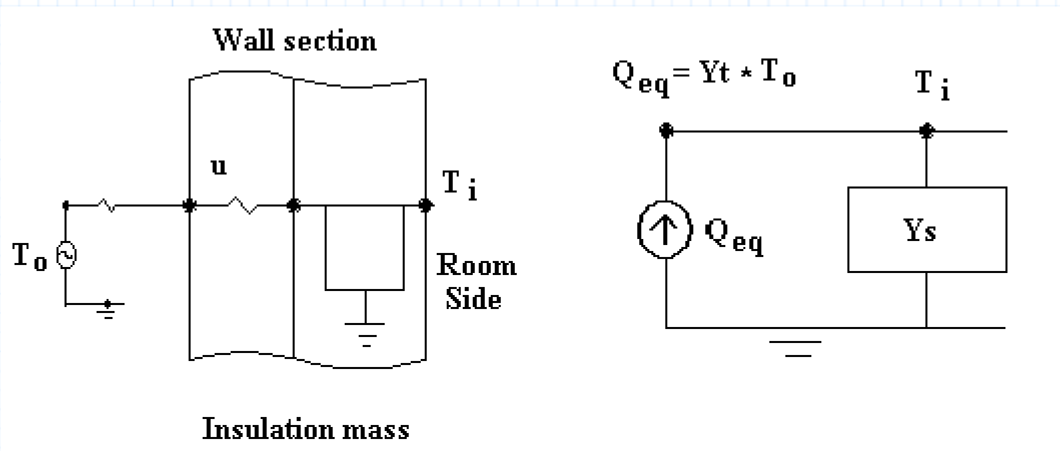
CHAPTER 4 PERIODIC HEAT FLOW IN MULTILAYERED WALLS

4.2 Thermal Admittance of a Multilayered Wall

The thermal admittance of a wall is a transfer function parameter useful for analysis of the effects of cyclical variations in weather variables such as solar radiation, outside temperature and dynamic heat flows under steady periodic conditions.

There are two transfer functions of primary interest: the self-admittance Ys relating the effect of a heat source at one surface to the temperature of that surface and the transfer admittance Yt relating the effect of an outside temperature variation to the resulting heat flow at the inside surface.

These two transfer functions are determined as demonstrated in the following example. The wall shown below consists of insulation and thermally nonmassive layers with conductance value U per unit area and a thermally massive layer of thickness L.



$i := 1, 2 .. 8$ $\qquad$ $j := \sqrt{-1}$

$A := 1\ m^2$ $\qquad$ analysis for unit wall area

$P := 86400\ s$ $\qquad$ period = 1 day

$n := 1, 2 .. 30$ $\qquad$ number of frequencies

$w_n := \dfrac{2 \cdot \pi \cdot n}{P}$ $\qquad$ frequency

$u := 0.4\ \dfrac{W}{m^2 \cdot \Delta°C}$ $\qquad$ conductance behind massive layer

$L_i := 0.04 \cdot i \cdot m$ thickness of massive layer

$c := 800 \frac{J}{kg \cdot \Delta°C}$ specific heat capacity of massive layer

$k := 1.7 \frac{W}{m \cdot \Delta°C}$ thermal conductivity

$\rho := 2200 \cdot \frac{kg}{m^3}$ density

$\alpha := \frac{k}{\rho \cdot c}$ thermal diffusivity

$$\gamma_n := \sqrt{\frac{j \cdot w_n}{\alpha}}$$

Self admittance:

$Ys = \frac{Q_{inside}}{T_{inside}}$ . . . all else constant

Transfer admittance:

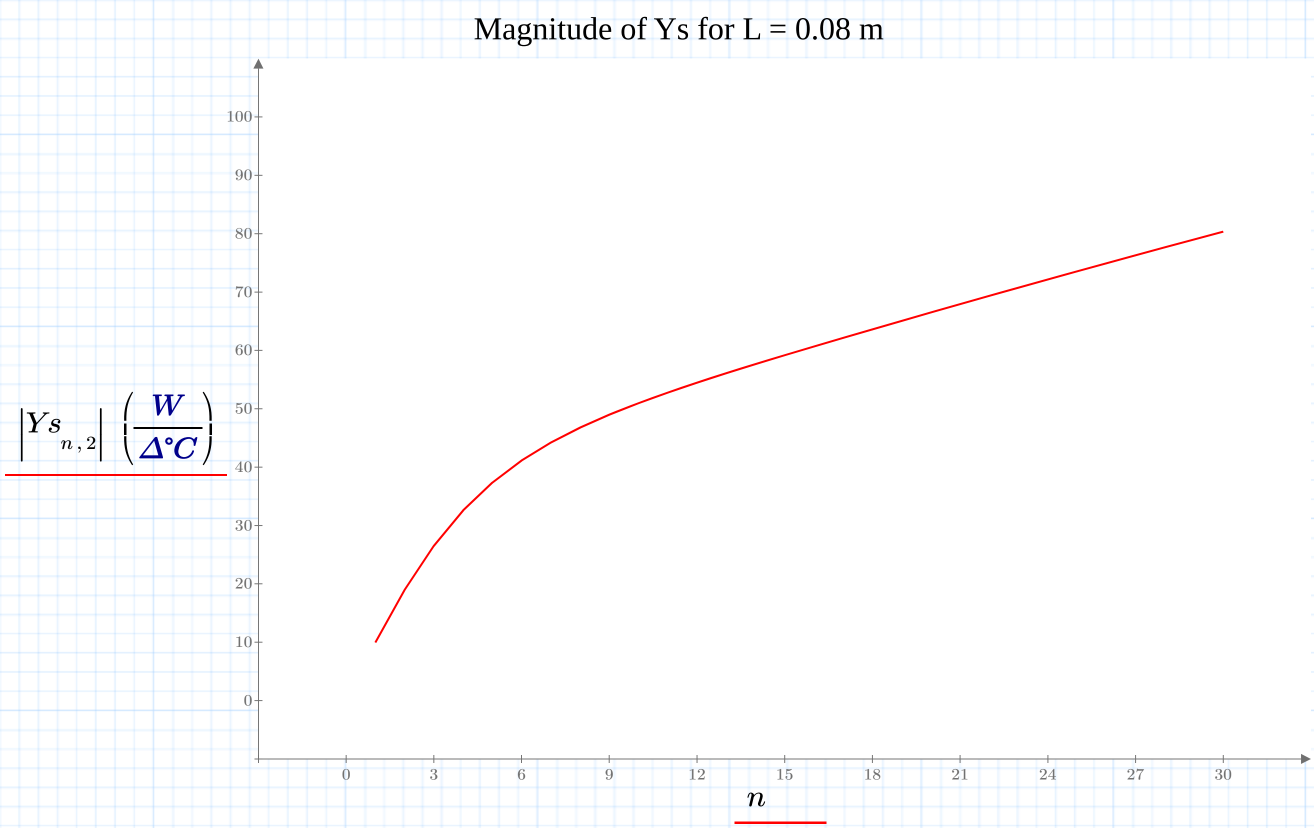
$Yt = \frac{Q_{inside}}{T_{outside}}$ . . . all else constant

