CHAPTER 1 STEADY-STATE HEAT CONDUCTION

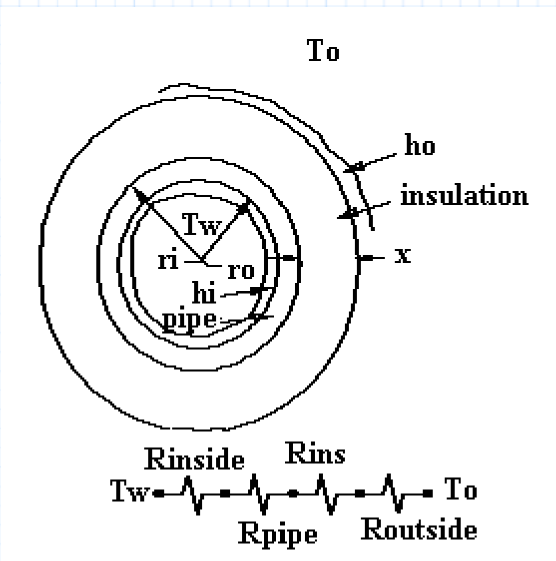
1.2 Heat Conduction Through Insulated Pipes

As in multilayered walls, conduction is assumed to be one-dimensional. The thermal resistance of an uninsulated cylinder of inner radius r_i and outer radius r_o is given by

$$R = \frac{ln\left(\frac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L}$$

For a pipe with one layer of insulation (conductivity k_{ins}), the thermal resistance of the insulation is calculated in a similar manner, but with r_i and r_o being the inner and outer radii of the insulation. The total thermal resistance of the insulated pipe is thus calculated by adding the thermal resistance of the interior and exterior films (surface resistances) to the thermal resistance of the pipe and the insulation.

Example: The insulated copper pipe shown below carries hot water at 80 degC from a solar collector to a hot water storage tank. Insulation of thermal conductivity 0.03 watt/m·degC is employed to reduce heat loss. Determine the heat loss per meter of length for thickness of insulation in the range 0.01 m - 0.06 m. What is the surface temperature in each case?



$L := 1\ m$

$T_w := 80\ \Delta°C$ $\quad$ $T_o := 10\ \Delta°C$

$r_i := 0.025\ m$ $\quad$ $r_o := 0.026\ m$

$k := 386\ \dfrac{W}{m \cdot \Delta°C}$ thermal conductivity of pipe and insulation

$k_{ins} := 0.03\ \dfrac{W}{m \cdot \Delta°C}$

$x := 0.01\ m, 0.02\ m .. 0.06\ m$

$h_i := 300\ \dfrac{W}{m^2 \cdot \Delta°C}$ interior heat transfer coefficient

$h_o := 14\ \dfrac{W}{m^2 \cdot \Delta°C}$ exterior heat transfer coefficient

$R_{inside} := \dfrac{1}{2 \cdot \pi \cdot r_i \cdot h_i \cdot L}$ interior film resistance

$R_{outside}(x) := \dfrac{1}{2 \cdot \pi \cdot (r_o + x) \cdot L \cdot h_o}$ exterior film resistance

$R_{ins}(x) := \dfrac{\ln\left(\dfrac{r_o + x}{r_o}\right)}{2 \cdot \pi \cdot k_{ins} \cdot L}$ insulation thermal resistance as a function of insulation thickness x

$R_{pipe} := \dfrac{\ln\left(\dfrac{r_o}{r_i}\right)}{2 \cdot \pi \cdot k \cdot L}$ pipe thermal resistance (usually very small compared to the other three resistances)

$R_{tot}(x) := R_{inside} + R_{ins}(x) + R_{pipe} + R_{outside}(x)$

$q(x) := \dfrac{T_w - T_o}{R_{tot}(x)}$ heat loss from pipe

$x = ?\ m$ $R_{tot}(x) = ?\ \dfrac{\Delta°C}{W}$ $q(x) = ?\ W$ variation of thermal resistance with insulation thickness x

$$T_{surface}(x) := T_o + q(x) \cdot R_{outside}(x)$$

$x = ?\ m$ $\qquad$ $T_{surface}(x) = ?\ \Delta°C$