CHAPTER 1 STEADY-STATE HEAT CONDUCTION

1.1 Heat Conduction in Multilayered Walls

The following example demonstrates calculation of the thermal resistance and temperature distribution within a wall assuming one-dimensional steady-state heat transfer. Note that in some cases different parts of the wall may have different layers, such as wood studs providing structural support. To determine a correct wall R-value in such cases, we need to calculate the correct value through each heat flow path and determine the overall R-value based on the relative area of each path.

Basic Equations

Heat flow q through a wall layer of thickness L, surface area A and thermal conductivity k is given by

$$q = \frac{T_{hot} - T_{cold}}{R} \qquad \text{with} \qquad R = \frac{L}{k \cdot A}$$

Heat flow q by convection or radiation described in approximate terms by a heat transfer coefficient h (convective, radiative, or combined):

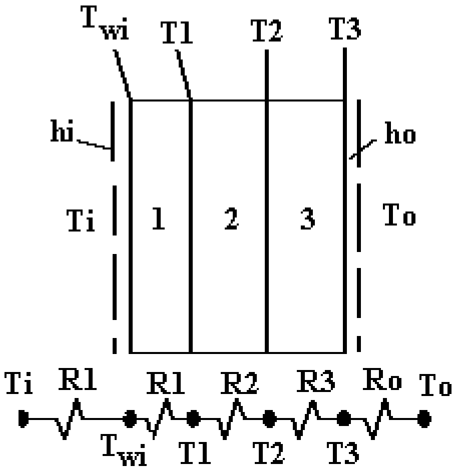
$$q = \frac{T_{hot} - T_{cold}}{R} \qquad \text{with} \qquad R = \frac{1}{h \cdot A}$$

Thermal resistance R_a through a path a is given by the sum of the resistances of the interior film, each wall layer and the exterior film. If a different path has thermal resistance Rb then the total wall resistance based on parallel heat flow paths is given by

$$R = \frac{A}{\frac{A_a}{R_a} + \frac{A_b}{R_b}} \qquad \text{where} \qquad A = A_a + A_b$$

In the example below, consider a wall with three layers:

1. Gypsum board
2. Insulation
3. Brick



Wall with three layers and interior and exterior films

Thermal network also shown

Input parameters:

$hi := 9 \frac{W}{m^2 \cdot \Delta°C}$ interior heat transfer (film) coefficient

$ho := 20 \frac{W}{m^2 \cdot \Delta°C}$ exterior heat transfer coefficient

$A := 1.0\ m^2$ heat transfer area (surface)

$L_1 := 0.013\ m$ L_i = thickness of layer i

$k_1 := 0.16 \frac{W}{m \cdot \Delta°C}$ k_i = thermal conductivity of layer i

$L_2 := 0.05\ m$

$k_2 := 0.025 \frac{W}{m \cdot \Delta°C}$

$L_3 := 0.10\ m$

$k_3 := 1.5\ \frac{W}{m \cdot \Delta°C}$

$N := 3$ N = number of layers (i denotes layer)

$i := 1..N$

$RI := \frac{1}{hi \cdot A}$ RI = interior film resistance

$RO := \frac{1}{ho \cdot A}$ RO = exterior film resistance

$R_i := \frac{L_i}{k_i \cdot A}$ R_i = resistance of wall layer i

$Rtot := RI + \sum_i R_i + RO$

$Rtot = 2.309\ \frac{\Delta°C}{W}$ Rtot = total resistance of wall

Calculation of heat flow Q from inside (temperature TI) to outside (temperature TO):

$TO := -20\ \Delta°C$ $TI := 20\ \Delta°C$

$m := 2..N$ $Q := \frac{TI - TO}{Rtot}$

$Twi := TI - Q \cdot RI$ Twi = wall room side surface temperature

$T_1 := Twi - Q \cdot R_1$ $T_m := T_{m-1} - Q \cdot R_m$

$Q = 17.323\ W$ $Twi = 18.075\ \Delta°C$

$i = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ $T_i = \begin{bmatrix} 16.668 \\ -17.979 \\ -19.134 \end{bmatrix} \Delta°C$