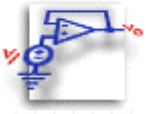


1 POWER DISTRIBUTION



Section 1.5b Power System Harmonic Analysis

Section 1.5.4 Harmonic Interactions

Data Preparation

Prepare an element parameter matrix that includes the resistance, capacitance, and inductance of the circuit elements. Compute the values of inductance and capacitance using the system fundamental frequency. Another Mathcad document can compute the values and generate the correct data file using the **WRITEPRN** function (see the *Mathcad User's Guide*). The Elements array would hold the values returned by a **READPRN** function eliminating the need to type in data by hand. You can also use the Input Table component.

The elements array holds the bus connection information and the component parameters for the system. There is a row in the array for every branch in the system. This row also includes the terminal buses of the branch in the system.

ORIGIN = 1 Reset origin

$$\text{Elements} := \begin{bmatrix} 1 & 1 & 0 & 0.051 & 10^{17} \\ 1 & 1 & 0 & 0 & 10 \cdot 10^{-6} \\ 1 & 2 & 0.1 & 0.02 & 10^{17} \\ 2 & 2 & 0 & 0 & 5 \cdot 10^{-6} \\ 2 & 2 & 0 & 0 & 7 \cdot 10^{-6} \\ 2 & 3 & 0.2 & 0.03 & 10^{17} \\ 3 & 3 & 0 & 0 & 8 \cdot 10^{-6} \\ 3 & 3 & 0 & 0.05 & 10^{17} \end{bmatrix}$$

The column definitions of Elements are

1. From bus
2. To bus (enter same value as from bus if shunt element)
3. Resistance (in ohms)
4. Inductance (in henrys)
5. Capacitance (in farads)

If R, L, and C are not present in each element, they must be eliminated from the model.

For R: enter 0 in column 3

For L: enter 0 in column 4

For C: enter large numerical values (1010 in this case)

Define the number of elements and buses in the system.

$$N_elements := \text{rows}(\text{Elements}) \quad N_bus := 3$$

Creation of the Matrices

Zero the branch admittance matrix and the connection matrix.

$$p := 1 .. N_elements$$

$$l := 1 .. N_elements$$

$$q := 1 .. N_bus$$

$$Y_{b,p,l} := 0 \quad A_{p,q} := 0$$

Define the terminal bus vectors from the Elements array.

$$Fb_p := Elements_{p,1} \quad Tb_p := Elements_{p,2}$$

Build the connection matrix, A, using Equations (1.5.8) and (1.5.9).

$$A_{p,Fb_p} := 1 \quad (1.5.8)$$

$$A_{p,Tb_p} := \text{if}(Tb_p = Fb_p, 1, -1) \quad (1.5.9)$$

Define the branch admittance matrix, **Yb**, and initialize it to zero.

$$Y_{b,p,q} := 0$$

Equation (1.5.10) converts the values of R, L, and C in the Element array into an admittance value given a base frequency. The skin effect associated with resistance in Equation (1.5.10) is ignored in this analysis to simplify the formulation of the problem. The accuracy of the solution remains largely unaffected since the inductive and capacitive elements are much larger than the resistance in practical power line configurations. (For more information on skin effect, see **Section 1.6a.**)

$$\omega_1 := 377 \quad \text{base frequency in radians/sec}$$

Define the values from the Elements array.

$$R_p := Elements_{p,3} \quad L_p := Elements_{p,4} \quad C_p := Elements_{p,5}$$

$$Y_{b,p,p} := \frac{1}{R_p + \left(1j \cdot \omega_1 \cdot L_p + \frac{1}{1j \cdot \omega_1 \cdot C_p} \right)} \quad (1.5.10)$$

Equation (1.5.11) is a function that permits the calculation of Zbus from the Yb and A matrices computed above.

$$F(X) := (A^T \cdot X \cdot A)^{-1} \quad (1.5.11)$$

Computation of Harmonic Impedances

Equation (1.5.12) is an iterative calculation that produces a number of harmonic impedance values used to plot the system frequency response. **This calculation is very time intensive** since it increases as the cube of the number of buses in the system. The iteration time also depends on the number of harmonics that are included in the study and the frequency step-size selection. Be prepared for a long wait if the system is larger than 4 buses.

