Section 2.3b Out-Of-Step Protection

Section 2.3.4 Out-of-Step Protection - Application

All impedance values are in per unit on 100 MVA base. All angles are in degrees.

System Data

$Z_{gs} := 0.05$ $\theta 1_1 := 90\ \mathbf{deg}$	impedance and impedance angle of generator at bus **S**
$Z_{sr} := 1.3$ $\theta 2_2 := 75\ \mathbf{deg}$	**Line impedance** and impedance angle
$Z_{rh} := 0.05$ $\theta 3_3 := 90\ \mathbf{deg}$	Impedance and impedance angle of generator at bus **R**
$M_r := 4$	angular momentum of generator at bus **R**
$M_s := 5$	angular momentum of generator at bus **S**
$P_{ss} := 0.61$	transmitted steady state power
$n := 1.0$	ratio of VG over VH

Relay Settings

$T := 0.9$	relay reach in percent of **SR**
$\theta 4_4 := 75\ \mathbf{deg}$	relay maximum torque angle

Fault Characteristic

$t_c := 0.06$	fault clearing time

Method of Solution

The procedure to verify the coordination of the protection scheme at point **b** is as follows:

From the system data, a graph is obtained on the R-X plane of the relay characteristic and the system impedance. From the system impedance and source voltage, the swing ohms trajectory is obtained and plotted on the R-X plain along with the relay characteristic. This curve indicates the ohm value seen by the relay as a result of varying line conditions. It is independent of the system loading conditions (i.e. prefault power transfer and transmission angle). The intersections of this trajectory with the relay characteristic determine the values of the transmission angle for which swing ohms are within that characteristic. These are the entry and exit angles. A stable swing generally will remain in the relay characteristic for some time and then exit from its entry angle. On the other hand, an unstable swing will traverse the relay characteristic through its entry and exit angles.

The time the swing ohms remain in the relay characteristic are found by solving the system dynamic equation and obtaining the transmission angle response. The values of the transmission angle are compared against the entry and exit angle of the swing and the above time is determined. This procedure can help in selecting time delays for relay operation.

Equation (2.3.1) is a polar-to-rectangular conversion function that is used throughout the example.

$$r(\theta) := \cos(\theta) + 1j \cdot \sin(\theta) \qquad \textbf{(2.3.1)}$$

Equations (2.3.2) to (2.3.4) allow the system impedance to be graphed on an impedance diagram along with the impedance-relay characteristic.

$$\theta 1_1 := \textbf{if}\left(\left(\theta 1_1 = 90\ deg\right), 88\ deg, \theta 1_1\right) \qquad \textbf{(2.3.2)}$$

$$\theta 2_2 := \textbf{if}\left(\left(\theta 2_2 = 90\ deg\right), 88\ deg, \theta 2_2\right) \qquad \textbf{(2.3.3)}$$

$$\theta 3_3 := \textbf{if}\left(\left(\theta 3_3 = 90\ deg\right), 88\ deg, \theta 3_3\right) \qquad \textbf{(2.3.4)}$$

The following relationships convert the polar impedance values to rectangular form.

$$z_{gs} := Z_{gs} \cdot r\left(\theta 1_1\right) \qquad z_{sr} := Z_{sr} \cdot r\left(\theta 2_2\right)$$

$$z_{rh} := Z_{rh} \cdot r\left(\theta 3_3\right) \qquad z_{relay} := T \cdot Z_{sr} \cdot r\left(\theta 4_4\right)$$

Combine the generator and line impedances to get the total impedance.

$$Z_T := z_{sr} + z_{rh} + z_{gs}$$

Define the R-X relay characteristic.

$Z_{r0} := \frac{z_{relay}}{2}$ center

$i := 0, 2 .. 360$ index for system impedance plots

Offset the characteristic to pass through zero.

$Z_{r_i} := |Z_{r0}| \cdot r(i \cdot \mathbf{deg}) + Z_{r0}$

Define the system impedance as a piece-wise linear function so it can be shown on the R-X diagram.

