Section 2.1b Power System Faults

Section 2.1.6 Faults - Application

For the system of Figure 2.1.5, the prefault voltage on bus 3 is 1.05 pu.

(a) Calculate the sequence components of the Zbus matrix,
(b) Calculate the pu subtransient fault-current for a single-line-to-ground fault occurring on bus 3,
(c) Calculate the bus voltages for the above fault, and
(d) Calculate the contribution of system components to the fault current for the conditions in (c).

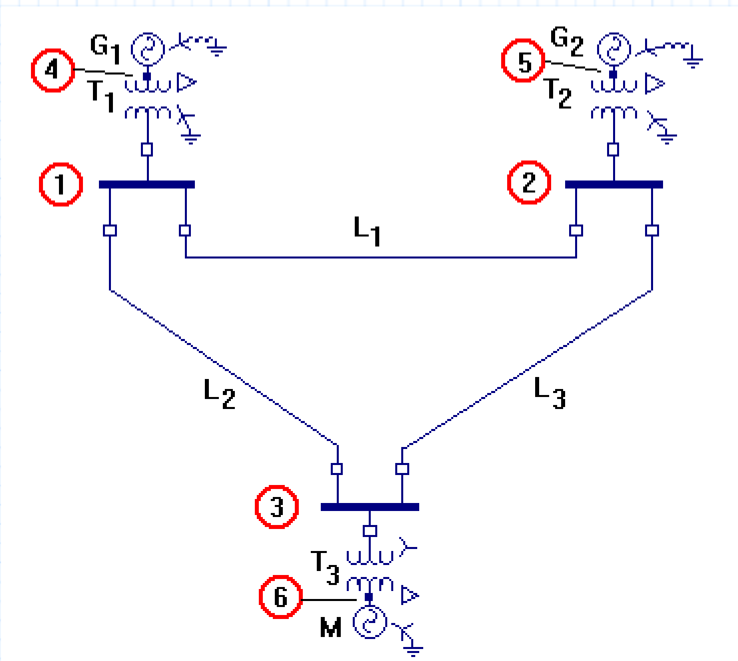

Fig. 2.1.5 Single line diagram of system

The line, transformer, and generator/motor data are entered into arrays. The columns of these arrays have the following meaning.

Line Data

	Line #	Sending End	Receiving End	Series impedance Z_1	Z_2	Z_0
Column #	1	2	3	4	5	6

For the system above, we have

$$Line := \begin{bmatrix} 1 & 1 & 2 & 0.04j & 0.04j & 0.07j \\ 2 & 1 & 3 & 0.06j & 0.06j & 0.08j \\ 3 & 2 & 3 & 0.06j & 0.06j & 0.08j \end{bmatrix}$$

Transformer Data

	Tr#	Prim.	Sec.	Leakage			Ground reactance		Con.
				X_1	X_2	X_0	Z_{n1}	Z_{n2}	
Column #	1	2	3	4	5	6	7	8	9

where Zn1 and Zn2 are, respectively, the primary and secondary ground impedances for Y-g sides.

ENTER a large value under **columns 7 or 8** if there is no ground connection at the corresponding side.

ENTER the following number under **column 9**

1 for Y-ground/Y-ground
2 for Y-ground/Delta
3 for Delta/Y-ground
-2 for Y/Δ
-3 for Δ/Y
0 for all other configurations

For this example,

$$Trans := \begin{bmatrix} 1 & 4 & 1 & 0.4j & 0.4j & 0.4j & 10^{15} & 0 & 3 \\ 2 & 5 & 2 & 0.4j & 0.4j & 0.4j & 10^{15} & 0 & 3 \\ 3 & 6 & 3 & 0.5j & 0.5j & 0.5j & 10^{15} & 10^{15} & -3 \end{bmatrix}$$

Generator/Motor Data

	Gen-Mot#	Bus#	Reactance			Ground
			X_1	X_2	X_0	Z_n
Column #	1	2	3	4	5	6

ENTER a large value under **column 6** if there is no connection to the ground.