The indices, i and j , allow the selection of a particular Z_{bus} element for plotting. Diagonal elements of Z_{bus} are the Thevenin's equivalent impedance of the system as seen from bus i . The off-diagonal elements are the mutual impedances at bus j with respect to bus i and give the pu voltage for a current injection at bus i .

Select the Thevenin's impedance at bus 3 to compute and plot.

$$i := 3 \quad j := 3$$

Define the frequency step used and the iteration counter.

$$\Delta\omega := 75.4 \quad \text{rad/sec}$$

Initialize frequency.

$$k := 1 .. 70 \quad \omega_1 := 377$$

Define a base impedance to normalize the calculations.

$$Z_{base} := 300$$

$$\omega_k := \omega_1 + (k-1) \cdot \Delta\omega$$

$$Z := \begin{array}{l} \text{for } k \in 1 .. 70 \\ \quad \text{for } p \in 1 .. N_elements \\ \quad \quad Y_{b_{p,p}} \leftarrow \frac{1}{R_p + \left(1j \cdot (\omega_k) \cdot L_p + \frac{1}{1j \cdot \omega_k \cdot C_p} \right)} \\ \quad \quad F(Y_b)_{i,j} \\ \quad \quad Z_k \leftarrow \frac{F(Y_b)_{i,j}}{Z_{base}} \\ \text{return } Z \end{array} \quad (1.5.12)$$

System Harmonic Calculations

Plotting the results of Equation (1.5.12) shows the natural system resonances. Figure 1.5.5 plots the magnitude of the harmonic impedances against the harmonic order.

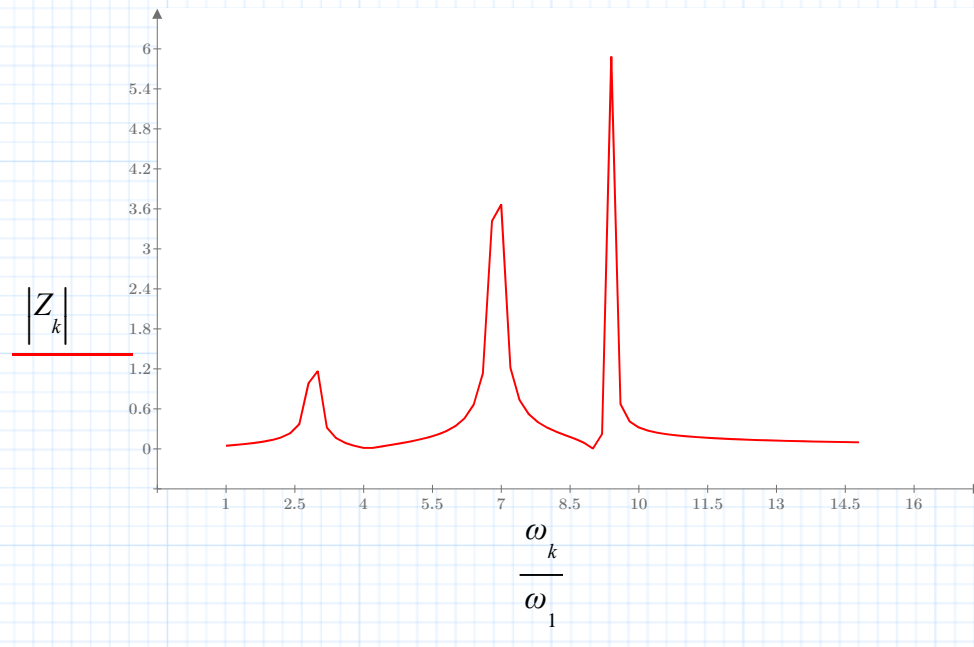


Fig. 1.5.5 Harmonic Impedance Plot

The plot in Figure 1.5.5 shows the resonances of the test system. A device that injects harmonic currents at or near these resonances will cause harmonic voltages from the interaction with the high magnitude harmonic impedances. To see the effect of harmonic current injections on system overvoltage, see **Section 1.3b**.

