Section 2.2 Mid-Line Fault Calculations

Section 2.2.1 Introduction

Section 2.1a outlines a method for computing bus faults on power systems. This type of fault causes the largest magnitude of fault current in a power system because several lines contribute current to the bus. In practice this type of fault rarely occurs. Most faults occur at a point on a line some distance from the substation bus. The current seen by a line protection system during a mid-line fault is typically much less than that for a bus fault, and varies in magnitude depending on the position of the fault relative to the end-point buses. Figure 2.2.1 shows an example system that has a mid-line fault applied at a location r per unit (pu) from breaker **A**.

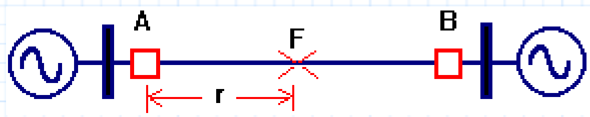

Fig. 2.2.1 Mid-line faults

The breaker relays at points **A** and **B** see varying magnitudes of fault current depending on the location of the fault point **F**. To analyze this topological change using the Zbus method of **Section 2.1b** requires three separate changes in the Zbus matrix. These are the removal of line **AB**, the addition of a new segment **AF**, and the addition of a new segment **FB**. These changes are computationally intensive. Also, a new bus is added to the system, increasing the dimension of Zbus by one.

In [1] Han develops formulae that allow the computation of mid-line faults without resorting to multiple changes in the original Zbus matrix of a system. Given a system Zbus matrix, and the general line segment in Figure 2.2.2,

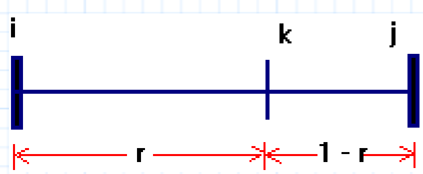

Fig. 2.2.2 Mid-line fault model

the new column and row entries are found from the existing column and rows at points i and j in the original matrix and the line impedance, zij. The mutual impedances of the new bus are

$$Z_{n,k} = (1-r)\cdot Z_{n,i} + r\cdot Z_{n,j} \qquad \textbf{(2.2.1)}$$

The Thevenin's equivalent impedance of the new bus is

$$Z_{k,k} = (1-r)\cdot Z_{i,i}\cdot r^2\cdot Z_{j,j} + 2\cdot r\cdot(1-r)\cdot Z_{i,j} + r\cdot(1-r)\cdot z_{i,j} \qquad \textbf{(2.2.2)}$$

where
Zmk = mutual impedances for the new bus
Zkk = Thevenin's impedance for the new bus
Zjj = Thevenin's impedance for bus j
Zii = Thevenin's impedance for bus i
Zij = mutual impedance between buses i and j
zij = line impedance between buses i and j
n = number of buses in the original system

Equations (2.2.1) and (2.2.2) assume that all impedances are in per unit on a common power base. Substituting the values of Zmk and Zkk into the fault current formulae given in **Section 2.1a** gives the current for any fault location.

Section 2.2.2 Protective Relaying Considerations

The fault currents seen by the breaker current transformers (CTs) determine the settings of the protective relays which control the operation of the breaker. Bus faults result in the maximum fault current a breaker CT will see, while midline faults produce currents of reduced magnitude. To protect the lines, breakers should reliably trip for all faults from their terminal to the remote line terminal, so maximum and minimum CT currents are important to relay coordination.

Ground fault protective devices on looped power systems use directional supervision. Line relays send a trip signal to a breaker only for faults that occur on the line that it protects. In Figure 2.2.1, breaker **A** should only trip for faults that occur between the two buses and not respond to faults that occur on the generator side of the bus. Protective relaying associated with the generator will clear a generator fault by opening a breaker at the plant represented by the single machine in the figure. Comparing the phase angle of the fault current to a fixed reference, a polarizing quantity, is the standard method of controlling the tripping direction of breakers.

Zero sequence voltages and currents polarize ground fault relays in most protection schemes. The polarizing quantity of the relay produces a restraining signal that prevents undesired tripping of the line breaker. The choice to use voltage or current polarization depends on the magnitude of the polarizing quantity for faults near the breaker. The polarizing quantity should be large enough to restrain the ground relay for a zero impedance fault at the breaker's terminals. Solving Equations 2.2.1 and 2.2.2 for a number of fault locations produces a zero sequence current and voltage profile for a line segment.

Using the impedances computed from Equations 2.2.1 and 2.2.2 along with the fault current formulae from **Section 2.1a** allows the computation of fault current magnitude seen by the terminal breakers for any fault location. This method finds the minimum and maximum fault current values for any line segment in a system.