For this example,

$$Gen := \begin{bmatrix} 1 & 4 & 0.3j & 0.3j & 0.6j & 0.2j \\ 2 & 5 & 0.3j & 0.3j & 0.6j & 0.2j \\ 3 & 6 & 0.4j & 0.4j & 0.8j & 0 \end{bmatrix}$$

SOLUTION

Start all arrays from 1.

$$\mathbf{ORIGIN} \equiv 1$$

(a) Zbus matrix

The bus-impedance matrix of the system is calculated by inverting the bus-admittance matrix of each sequence.

Define the number of system buses and the number of lines, transformers, and generators.

$$N\_bus := 6$$

$$N\_line := \text{rows}(Line)$$

$$N\_trans := \text{rows}(Trans)$$

$$N\_gen := \text{rows}(Gen)$$

Define arrays holding the configuration information of lines, transformers, and generators.

$Il := \text{Re}\left(Line^{\langle 2 \rangle}\right)$ line sending end

$Jl := \text{Re}\left(Line^{\langle 3 \rangle}\right)$ line receiving end

$Ig := \text{Re}\left(Gen^{\langle 2 \rangle}\right)$ generator terminal

$It := \text{Re}\left(Trans^{\langle 2 \rangle}\right)$ transformer primary

$Jt := \text{Re}\left(Trans^{\langle 3 \rangle}\right)$ transformer secondary

Calculate the contribution of generator impedance to bus shunt admittance and include it into the arrays, Ysh1, Ysh2, Ysh0, for positive, negative, and zero sequence respectively.

$$i := 1 .. N\_bus$$

$$Ysh1_i := 0 \qquad Ysh2_i := 0 \qquad Ysh0_i := 0$$

$$k := 1 .. N\_gen$$

$$Ysh1_{Ig_k} := Ysh1_{Ig_k} + \frac{1}{Gen_{k,3}} \qquad Ysh2_{Ig_k} := Ysh2_{Ig_k} + \frac{1}{Gen_{k,4}}$$

$$Ysh0_{Ig_k} := Ysh0_{Ig_k} + \frac{1}{Gen_{k,5} + Gen_{k,6}}$$

Add to Ysh0 the transformer contribution for Yg/Δ and Δ/Yg connections.

$$m := 1..N\_trans$$

$$Ysh0_{It_m} := \mathbf{if}\left(Trans_{m,9} = 2, \frac{1}{Trans_{m,6} + 3 \cdot \left(Trans_{m,7}\right)}, 0\right) + Ysh0_{It_m}$$

$$Ysh0_{Jt_m} := \mathbf{if}\left(Trans_{m,9} = 3, \frac{1}{Trans_{m,6} + 3 \cdot Trans_{m,8}}, 0\right) + Ysh0_{Jt_m}$$

Calculate the zero sequence series admittance of Y-g/Y-g transformers and store them in arrays Ys0.

$$Ys0_{m} := \mathbf{if}\left(Trans_{m,9} = 1, \frac{1}{Trans_{m,6} + 3 \cdot \left(Trans_{m,7} + Trans_{m,8}\right)}, 0\right)$$

Build the bus-admittance matrix for each sequence, Yb1, Yb2, Yb0, from system configuration and data.

$$j := 1..N\_bus \qquad Yb1_{i,j} := 0 \qquad Yb2_{i,j} := 0 \qquad Yb0_{i,j} := 0$$

Add lines.

$$k := 1..N\_line$$

Add transformers.

$$m := 1..N\_trans$$

Positive:

$$Yb1_{Il_k, Il_k} := Yb1_{Il_k, Il_k} + \frac{1}{Line_{k,4}} \qquad Yb1_{It_m, It_m} := Yb1_{It_m, It_m} + \frac{1}{Trans_{m,4}}$$

$$Yb1_{Jl_k, Jl_k} := Yb1_{Jl_k, Jl_k} + \frac{1}{Line_{k,4}} \qquad Yb1_{Jt_m, Jt_m} := Yb1_{Jt_m, Jt_m} + \frac{1}{Trans_{m,4}}$$

$$Yb1_{Il_k, Jl_k} := Yb1_{Il_k, Jl_k} - \frac{1}{Line_{k,4}} \qquad Yb1_{It_m, Jt_m} := Yb1_{It_m, Jt_m} - \frac{1}{Trans_{m,4}}$$

$$Yb1_{Jl_k, Il_k} := Yb1_{Jl_k, Il_k} - \frac{1}{Line_{k,4}} \qquad Yb1_{Jt_m, It_m} := Yb1_{Jt_m, It_m} - \frac{1}{Trans_{m,4}}$$