The installation of a 6-pulse phase-control thyristor converter on bus 3 in the test system will inject pu currents of the order and magnitude given in Section 1.5.2.

For harmonic order from

$$h := 1 .. 13$$

find the harmonic voltage produced by the current injections.

$$I_{inj} := \begin{bmatrix} 1.000 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.200 \\ 0.000 \\ 0.143 \\ 0.000 \\ 0.000 \\ 0.000 \\ 0.091 \\ 0.000 \\ 0.077 \end{bmatrix}$$

theoretical values of
harmonic current injections
from a 6-pulse converter.

Find the harmonic impedance corresponding to the harmonic orders of interest from the list computed using Equation (1.5.12).

$$Z_{th_h} := Z_{\text{floor}\left(\frac{\omega_1 \cdot h}{\Delta\omega}\right)}$$

The floor function selects the correct index from the bus impedance array. The argument of the floor function is the product of the fundamental frequency and the harmonic order divided by the frequency step.

Equation (1.5.13) finds the harmonic voltage for the current injections.

$$V_h := Z_{th_h} \cdot I_{inj_h} \quad (1.5.13)$$

Figure 1.5.6 shows the harmonic voltages produced by the converter current injections. The graph plots pu voltage magnitudes against the harmonic order of the voltage.

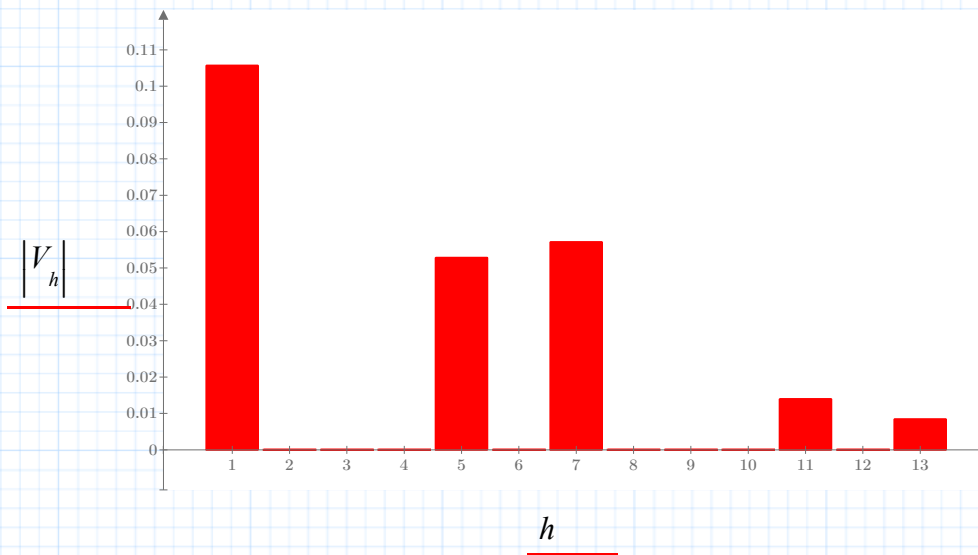


Fig. 1.5.6 Harmonic Voltage Plot

Time Plot of Harmonic Voltage Distortion

The harmonic voltages add to the voltage at the fundamental frequency to produce voltage distortion. Severe voltage distortion causes multiple zero-crossings of the supply voltage sine wave. Excessive oscillation of the voltage causes the malfunction which, in turn, produces more harmonic distortion by creating uncharacteristic harmonics on the ac side of the converter.