Section 2.2.3 Mathcad Formulation

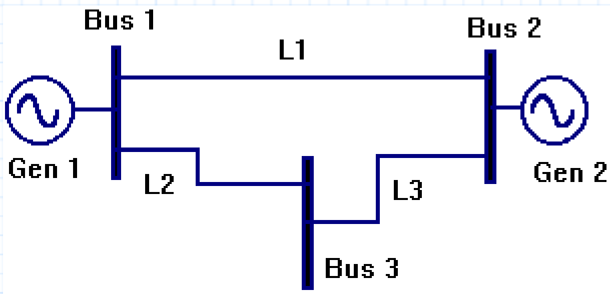

Fig. 2.2.3 Example system

What follows is a demonstration of computing mid-line faults on power systems. Computing the value of a mid-line fault determines the fault current that a circuit breaker's protective relaying sees when a fault occurs on the line that it protects. Knowledge of the fault current contributions from both ends of the line allows the interrupting capacity of the breakers to be determined. The fault current contributions from each end of the line are also used to coordinate the protective relays that control the line breaker tripping.

This example shows a fault current and fault voltage profile for a line segment. The profiles determine the magnitude of the polarizing current and voltage available to restrain the operation of ground-fault protective relays. The data in the test system in Figure 2.2.3 illustrate this technique. The example is small, but analysis of any size system that does not exceed the storage limits of Mathcad is possible. Use the Zbus computation in Section 2.1 to compute the system impedance matrix and import the Zbus matrix into this section using either a Reference, or Input and Output Tables. To access the **Reference...** option in the **Insert** menu, this file must be resaved under another name and opened in the Mathcad worksheet. Inserting a Reference makes the variables in the referenced document available in the current document. Input and Output Tables are another way to transfer data between files. To insert Input and Output Tables both files must be resaved under other names and opened in the Mathcad worksheet. Input and Output Tables can then be accessed through **Insert/Component...**.

All impedance and voltage information in this example is given in **per unit** on a 100 MVA base. The prefault voltage is 1.0 per unit for all system buses.

Data Structure

The array called Lines holds the line data for the system in Figure 2.2.3.

$$Lines := \begin{bmatrix} 1 & 2 & 0.04j & 0.04j & 0.12j \\ 2 & 3 & 0.06j & 0.06j & 0.18j \\ 3 & 1 & 0.03j & 0.03j & 0.09j \end{bmatrix}$$

The columns of the Lines array are defined as
1. from bus,
2. to bus,
3. positive sequence Z,
4. negative sequence Z,
5. zero sequence Z.

Generator Impedance Data

The generator subtransient reactance is the same for both generators 1 and 2 for both positive and negative sequence.

$X_{d1} := 0.15j$ positive sequence

$X_{d2} := X_{d1}$ negative sequence

$X_{d0} := 0.30j$ zero sequence

Using the method in **Section 2.1b** to find the Zbus matrices for the system in Figure 2.2.3 yields

$$Z_{bus1} := \begin{bmatrix} 0.081j & 0.069j & 0.077j \\ 0.069j & 0.081j & 0.073j \\ 0.077j & 0.073j & 0.096j \end{bmatrix}$$ positive sequence

For this case, the positive sequence impedances are equal to the negative sequence so

$Z_{bus2} := Z_{bus1}$

The matrix Zbus0 is the zero sequence impedance matrix.

$$Z_{bus0} := \begin{bmatrix} 0.168j & 0.132j & 0.156j \\ 0.132j & 0.168j & 0.144j \\ 0.156j & 0.144j & 0.212j \end{bmatrix}$$ Zero sequence

Reset the origin of all arrays to match the bus numbering.

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Thevenin's Impedances

Define the Thevenin's impedance of the mid-line fault point as a function of the per unit distance, r, from bus, i, the line terminal buses, i and j, and the line impedance.

For positive sequence,

$$Z_{k1}(r, i, j, Z_L) := (1-r)^2 \cdot Z_{bus1_{i,i}} + r^2 \cdot Z_{bus1_{j,j}} + 2 \cdot r \cdot (1-r) \cdot Z_{bus1_{i,j}} + r \cdot (1-r) \cdot Z_L \quad \textbf{(2.2.3)}$$

For negative sequence,

$$Z_{k2}(r, i, j, Z_L) := (1-r)^2 \cdot Z_{bus2_{i,i}} + r^2 \cdot Z_{bus2_{j,j}} + 2 \cdot r \cdot (1-r) \cdot Z_{bus2_{i,j}} + r \cdot (1-r) \cdot Z_L \quad \textbf{(2.2.4)}$$

For zero sequence,

$$Z_{k0}(r, i, j, Z_L) := (1-r)^2 \cdot Z_{bus0_{i,i}} + r^2 \cdot Z_{bus0_{j,j}} + 2 \cdot r \cdot (1-r) \cdot Z_{bus0_{i,j}} + r \cdot (1-r) \cdot Z_L \quad \textbf{(2.2.5)}$$

Mutual Impedances

Define the mutual impedances as a function of the pu distance from bus, i, the terminal buses of the line segment, i,j, and the bus of interest in the system, n.