Negative:

$$Yb2_{Il_k,Il_k} := Yb2_{Il_k,Il_k} + \frac{1}{Line_{k,5}}$$

$$Yb2_{It_m,It_m} := Yb2_{It_m,It_m} + \frac{1}{Trans_{m,5}}$$

$$Yb2_{Jl_k,Jl_k} := Yb2_{Jl_k,Jl_k} + \frac{1}{Line_{k,5}}$$

$$Yb2_{Jt_m,Jt_m} := Yb2_{Jt_m,Jt_m} + \frac{1}{Trans_{m,5}}$$

$$Yb2_{Il_k,Jl_k} := Yb2_{Il_k,Jl_k} - \frac{1}{Line_{k,5}}$$

$$Yb2_{It_m,Jt_m} := Yb2_{It_m,Jt_m} - \frac{1}{Trans_{m,5}}$$

$$Yb2_{Jl_k,Il_k} := Yb2_{Jl_k,Il_k} - \frac{1}{Line_{k,5}}$$

$$Yb2_{Jt_m,It_m} := Yb2_{Jt_m,It_m} - \frac{1}{Trans_{m,5}}$$

Zero:

$$Yb0_{Il_k,Il_k} := Yb0_{Il_k,Il_k} + \frac{1}{Line_{k,6}}$$

$$Yb0_{It_m,It_m} := Yb0_{It_m,It_m} + \left(Ys0_m\right)$$

$$Yb0_{Jl_k,Jl_k} := Yb0_{Jl_k,Jl_k} + \frac{1}{Line_{k,6}}$$

$$Yb0_{Jt_m,Jt_m} := Yb0_{Jt_m,Jt_m} + \left(Ys0_m\right)$$

$$Yb0_{Il_k,Jl_k} := Yb0_{Il_k,Jl_k} - \frac{1}{Line_{k,6}}$$

$$Yb0_{It_m,Jt_m} := Yb0_{It_m,Jt_m} - \left(Ys0_m\right)$$

$$Yb0_{Jl_k,Il_k} := Yb0_{Jl_k,Il_k} - \frac{1}{Line_{k,6}}$$

$$Yb0_{Jt_m,It_m} := Yb0_{Jt_m,It_m} - \left(Ys0_m\right)$$

Add the Ysh arrays to the diagonal of Yb for each sequence.

$$Yb1_{i,i} := Yb1_{i,i} + Ysh1_{i}$$

$$Yb2_{i,i} := Yb2_{i,i} + Ysh2_{i}$$

$$Yb0_{i,i} := Yb0_{i,i} + Ysh0_{i}$$

Find the bus-impedance matrix by inverting the sequential Yb matrices.

$$Z_{bus1} := Yb1^{-1} \qquad Z_{bus2} := Yb2^{-1} \qquad Z_{bus0} := Yb0^{-1}$$

(b) Fault Current

The fault current is calculated using the sequence The venin impedance from the bus-impedance matrix.

$k := 3$ bus location of fault

$Z_f := 0.0$ fault impedance

$VF := 1.05$ pre-fault voltage on bus 3

Define the transformation matrix T with a1 and a2 defined as

$$a1 := e^{2j \cdot \frac{\pi}{3}} \qquad a2 := a1^{2} \qquad T := \begin{bmatrix} 1 & 1 & 1 \\ 1 & a2 & a1 \\ 1 & a1 & a2 \end{bmatrix}$$

For a single-line-to-ground fault, define

$$I := \frac{VF}{Z_{bus1_{k,k}} + Z_{bus2_{k,k}} + Z_{bus0_{k,k}} + 3 \cdot Z_f}$$

In this case, all three currents, the zero sequence fault current, the positive sequence fault current, and the negative sequence fault current, are equal to I. Therefore, the sequence fault current is

$$I_f := \begin{bmatrix} I \\ I \\ I \end{bmatrix}$$

The abc fault current is

$$I_{abc} := T \cdot I_f$$

For the other fault types, the user may activate these equations by first dragging the expressions and all defined variables to the Worksheet window.

