delta)\right)^2\right)\right)\right)^{\frac{1}{2}} = 1.41 \cdot 10^{-6}$$

The bus voltage magnitudes and phase angles are

$$V = \begin{bmatrix} 1.04 \\ 0.961 \\ 1.02 \\ 0.926 \\ 0.971 \end{bmatrix} \qquad \frac{\delta}{deg} = \begin{bmatrix} 0 \\ -6.312 \\ -3.698 \\ -10.9 \\ -6.18 \end{bmatrix}$$

The real and reactive power of the slack bus are respectively

$$Ps := f_p(1, V, \delta) + Bus_{1,3} = 2.345 \qquad Qs := f_q(1, V, \delta) + Bus_{1,4} = 0.979$$

Calculation of Line Losses

The line losses for the second line are calculated as follows:

Define line number as in the array **Series**.

$$m := 2$$

$$i := Is_m \qquad j := Js_m$$

$$ys := \frac{1}{Series_{m,3}} \qquad ysh := Series_{m,4}$$

Power flow from sending to receiving terminal:

$$Pij := -\left(V_i \cdot V_j \cdot |ys| \cdot \cos\left(\arg(ys) + \delta_j - \delta_i\right)\right) + \left(V_i\right)^2 \cdot |ys| \cdot \cos(\arg(ys)) + \left(V_i\right)^2 \cdot |ysh| \cdot \cos(\arg(ysh))$$

Power flow from receiving to sending terminal:

$$Pji := -\left(V_j \cdot V_i \cdot |ys| \cdot \cos\left(\arg(ys) + \delta_i - \delta_j\right)\right) + \left(V_j\right)^2 \cdot |ys| \cdot \cos(\arg(ys)) + \left(V_j\right)^2 \cdot |ysh| \cdot \cos(\arg(ysh))$$

The real power losses in the second line are

$$Ploss := Pij + Pji \qquad Ploss = 0.03$$

Reactive power flow from sending to receiving terminal:

$$Qij := V_i \cdot V_j \cdot |ys| \cdot \sin\left(\arg(ys) + \delta_j - \delta_i\right) - \left(V_i\right)^2 \cdot |ys| \cdot \sin(\arg(ys)) + -\left(\left(V_i\right)^2 \cdot |ysh| \cdot \sin(\arg(ysh))\right)$$

Reactive power flow from receiving to sending terminal:

$$Qji := V_j \cdot V_i \cdot |ys| \cdot \sin\left(\arg(ys) + \delta_i - \delta_j\right) - \left(V_j\right)^2 \cdot |ys| \cdot \sin(\arg(ys)) + -\left(\left(V_j\right)^2 \cdot |ysh| \cdot \sin(\arg(ysh))\right)$$

The reactive power losses in the second line are

$$Qloss := Qij + Qji = 0.095$$