For positive sequence,

$$Z_{mk1}(r,i,j,n) := (1-r)\cdot Z_{bus1_{n,i}} + r\cdot Z_{bus1_{n,j}} \quad \textbf{(2.2.6)}$$

For negative sequence,

$$Z_{mk2}(r,i,j,n) := (1-r)\cdot Z_{bus2_{n,i}} + r\cdot Z_{bus2_{n,j}} \quad \textbf{(2.2.7)}$$

For zero sequence,

$$Z_{mk0}(r,i,j,n) := (1-r)\cdot Z_{bus0_{n,i}} + r\cdot Z_{bus0_{n,j}} \quad \textbf{(2.2.8)}$$

The Znk values indicate the degree of coupling between the faulted bus and any other bus of interest. Postfault voltage computation is possible for any bus in the system with knowledge of the fault current and the values of Znk.

Equations (2.2.9) and (2.2.10) define the three-phase and single-line-to-ground faults as functions of the distance r.

For three-phase faults,

$$I_{3\phi}(r,i,j,Z_L) := \frac{1}{Z_{k1}(r,i,j,Z_L)} \quad \textbf{(2.2.9)}$$

For single-line-to-ground faults,

$$I_{slg}(r,i,j,Z_{L1},Z_{L2},Z_{L0}) := \frac{3}{Z_{k1}(r,i,j,Z_{L1}) + Z_{k2}(r,i,j,Z_{L2}) + Z_{k0}(r,i,j,Z_{L0})} \quad \textbf{(2.2.10)}$$

Zero Sequence Currents

The polarizing current seen by the ground fault protective relays is the zero sequence current. This current is defined as

$$I_0(r,i,j,Z_{L1},Z_{L2},Z_{L0}) := \frac{I_{slg}(r,i,j,Z_{L1},Z_{L2},Z_{L0})}{3} \quad \textbf{(2.2.11)}$$

The following example shows the relationship between line fault location and fault current magnitude.

Plot the three-phase fault current profile for the line 1-2. Let r vary over the range

$$r := 0.05, 0.1 .. 0.95$$

For Line 1-2,

$z_{L1} := 0.04j$ positive sequence line Z $\qquad z_{L2} := 0.04j$ negative sequence line Z

$z_{L0} := 0.12j$ zero sequence line Z

$i := 1$ from bus $\qquad j := 2$ to bus

Use Equation (2.2.9) to plot the three-phase fault current magnitude over the range of r for line 1-2.

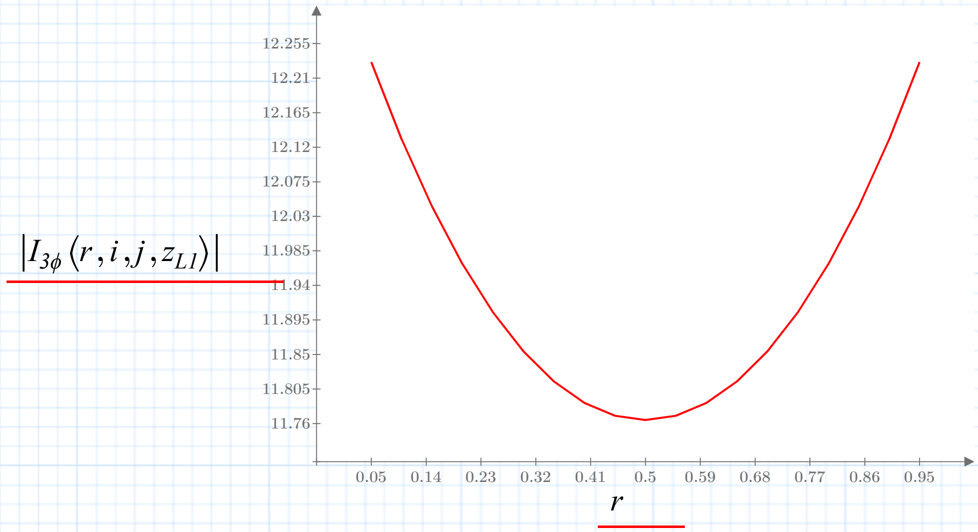

Fig. 2.2.4 Three-phase fault current magnitude for sliding fault position

The fault current magnitude is minimum for line faults located at the center of the line. The current magnitude increases as the fault location approaches the terminal buses.