Activating Equations

Three phase fault

$I_0 := 0$ zero sequence fault current

$I_1 := \frac{VF}{Z_f + Z_{bus1_{k,k}}}$ positive sequence fault current

$I_2 := 0$ negative sequence fault current

The sequence fault current is

$$I_f := \begin{bmatrix} I_0 \\ I_1 \\ I_2 \end{bmatrix}$$

Double Line

Define

$$I := \frac{VF}{Z_{bus1_{k,k}} + Z_{bus2_{k,k}} + Z_f}$$

$$I_f := \begin{bmatrix} 0 \\ I \\ -I \end{bmatrix}$$

Double-Line-to-Ground

Define

$$Zeq := \frac{Z_{bus0_{k,k}} + 3 \cdot Z_f}{Z_{bus2_{k,k}} + Z_{bus0_{k,k}} + 3 \cdot Z_f}$$

$$I := \frac{VF}{Z_{bus1_{k,k}} + Z_{bus2_{k,k}} \cdot Zeq}$$

The sequence fault current is

$$I_f := \begin{bmatrix} -I \cdot \left(\frac{Z_{bus2_{k,k}}}{Z_{bus2_{k,k}} + Z_{bus0_{k,k}} + 3 \cdot Z_f} \right) \\ I \\ -I \cdot Zeq \end{bmatrix}$$

(c) Bus Voltage

The bus voltage for each sequence is calculated using Equations (2.1.8).

For the case of single-line-to-ground fault, we solve as follows.

Calculate the sequential bus voltages.

$i := 1 .. N\_bus$

$V0_i := -Z_{bus0_{i,k}} \cdot I_{f_1}$ zero sequence

$V1_i := VF - Z_{bus1_{i,k}} \cdot I_{f_2}$ positive sequence

$V2_i := -Z_{bus2_{i,k}} \cdot I_{f_3}$ negative sequence

The positive and negative sequence voltages of the primary buses of Y/D and D/Y transformers must be phase-shifted. Define phase-shift angle from Y to D.

$ph_{YD} := -1j$ for 90 degree

Activating Equations

$ph_{YD} := e^{-1j \cdot \frac{\pi}{6}}$ for 30 degree

$f1_i := 1$ $f2_i := 1$

$m := 1 .. N\_trans$

$f1_{It_m} := \mathbf{if}\left(\left|Trans_{m,9}\right| = 2, ph_{YD}, f1_{It_m}\right)$ for Y/D

$f2_{It_m} := \mathbf{if}\left(\left|Trans_{m,9}\right| = 2, -ph_{YD}, f2_{It_m}\right)$

$f1_{It_m} := \mathbf{if}\left(\left|Trans_{m,9}\right| = 3, -ph_{YD}, f1_{It_m}\right)$ for D/Y

$f2_{It_m} := \mathbf{if}\left(\left|Trans_{m,9}\right| = 3, ph_{YD}, f2_{It_m}\right)$

The abc components of the bus voltages are

$$\begin{bmatrix} V_{a_i} \\ V_{b_i} \\ V_{c_i} \end{bmatrix} := T \cdot \begin{bmatrix} V0_i \\ V1_i \cdot f1_i \\ V2_i \cdot f2_i \end{bmatrix}$$

$$V_{a_i} = \begin{bmatrix} 0.985 \\ 0.985 \\ 1.012 \\ -0.55+0.318i \\ -0.55+0.318i \\ -0.505+0.292i \end{bmatrix} \quad V_{b_i} = \begin{bmatrix} -0.071-0.072i \\ -0.071-0.072i \\ 1.11i \cdot 10^{-16} \\ 0.666+0.517i \\ 0.666+0.517i \\ 0.635+0.517i \end{bmatrix} \quad V_{c_i} = \begin{bmatrix} -0.071+0.072i \\ -0.071+0.072i \\ 1.11i \cdot 10^{-16} \\ -0.115-0.835i \\ -0.115-0.835i \\ -0.13-0.808i \end{bmatrix}$$

(d) Component Currents

The procedure for calculating the current contribution of the components is as follows.

