CHAPTER 2 TRANSIENT HEAT CONDUCTION

2.4 Semi-Infinite Slab: Convective Boundary Condition

A thick floor slab is initially at a uniform temperature Ti. Suddenly, one of its surfaces is subjected to convective cooling with a heat transfer coefficient h into an environment at Te. Calculate the temperature at a depth x from the surface at time t after the change.

Properties of floor slab:

$\rho := 1700 \, \frac{kg}{m^3}$ density

$k := 1.7 \, \frac{W}{m \cdot \Delta°C}$ thermal conductivity

$c := 800 \, \frac{J}{kg \cdot \Delta°C}$ specific heat capacity

$\alpha := \frac{k}{\rho \cdot c}$ thermal diffusivity

$h := 14 \, \frac{W}{m^2 \cdot \Delta°C}$ film coefficient at surface

$Te := 15 \, \Delta°C$ environment T

$Ti := 30 \, \Delta°C$ initial floor temperature

$i := 1 .. 10$ $t_i := i \cdot 1.0 \, hr$

$j := 0, 1 .. 5$ $x_j := j \cdot 0.1 \, m$

The temperature at depth x and time t for a semi-infinite slab with convective boundary condition is given by

$$T_{i,j} := (Te - Ti) + \left(\left(1 - \operatorname{erf}\left(\frac{x_j}{2 \cdot \sqrt{\alpha \cdot t_i}}\right)\right) - \exp\left(\frac{h \cdot x_j}{k} + \frac{h^2 \cdot \alpha \cdot t_i}{k^2}\right) \cdot \left(1 - \operatorname{erf}\left(\frac{x_j}{2 \cdot \sqrt{\alpha \cdot t_i}} + \frac{h \cdot \sqrt{\alpha \cdot t_i}}{k}\right)\right)\right) \cdot \Delta°C + Ti$$

Heat flow at surface at time t:

$$qs_i := h \cdot (Te - Ti) \cdot \exp\left(\frac{h^2 \cdot \alpha \cdot t_i}{k^2}\right) \cdot \left(1 - \operatorname{erf}\left(\frac{h \cdot \sqrt{\alpha \cdot t_i}}{k}\right)\right)$$