CHAPTER 2 TRANSIENT HEAT CONDUCTION

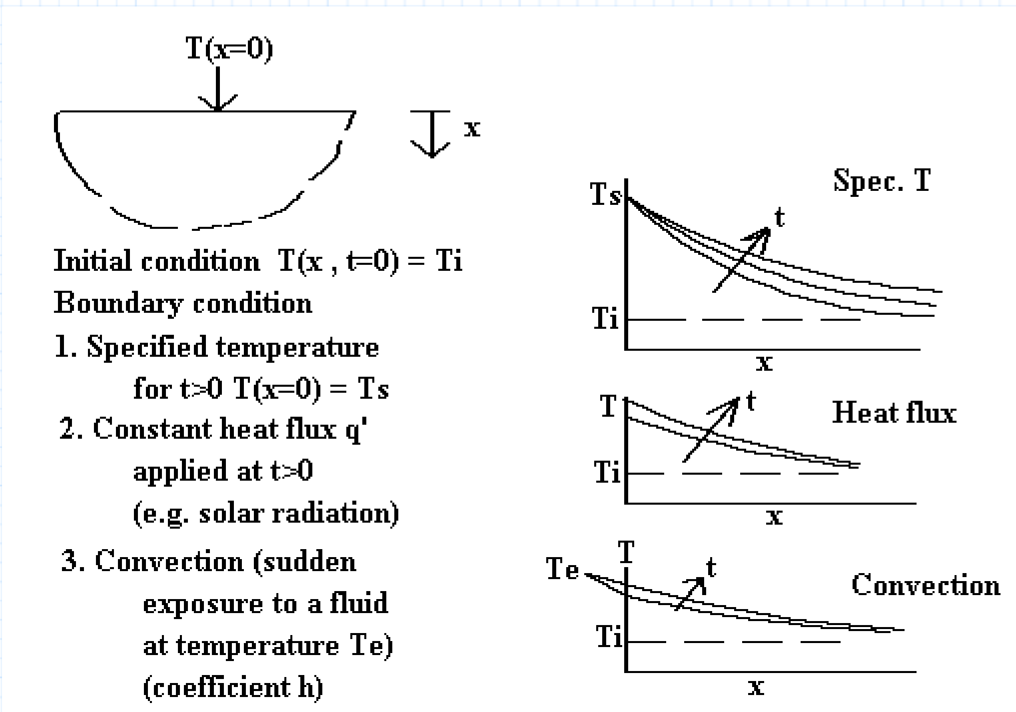
2.2 Transient Conduction in Semi-Infinite Slab

A semi-infinite slab is a model for a body with a single plane surface (x = 0) and its other surfaces far enough to ignore for time periods of interest in transient analysis. If a uniform boundary condition is applied at x = 0, it is reasonable to assume that this case can be analyzed as transient one-dimensional conduction. One case that closely fits this model is the ground with a uniform surface or air temperature; if we measure the soil temperature deep into the ground away from the surface, one would expect that the temperature is not significantly affected by what is happening at the surface.

Assuming no heat generation, the governing equation for T(x,t) is

$$\partial T/dt \ = \ \alpha \ \partial 2T/\partial x2$$

with three possible boundary conditions as indicated below:



Case 1: Specified temperature T_s at x = 0 imposed at time t = 0 with initial uniform body temperature T_i.

The solution is given by

$$\frac{T(x,t)-T_s}{T_i-T_s}=erf\left(\frac{x}{2\cdot\sqrt{\alpha\cdot t}}\right)$$

Example:

Consider a buried pipe in soil with the following properties:

$\rho := 2050\ \frac{kg}{m^3}$ density

$k := 0.52\ \frac{W}{m\cdot K}$ thermal conductivity

$c := 1840\ \frac{J}{kg\cdot K}$ specific heat capacity

$\alpha := \frac{k}{\rho\cdot c} = \left(1.379\cdot 10^{-7}\right)\ \frac{m^2}{s}$ thermal diffusivity

The soil is initially at a uniform temperature

$T_i := 20\ \Delta°C$

It is then subjected to surface temperature

$T_s := -15\ \Delta°C$

for 60 days i.e.

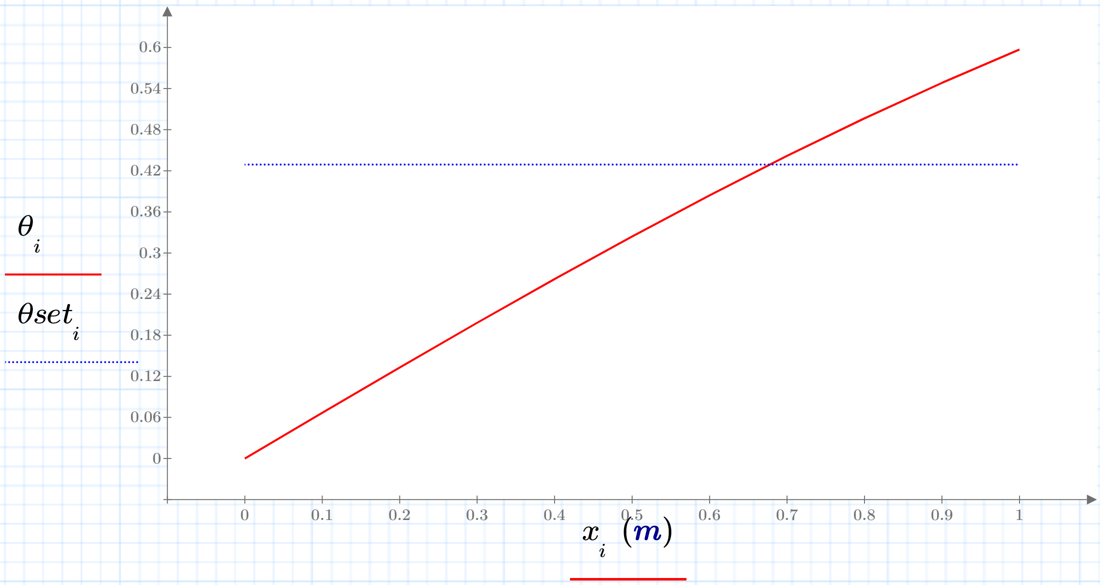
$t := 60\cdot 86400\ s$

Let's say that we would like to determine the depth xm below the surface at which one must place a water main to avoid freezing(T(x,t) = 0).

$$\theta := \frac{0 \cdot \Delta°C - T_s}{T_i - T_s} = 0.429$$

$i := 0 .. 10$ $x_i := i \cdot 0.1\ m$

$\theta_i := \text{erf}\left(\frac{x_i}{\sqrt{\alpha \cdot t} \cdot 2}\right)$ $\theta set_i := 0.429$



As can be seen from the above graph, the depth at which the temperature will reach 0 degC is at the intersection of the two lines, xm = 0.68 m. Alternatively, we can use the following technique:

$xm := 0.8\ m$ just a guess

$$answer := \textbf{root}\left(\text{erf}\left(\frac{xm}{\sqrt{\alpha \cdot t} \cdot 2}\right) - \theta set_1, xm\right) = 0.677\ m$$

Calculation of heat flux at surface:

$$q := k \cdot \frac{T_s - T_i}{\sqrt{\pi \cdot \alpha \cdot t}} = -12.146\ \frac{W}{m^2}$$