Three-Phase Fault Contribution from Bus i

Equation (2.2.12) defines the current that the breaker on bus i will see for a three-phase fault a distance r pu from the bus.

$$I_{ik}(r,i,j,n,z_{L1}) := \frac{Z_{k1}(r,i,j,z_{L1}) - Z_{mk1}(r,i,j,n)}{Z_{k1}(r,i,j,z_{L1}) \cdot r \cdot z_{L1}} \qquad \textbf{(2.2.12)}$$

Use Equation (2.2.12) to plot the three-phase fault current contribution from Bus 1 over the range of r for line 1-2.

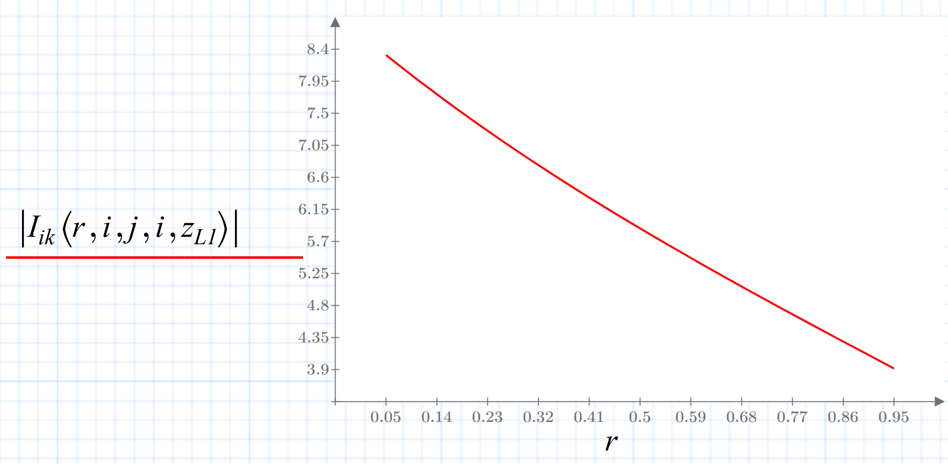

Fig. 2.2.5 Three-phase fault current contribution from Bus 1

The current contribution from bus 1 increases as the fault location approaches bus 1. As the distance from the fault to bus 1 increases, the current contribution from the bus 1 terminal decreases.

Use Equation (2.2.10) to compute the single-line-to-ground fault current magnitude as a function of the fault distance from bus 1.

$$\left|I_{slg}\left(r, i, j, z_{L1}, z_{L2}, z_{L0}\right)\right|$$

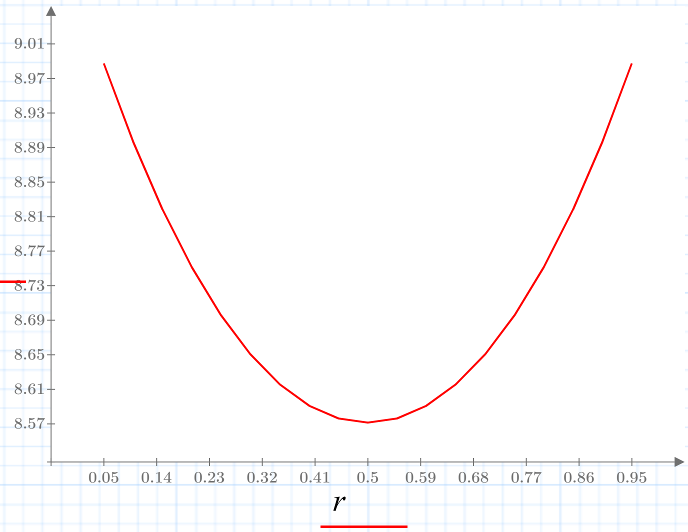

Fig. 2.2.6 Single-line-to-ground fault current magnitude for line 1-2

Ground Fault Current at Terminal i

Computing the single-line-to-ground fault current that breaker 1 sees on line 1-2 requires the use of all three sequence mutual impedances and Thevenin's impedances.