(d1) Transmission lines

For line #2,

$m := 2$ line number

Define sequential impedances.

$X0 := Line_{m,6}$ $\quad X1 := Line_{m,4}$ $\quad X2 := Line_{m,5}$

Calculate sequence current.

$$I0 := \frac{V0_{Il_m} - V0_{Jl_m}}{X0} \quad I1 := \frac{V1_{Il_m} - V1_{Jl_m}}{X1} \quad I2 := \frac{V2_{Il_m} - V2_{Jl_m}}{X2}$$

Calculate abc current.

$$I_{line} := T \cdot \begin{bmatrix} I0 \\ I1 \\ I2 \end{bmatrix} \quad I_{line} = \begin{bmatrix} 0.209i \\ -1.196+0.95i \\ 1.196+0.95i \end{bmatrix}$$

(d2) Generators

For generator #1,

$$m := 1$$

Define the sequence impedance of the generator.

$$X0 := Gen_{m,5} + (3 \cdot Gen)_{m,6}$$

$$X1 := Gen_{m,3} \qquad X2 := Gen_{m,4}$$

Calculate sequence current.

$$I0 := -\frac{V0_{Ig_m}}{X0} \qquad I2 := -f2_{Ig_m} \cdot \frac{V2_{Ig_m}}{X2}$$

$$I1 := -f1_{Ig_m} \cdot \frac{V1_{Ig_m} - (VF)}{X1}$$

Calculate abc current.

$$I_{gen} := T \cdot \begin{bmatrix} I0 \\ I1 \\ I2 \end{bmatrix} \qquad I_{gen} = \begin{bmatrix} 0.691 + 1.196i \\ 0.025 - 0.812i \\ -0.716 - 0.384i \end{bmatrix}$$

(d3) Transformers

The primary side current for transformer #2 is calculated as follows.

$$m := 2$$

Define the positive, negative, and zero sequence impedances.

$$X1 := Trans_{m,4} \qquad X2 := Trans_{m,5}$$

$$X0 := \textbf{if}\left(\left|Yb0_{It_m, Jt_m}\right| = 0, 10^{15}, -\frac{1}{Yb0_{It_m, Jt_m}}\right)$$

The sequence currents are

$$I0 := \frac{V0_{It_m} - V0_{Jt_m}}{X0} \qquad I1 := f1_{It_m} \cdot \frac{V1_{It_m} - V1_{Jt_m}}{X1} \qquad I2 := f2_{It_m} \cdot \frac{V2_{It_m} - V2_{Jt_m}}{X2}$$

The abc currents of the transformer primary are

$$I_{ptrans} := T \cdot \begin{bmatrix} I0 \\ I1 \\ I2 \end{bmatrix} \qquad I_{ptrans} = \begin{bmatrix} 0.691 + 1.196i \\ 0.025 - 0.812i \\ -0.716 - 0.384i \end{bmatrix}$$

The secondary currents can be obtained if the phase-shift is omitted.

$$I0 := \frac{V0_{It_m} - V0_{Jt_m}}{X0} \qquad I1 := \frac{V1_{It_m} - V1_{Jt_m}}{X1} \qquad I2 := \frac{V2_{It_m} - V2_{Jt_m}}{X2}$$

The abc currents of the transformer secondary are

$$I_{strans} := T \cdot \begin{bmatrix} I0 \\ I1 \\ I2 \end{bmatrix} \qquad I_{strans} = \begin{bmatrix} -2.811 \cdot 10^{-16} - 0.494i \\ -1.196 + 0.247i \\ 1.196 + 0.247i \end{bmatrix}$$