$F_1(x) := \tan(\theta 1_1) \cdot x$ **(2.3.5)**

$F_2(x) := \tan(\theta 2_2) \cdot x$ **(2.3.6)**

$F_3(x) := \tan(\theta 3_3) \cdot (x - \mathrm{Re}(z_{sr})) + \mathrm{Im}(z_{sr})$ **(2.3.7)**

$c_1 := 0$ Used by following **if** statements to produce zero traces of system impedance on graphs.

Substitute Equations (2.3.5)-(2.3.7) to create the following piece-wise linear functions.

$G_1(x) := \mathbf{if}((x < -\mathrm{Re}(z_{gs})), c_1, F_1(x))$ **(2.3.8)**

$G_2(x) := \mathbf{if}((x < 0), G_1(x), F_2(x))$ **(2.3.9)**

$G_3(x) := \mathbf{if}((x < \mathrm{Re}(z_{sr})), G_2(x), F_3(x))$ **(2.3.10)**

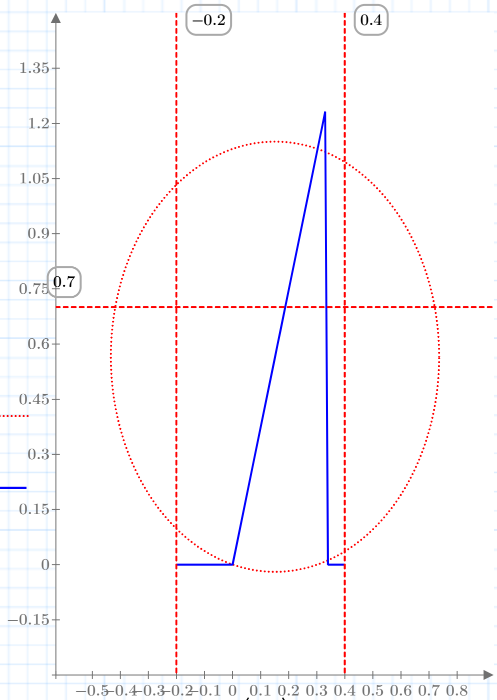
Create the line impedance function.

$Z_{line}(x) := \mathbf{if}((x < \mathrm{Re}(z_{sr} + z_{rh})), G_3(x), c_1)$ **(2.3.11)**

$x := -0.2, -0.19 .. 0.4$ range to plot

Line Impedance and Relay Characteristic

The figure below shows the relay characteristic with the line impedance superimposed. The maximum torque angle of the relay is assumed to be the same as the line impedance angle.



$\mathrm{Im}\left(Z_{r_i}\right)$

$Z_{line}(x)$

$\mathrm{Re}\left(Z_{r_i}\right)$

x

Fig. 2.3.2 Relay and system impedance plots

Determine the Swing Ohm Trajectory

The swing ohm trajectory appears as a circle on an R-X diagram. The radius for this circle is given by

$$n := \textbf{if}\,(n = 1, 0.999, n)$$

$$R := \frac{Z_T \cdot n}{n^2 - 1} \qquad \textbf{(2.3.12)}$$

The **if** statement removes the singularity for a circle of infinite radius.

Define the trajectory of the swing ohms in rectangular form.

$$c_{swing} := Z_T + \frac{Z_T}{n^2 - 1} - z_{gs}$$ center of the swing impedance circle

$$r_{swing} := n \cdot \frac{|Z_T|}{|n^2 - 1|}$$ radius of the swing impedance circle

$$Y_{swing}(s) := \left(\sqrt{r_{swing}{}^2 - \mathrm{Re}(s - c_{swing})^2}\right) + \mathrm{Im}(c_{swing})$$