Create a function to sum the mutual impedances for all sequences.

$$Z_m(r, i, j, n) := Z_{mk1}(r, i, j, n) + Z_{mk2}(r, i, j, n) + Z_{mk0}(r, i, j, n) \quad \textbf{(2.2.13)}$$

Create a function to sum the Thevenin's impedances of all sequences.

$$Z_{th}\left(r, i, j, Z_{L1}, Z_{L2}, Z_{L0}\right) := Z_{k1}\left(r, i, j, Z_{L1}\right) + Z_{k2}\left(r, i, j, Z_{L2}\right) + Z_{k0}\left(r, i, j, Z_{L0}\right) \quad \textbf{(2.2.14)}$$

Compute the current through breaker 1 CTs with the formula below.

$$I_{sik}\left(r, i, j, Z_{L1}, Z_{L2}, Z_{L0}\right) := \left(\frac{1 - \dfrac{Z_m(r, i, j, i)}{Z_{th}\left(r, i, j, Z_{L1}, Z_{L2}, Z_{L0}\right)}}{Z_{L1} \cdot r}\right) \quad \textbf{(2.2.15)}$$

This relationship will compute the breaker currents for any distance down the line.

Use Equation (2.2.15) to compute the single-line-to-ground fault current magnitude for the range of r.

Fig. 2.2.7 Single-line-to-ground current contribution from Bus 1

Bus Fault Voltage (Single-Line-to-Ground Fault)

Compute the bus voltage that the breaker at bus i sees as a function of the fault location. Equation (2.2.16) defines the relationship between bus voltage a bus i and the single-line-to-ground fault position, r.

$$V_b(r,i,j,n,Z_{L1},Z_{L2},Z_{L0}) := \left(1 - \frac{Z_m(r,i,j,n)}{Z_{th}(r,i,j,Z_{L1},Z_{L2},Z_{L0})}\right) \qquad \textbf{(2.2.16)}$$

where
r = per unit fault location
i = from bus
j = to bus
n = bus of interest
ZL1 = positive sequence line Z
ZL2 = negative sequence line Z
ZL0 = zero sequence line Z

To plot the magnitude of bus voltage against the fault location, define the bus voltage magnitude as a function of the per unit fault distance, r.

$$V_{bb}(r) := \left| V_b\left(r, i, j, i, z_{L1}, z_{L2}, z_{L0}\right) \right| \qquad \textbf{(2.2.17)}$$

Use Equation (2.2.17) to create a plot of the bus voltage at bus 1 as a function of the fault location.

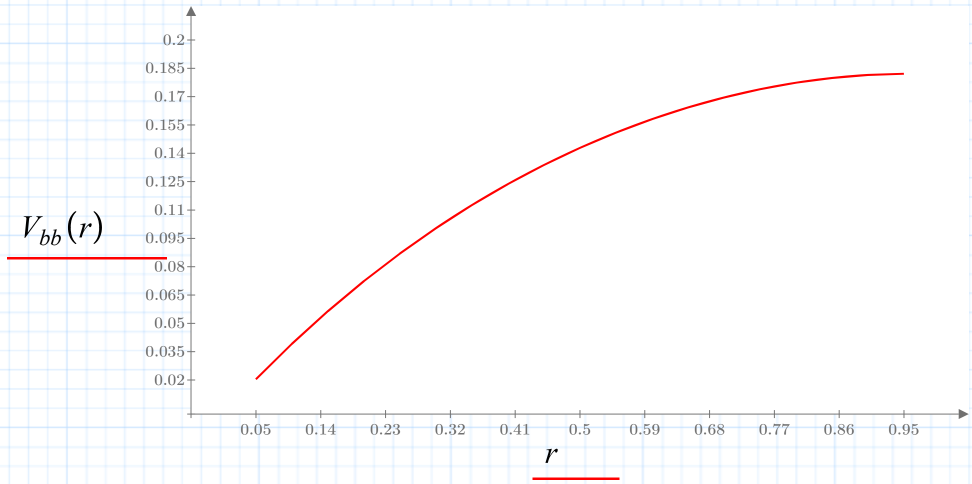

Fig. 2.2.8 Voltage at Bus 1 for a single-line-to- ground fault

The previous example shows how the total fault current magnitude of both three-phase and single-line-to-ground faults decreases as the fault location moves toward the center of a line segment. The fault current contributions seen by terminal breakers are also a function of the fault location. The largest line current is produced by a fault that is just beyond the bus that the breaker protects. The minimum current occurs when the fault is just outside the remote terminal bus. Bus fault voltages on a bus increase as the location of the fault moves away from the bus of interest.

Section 2.2.4 References

1. Han, Z. X., "Generalized Method of Analysis of Simultaneous Faults in Electric Power Systems," *IEEE Transactions on Power Apparatus and Systems*, Vol. PAS-101, No. 10, October 1982.