Equation (1.5.14) is a function that represents the fundamental frequency sine wave and the first 13 voltage harmonics generated by the 6-pulse converter. Converters operate at a lagging power factor. The power factor angle is related to the firing angle of the converter. The angle α represents the power factor angle of a converter.

$$\alpha := 15 \cdot \text{deg} \quad \text{converter firing angle.}$$

Define the time step of the plot to be 1/2 the period of the 13th harmonic.

$$\Delta t := \frac{0.5}{13 \cdot \omega_1}$$

Plot 1.5 periods of the fundamental frequency.

$$t := 0, \Delta t .. \frac{3 \cdot \pi}{\omega_1} \quad (1.5.14)$$

$$v(t) := 1 \cdot \cos(\omega_1 \cdot t + \alpha) + \sum_h \text{if}(V_h \neq 0, |V_h| \cdot \cos(h \cdot \omega_1 \cdot t + \arg(V_h)), 0)$$

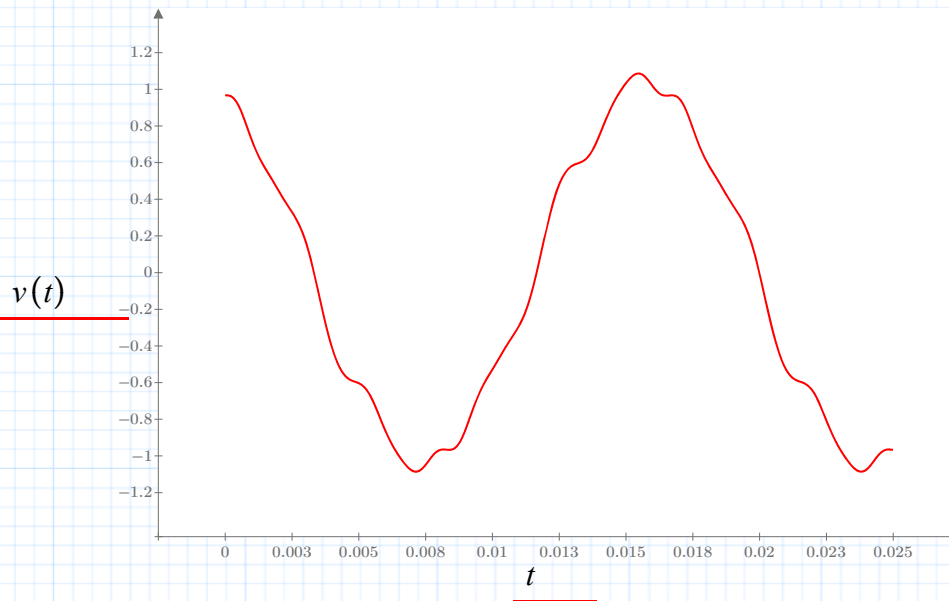


Fig. 1.5.7 Harmonic Voltage Time Plot

Computation of Harmonic Factors

Equations (1.5.15) and (1.5.16) give the values for the voltage and current harmonic factors defined in Section 1.5.2.

Given

$$V_1 := 1.0 \quad \text{p.u.} \quad h := 2..13$$

$$HF_V := \sqrt{\frac{\sum_h (|V_h|)^2}{V_1}} = 0.079 \quad (1.5.15)$$

Given the fundamental converter current

$$I_1 := 1.0 \quad \text{p.u.}$$

$$HF_I := \sqrt{\frac{\sum_h (|I_{inj_h}|)^2}{I_1}} = 0.273 \quad (1.5.16)$$

Section 1.5.5 References

1. *IEEE Guide for Harmonic Control and Reactive Compensation of Static Power Converters*, ANSI/IEEE STD 519-1991, Institute of Electrical and Electronic Engineers, Inc.
2. Kimbark, E. W., *Direct Current Transmission, Vol. 1*, Wiley-Interscience, New York, 1971.