$x := -0.5, -.45 .. 0.8$ plot range

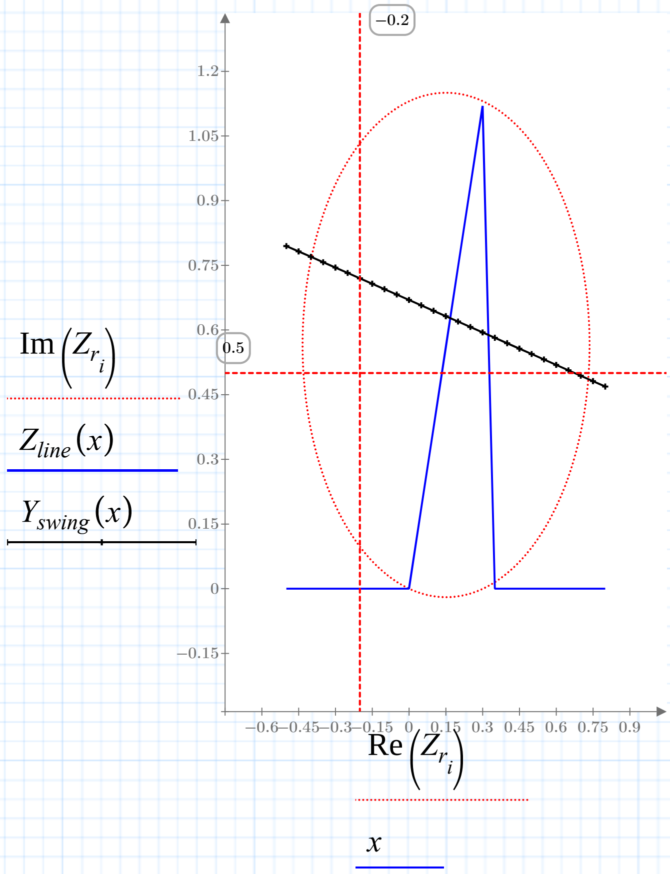


Fig 2.3.3 Plot of line Z, relay characteristic, and system swing ohms

Determination of the Entry and Exit Angles

System power swings will follow the trajectory of the swing ohms plot. The intersections of this trajectory with the relay characteristic provide the relative rotor angles at the entry and exit of the swing through the relay characteristic. The intersection points, (xo, yo), are found using the following solve block:

$$\left(x_o - Re\left(c_{swing}\right)\right)^2 + \left(y_o - Im\left(c_{swing}\right)\right)^2 = r_{swing}^{\;2}$$

$$\left(x_o - Re\left(Z_{r0}\right)\right)^2 + \left(y_o - Im\left(Z_{r0}\right)\right)^2 = \left(\left|Z_{r0}\right|\right)^2$$

Define a function **Int**(xo, yo) of the intersection point coordinates, where xo and yo are the initial guesses of these points for the solve block.

$$Intsct\left(x_o, y_o\right) := \mathbf{Find}\left(x_o, y_o\right)$$

Find the two intersection points by giving different initial guesses to **Int**.

$P_1 := Intsct(0.4, 0.2)$ intersection at entry of swing trajectory

$P_2 := Intsct(-0.2, 0.1)$ intersection at exit of swing trajectory

$$P_1 = \begin{bmatrix} 0.731 \\ 0.486 \end{bmatrix} \qquad P_2 = \begin{bmatrix} -0.397 \\ 0.769 \end{bmatrix}$$

Calculate the corresponding rotor angle at **P1** and **P2**.

$O(P) := P_0 + 1j \cdot P_1$ polar impedance at intersections

$H := z_{sr} + z_{rh}$ polar impedance between points **H** and **R**

$G := -z_{gs}$ polar impedance between points **S** and **G**

Compute the value of power angle where the swing trajectory intersects the characteristics using the law of cosines:

$$\delta_{int}(P) := \operatorname{acos}\left(\frac{\left(\left|O(P)-G\right|\right)^2 + \left(\left|O(P)-H\right|\right)^2 - \left(\left|G-H\right|\right)^2}{2 \cdot \left(\left(\left|O(P)-G\right|\right) \cdot \left|O(P)-H\right|\right)}\right)$$

$\delta_1 := \delta_{int}\left(P_1\right)$ entry angle

$\delta_2 := 360\ \mathbf{deg} - \delta_{int}\left(P_2\right)$ exit angle

$$\delta_1 = 100.577\ \mathbf{deg} \qquad \delta_2 = 259.626\ \mathbf{deg}$$

Computation of the Transient Response

After the swing entry and exit angles through the relay characteristic are determined, a transient simulation of the swing is computed. This simulation determines the time the swing ohms stay within the relay characteristic.

