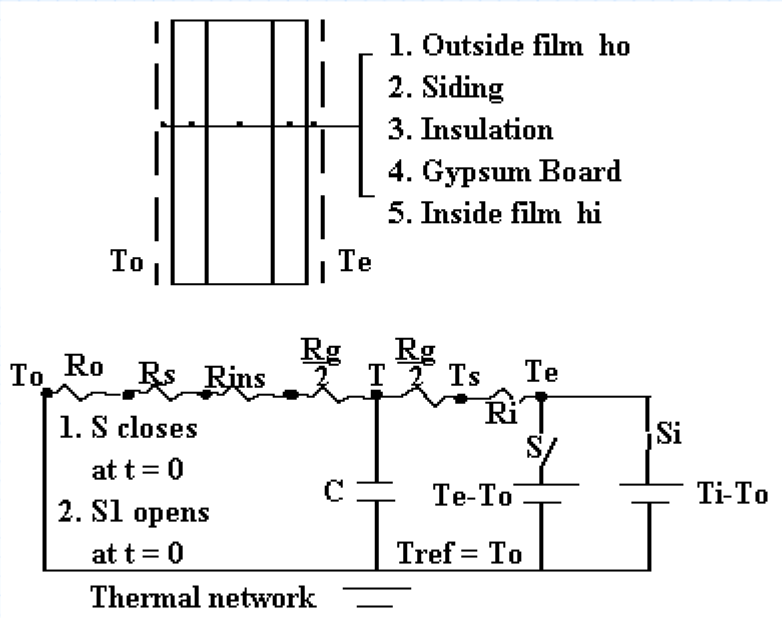
CHAPTER 2 TRANSIENT HEAT CONDUCTION

2.5 Simple Transient Model for a Wall

Wall transient thermal response can be determined in an approximate manner analytically by modeling the wall with two resistances and a thermal capacitance. Consider, for example, the response of a wall when convective heating is suddenly applied in a room. Although the room air temperature may be raised very fast, the wall temperature responds slowly if it stores significant quantities of heat.

Example: Consider a wall consisting of a layer of gypsum board, insulation and exterior siding with parameters shown below. The room is initially unoccupied at temperature T1. The temperature of the air is raised to Te by convective heating. The outside temperature is To. Determine the change of the wall surface temperature with time.



$A := 1 \cdot m^2$ (unit area)

$R_s := 0.3 \, \dfrac{m^2 \cdot \Delta°C}{W}$ siding

$R_{ins} := 2.3 \, \dfrac{m^2 \cdot \Delta°C}{W}$ insulation

Gypsum board:

$L := 0.013\ m$ thickness

$k := 0.16\ \frac{W}{m \cdot \Delta°C}$ conductivity

$c := 750\ \frac{J}{kg \cdot \Delta°C}$ specific heat

$\rho := 800\ \frac{kg}{m^3}$ density

$hi := 9\ \frac{W}{m^2 \cdot \Delta°C}$

$ho := 15\ \frac{W}{m^2 \cdot \Delta°C}$

interior and exterior film coefficients

$C := c \cdot \rho \cdot L \cdot A$

$Rg := \frac{L}{k}$

Let

$$Ra := \frac{\frac{1}{ho} + R_s + R_{ins} + \left(\frac{Rg}{2}\right)}{A}$$

and

$$Rb := \frac{\frac{Rg}{2} + \frac{1}{hi}}{A}$$

$$Rtot := Ra + Rb = 2.859\ \frac{\Delta°C}{W}$$

Note that the gypsum board is modeled by a thermal capacitance connected to two resistances, each equal to half the total thermal resistance Rg. Only the thermal capacity of the gypsum board is considered in this problem because it is the interior layer, and it thus stores or releases heat due to thermal changes in the room much more than the outer layers. For a more accurate analysis of transient heat transfer, the finite difference method may be employed (**Section 3.3**).

An energy balance at the thermal capacitance yields

$$C \cdot \frac{d}{dt}(T - To) + \frac{T - To}{Ra} + \frac{T - Te}{Rb} = 0$$

with the initial condition

$$T(t = 0) = Ti$$

where Ti may be determined from To and Te. The above equation can be written in the form

$$C \cdot \frac{d}{dt}\theta + \theta \cdot \frac{Ra + Rb}{Ra \cdot Rb} = Qeq$$

with

$$Qeq = \frac{Te - To}{Rb} \qquad \theta = T - To$$

We will solve this problem for the following conditions:

$i := 0, 1 .. 10$ $\qquad t_i := i \cdot 0.5\ hr$

$To := -10\ \Delta°C$ outside temperature (constant)

$T1 := 10\ \Delta°C$ initial room air temperature

$Te := 20\ \Delta°C$ room air temperature at t > 0

$T_0 := T1 - \frac{Rb \cdot (T1 - To)}{Rtot}$ initial temperature of C in model

Also

$\theta_0 := T_0 - To$ initial excess temperature

Let

$$m := \frac{Ra + Rb}{Ra \cdot Rb \cdot C}$$

and

$$Qeq := \frac{Te - To}{Rb}$$

The solution to the differential equation representing the energy balance is given by

$$\theta_i := \theta_0 \cdot \exp\left(-m \cdot t_i\right) + \left(1 - \exp\left(-m \cdot t_i\right)\right) \cdot \frac{Qeq}{m \cdot C}$$

$$T_i := \theta_i + To$$

Wall surface temperature:

$$Ts_i := Te - \frac{Te - T_i}{Rb \cdot A \cdot hi}$$