Section 1.4b Least-Cost Power Transformer Sizing

1.4.3 Cost Estimation

To illustrate the costs involved in transformer operation, select a 750 KVA three-phase 12,470Y-g/480Y transformer that has the following total and core losses at rated load and unity power factor. All values are in watts.

$P_T := 10200\ W$ total losses

$P_c := 1800\ W$ core losses

$kW \equiv 1000\ W$ scaling factor

Short circuit and open circuit test results give the values for the transformer power lose values.

The transformer will serve a customer that has an estimated load factor of

$F_{LD} := 0.652$

For more information on how to calculate the load factor, see **Section 1.4a**.

Using this information, and the costs provided below, compute:
1. The loss ratio of the transformer
2. The pu load for maximum efficiency
3. Maximum efficiency for the estimated customer load factor
4. Total cost to own the transformer over a 30 year life time

Fixed Costs

Three cost components add to give the total cost to own a power transformer. These cost components are 1) the cost of investment, 2) the cost of the energy losses due to the transformer, 3) the demand costs due to the loss of transformer capacity. The cost of investment is usually the largest component of the three. The investment costs include the cost of the transformer itself, the installation costs, and labor involved in the installation. Equation (1.4.7) from [1] gives the annual fixed cost of owning a transformer.

$$FC(r_i, r_t, r_d, C) := (r_i + r_t + r_d) \cdot C \qquad \textbf{(1.4.7)}$$

where ri is the annual rate of interest, rt is the rate of insurance, rd is the percent depreciation, and C is the cost of transformer.

First, we'll need some units:

$KVA := kW \qquad cent := 1 \qquad dollar := 100 \cdot cent$

Suppose the transformer has a purchase price of $34,100,

$$C := 34100\ \textbf{dollar}$$

and the utility related charges are as follows:

$$d := 7.50\ \frac{\textbf{dollar}}{\textbf{kW}}$$ utility demand charge

$$q := .048\ \frac{\textbf{dollar}}{\textbf{kW}}$$ energy charge

No load growth is expected over the life of the transformer. The load is served continuously with no extended outages. The utility rate-of-return, insurance rate, and depreciation rate are

$$r_i := 10 \cdot \frac{\%}{\textbf{yr}}$$ annual rate of interest

$$r_t := 2 \cdot \frac{\%}{\textbf{yr}}$$ rate of insurance

$$r_d := 6.5 \cdot \frac{\%}{\textbf{yr}}$$ percent depreciation

Compute the fixed capital cost associated with owning the transformer using the rates and transformer cost given.

$$FC\left(r_i, r_t, r_d, C\right) = 6308.5\ \frac{dollar}{\textbf{yr}}$$ fixed costs

The value of energy losses depends on the transformer efficiency and the load factor. The load factor determines the total losses dissipated by the machine over its operating life. An approximate relationship between the load factor and the loss factor of a device is given in [2] which is shown in Equation (1.4.8). In general, the value of loss factor is

$$F_{LD}^{\ 2} < F_{LS} < F_{LD}$$

where $F_{LD} = 0.652$ is the load factor,

F_{LS} is the loss factor, and

$$F_{LS}\left(F_{LD}\right) := 0.3 \cdot F_{LD} + 0.7 \cdot F_{LD}^{\ 2} \qquad \textbf{(1.4.8)}$$

Compute the loss factor from the given load factor.

$$F_{LS}\left(F_{LD}\right) = 0.493 \qquad LsF := F_{LS}\left(F_{LD}\right)$$

These energy losses may have different values each year during the operational lifetime of the equipment.

Cost of Core Losses

The energy losses have two cost components--the core loss component and the load loss component. Equation (1.4.9) gives the annual core loss value.

Suppose the number of operational hours in a year is

$$hpy := 8760$$

Then,

$$C_c := \frac{P_c \cdot (d + hpy \cdot q)}{\mathbf{yr}} \qquad C_c = 770.364 \ \frac{dollar}{\mathbf{yr}} \qquad \textbf{(1.4.9)}$$

where Pc is the core loss at the rated voltage, as given above.

Cost of Load Losses

Equation (1.4.10) gives the annual cost of the load losses.

$$C_l\left(P_l, F_{LD}, F_{LS}\right) := \frac{P_l \cdot F_{LD} \cdot \left(d + hpy \cdot q \cdot F_{LS}\right)}{\mathbf{yr}} \qquad \textbf{(1.4.10)}$$

where Pl is the full-load load loss, FLD is the transformer load factor, FLS and is the transformer loss factor from (1.4.8).