The steady-state power transfer determines the initial power angle of the system before the disturbance occurs.

$$\delta o_o := \mathrm{asin}\left(\frac{P_{ss} \cdot |Z_T|}{n}\right) \qquad \text{initial angle}$$

$$\frac{\delta o_o}{c} = \left(3.411 \cdot 10^{-9}\right) \frac{s}{m}$$

The equivalent angular momentum is

$$M := \frac{M_r \cdot M_s}{M_r + M_s}$$

$$M = 2.222$$

A simple numerical integration is used to solve the differential equation describing the system.

Define the integration index.

$$i := 1 .. 60$$

Define the time step of the integration.

$$dt := 0.01$$

Define the initial conditions.

$$\omega_0 := 0.0 \qquad \omega_1 := 0.0 \qquad \delta_0 := \delta o_o$$

$$\delta_1 := \delta o_o \qquad t_0 := 0.0 \qquad t_1 := 0.0$$

$$t_i := i \cdot dt \qquad \text{increment time by the integration time interval}$$

These are the coefficients of the Adams-Bashford two-step numerical integration formula. This formula will produce the solution to the system.

$$h_1 := 1.5 \cdot dt \qquad h_2 := 0.5 \cdot dt$$

Define a function for power with respect to power angle and clearing time. Assuming a three-phase fault, the line power is zero during the fault. Therefore,

$$P(\delta, t) := \mathbf{if}\left(\left(t < t_c\right), 0, \left(n \cdot \frac{\sin(\delta)}{|Z_T|}\right)\right)$$

Solution of System State-Equations

Define the necessary parameters.

$D\delta(\omega) := 377 \cdot \omega$ derivative of the rotor angle

$D\omega(\delta, t) := \dfrac{P_{ss} - P(\delta, t)}{M}$ derivative of the rotor speed

Integration formula for finding the change in frequency and power angle from the previous two derivatives:

$$\Delta\omega(\delta1, \delta2, t1, t2) := h_1 \cdot D\omega(\delta1, t1) - h_2 \cdot D\omega(\delta2, t2)$$

$$\Delta\delta(\omega1, \omega2) := h_1 \cdot D\delta(\omega1) - h_2 \cdot D\delta(\omega2)$$

Vectorize and solve.

$$\begin{bmatrix} \omega_{i+1} \\ \delta_{i+1} \end{bmatrix} := \begin{bmatrix} \omega_i + \Delta\omega\left(\delta_i, \delta_{i-1}, t_i, t_{i-1}\right) \\ \delta_i + \Delta\delta\left(\omega_i, \omega_{i-1}\right) \end{bmatrix}$$

Plot the dynamic response of the system to determine the system stability and the time interval inside the relay trip region. This interval is determined by the time the relative rotor angle is between the entry and exit angles calculated previously.

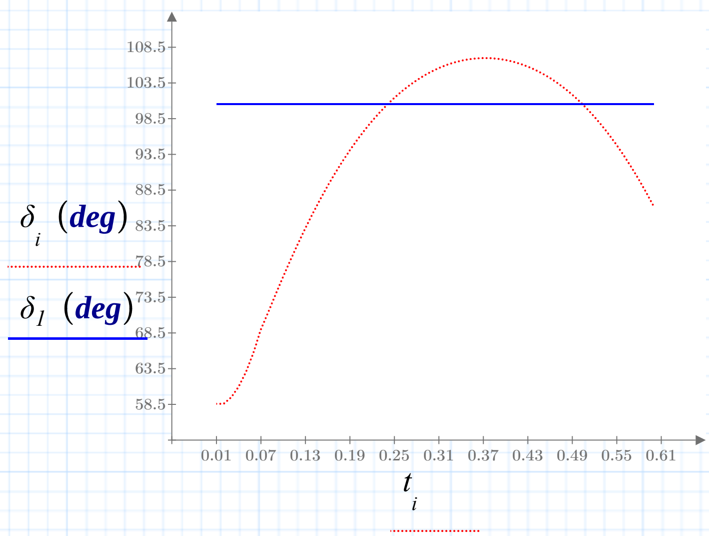


Fig. 2.3.5 Dynamic response of the generator rotor angle

For the given system operating conditions and fault clearing time, the system generators remain stable. If the damping effects are included in the computations, the oscillations would decay to zero. The power swing causes the swing ohms to enter the relay characteristic and remain there for approximately 0.25 seconds. A blocking relay should be added to this system to improve the tripping selectivity of the protection scheme. The above time is greatly affected by the system inertia.

Unstable System Operation

Different loading conditions may produce unstable operation. If the steady state power is increased as

$$P_{ss} := 0.7$$

then the system transient can be calculated again.

$$\delta o_o := \operatorname{asin}\left(\frac{P_{ss} \cdot |Z_T|}{n}\right) \qquad \text{initial angle}$$

$$\frac{\delta o_o}{c} = \left(4.56 \cdot 10^{-9}\right) \frac{s}{m}$$

Define the initial conditions.

$$\omega_0 := 0.0 \qquad \omega_1 := 0.0 \qquad \delta_0 := \delta o_o$$

$$\delta_1 := \delta o_o \qquad t_0 := 0.0 \qquad t_1 := 0.0$$

$$t_i := i \cdot dt \qquad \text{increment time by the integration time interval}$$

Vectorize and solve.

$$\begin{bmatrix} \omega_{i+1} \\ \delta_{i+1} \end{bmatrix} := \begin{bmatrix} \omega_i + \Delta\omega\left(\delta_i, \delta_{i-1}, t_i, t_{i-1}\right) \\ \delta_i + \Delta\delta\left(\omega_i, \omega_{i-1}\right) \end{bmatrix}$$

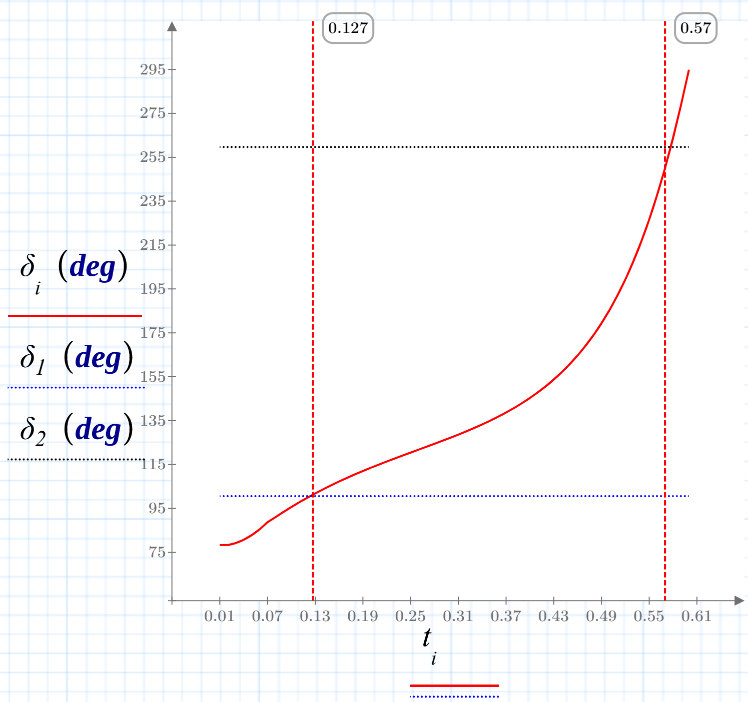


Fig. 2.3.6 Unstable system response

For this case, the unstable swing enters the relay characteristic at t = 0.13 sec and exits at t = 0.57 sec. Therefore, for the worst case of swing, the swing ohms remain in the relay characteristic for 0.44 sec.