Find the load losses at rated load from the core and total loss values.

$$P_l := P_T - P_c = 8.4 \ \mathbf{kW} \qquad \text{load loss}$$

$$C_l\left(P_l, F_{LD}, LsF\right) = 1176.796 \ \frac{dollar}{yr} \qquad \text{cost of load losses}$$

The total annual cost of ownership is given by Equation (1.4.11).

$$AC := FC\left(r_i, r_t, r_d, C\right) + C_c + C_l\left(P_l, F_{LD}, LsF\right) = 8255.66 \ \frac{\mathbf{dollar}}{\mathbf{yr}} \qquad \textbf{(1.4.11)}$$

For intermittent transformer operation, change the variable hpy in Equations (1.4.9) and (1.4.10) from 8760 to reflect the actual transformer service hours.

If the load factor differs from year to year, the annual cost will differ. Finding the present worth of the annual costs gives a comparison for the total cost of owning two different transformers. Define the present-worth factor for an interest rate and a time period.

$$pwf(i, n) := \frac{(1+i)^n - 1}{i \cdot (1+i)^n}$$

where n is the number of time periods and i is the interest per period.

Equation (1.4.12) gives the total (present) cost to own the transformer over its operational life

with

$C = 34100\ \textbf{dollar}$	purchase price in dollars
$i := 0.1$	rate of return
$n := 30$	number of periods in years
$AC = 8255.66\ \dfrac{dollar}{\textbf{yr}}$	annual cost
$TC := C + AC \cdot pwf(i, n)\ \textbf{yr} = 111925.402\ \textbf{dollar}$	total cost in today's dollars to own over $n = 30$ years **(1.4.12)**

Equation (1.4.12) computes the total cost of owning a transformer with an initial cost, C, and an equal stream of annual costs, AC.

Generally the annual costs vary each year. The method of levelized annual costs creates an equivalent stream of annual costs with the same value. To compute the levelized annual cost for a stream of unequal annual costs, compute the present worth of each annual cost. Add these present worths at time zero, and distribute the lumped sum in time as costs of equal value. This procedure requires the single-payment present-worth factor and the capital recovery factor.

$$crf(i, n) := \frac{i \cdot (1+i)^n}{(1+i)^n - 1}$$ capital recovery factor

$$crf(i, 30) = 0.106$$

$$sppw(i, n) := \frac{1}{(1+i)^n}$$ single-payment present-worth factor

$$sppw(i, 30) = 0.057$$

For p unequal annual costs, the levelized annual cost is given by Equation (1.4.13).

$$LAC = \left(\sum_p \left(AC_p \cdot sppw(i, p) \right) \right) \cdot crf(i, n) \qquad \textbf{(1.4.13)}$$

Using the levelized annual cost in Equation (1.4.13) in place of the AC variable computes the total cost to own the transformer in a more realistic way.

Compute Loss Ratio

The load and core losses from above give the loss ratio of the transformer.

$$LR := \frac{P_l}{P_c} = 4.667$$

Compute pu values of load and core losses. The power base is the rated power of the transformer.

$$S_{base} := 750 \cdot KVA$$

$$P_{cpu} := \frac{P_c}{S_{base}} \qquad P_{tpu} := \frac{P_T}{S_{base}} \qquad P_{lpu} := \frac{P_l}{S_{base}}$$

Compute the transformer efficiency for the customer load factor and the pu losses.

$$\eta_{TL}\left(F_{ld}, P_c, P_l\right) := \left(\frac{1}{1 + \frac{P_c}{F_{ld}} + F_{ld} \cdot P_l} \cdot 100\right)\% \qquad \eta_{TL}\left(F_{LD}, P_{cpu}, P_{lpu}\right) = 98.914\ 1\%$$

Compute the pu load that produces the maximum efficiency. Either the pu values of the losses or the values in watts can be used to compute this value since the maximum efficiency depends on the ratio of the losses. Using the pu values for losses, the pu of rated load for maximum efficiency is

$$L_{max}\left(P_c, P_l\right) := \sqrt{\frac{P_c}{P_l}}$$

$$L_{max}\left(P_{cpu}, P_{lpu}\right) = 0.463$$

$$LF_m := L_{max}\left(P_{cpu}, P_{lpu}\right)$$

Loading for Maximum Efficiency

This transformer produces maximum efficiency at 0.463 of rated load. In term of the power base,

$$L_{max}\left(P_{cpu}, P_{lpu}\right) \cdot S_{base} = 347.183\ KVA$$

A transformer with a lower loss ratio will produce a higher efficiency for this load. The function below computes the maximum transformer efficiency.

$$\eta_{max}\left(LF, P_t\right) := \left(\frac{LF}{LF + \left(\frac{P_t \cdot LF^2}{1 + LF^2}\right) \cdot 2} \cdot 100\right)\%$$

$$\eta_{max}\left(LF_m, P_{tpu}\right) = 98.974\ 1\%$$

Section 1.4.4 References

1. D. P. Franklin and A. C. Franklin, *J&P Transformer Book,* 11th Edition, Butterworths, London, 1983.

2. Turan Gonen, *Electric Power Distribution System Engineering*, McGraw-Hill, New York, 1986.