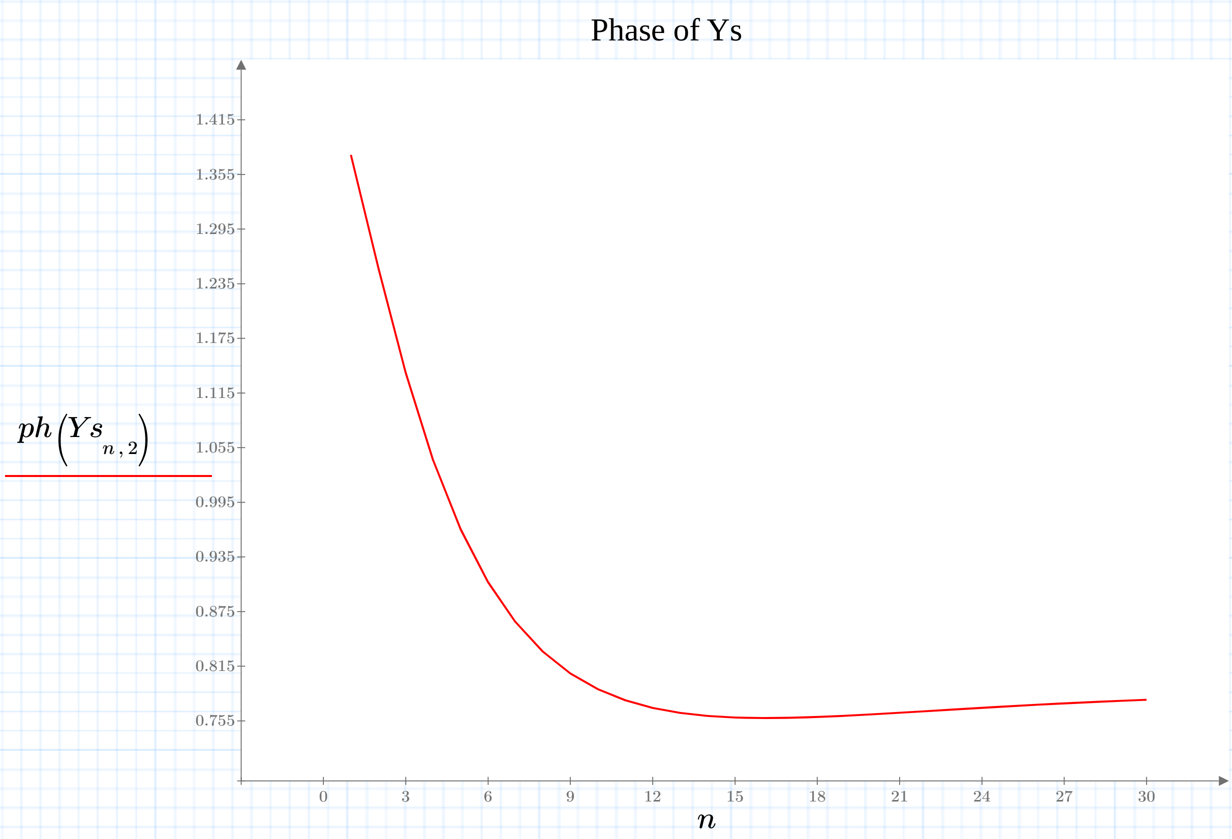
$$Ys_{n,i} := A \cdot \frac{u + k \cdot \gamma_n \cdot \tanh\left(\gamma_n \cdot L_i\right)}{\frac{u}{k \cdot \gamma_n} \cdot \tanh\left(\gamma_n \cdot L_i\right) + 1}$$

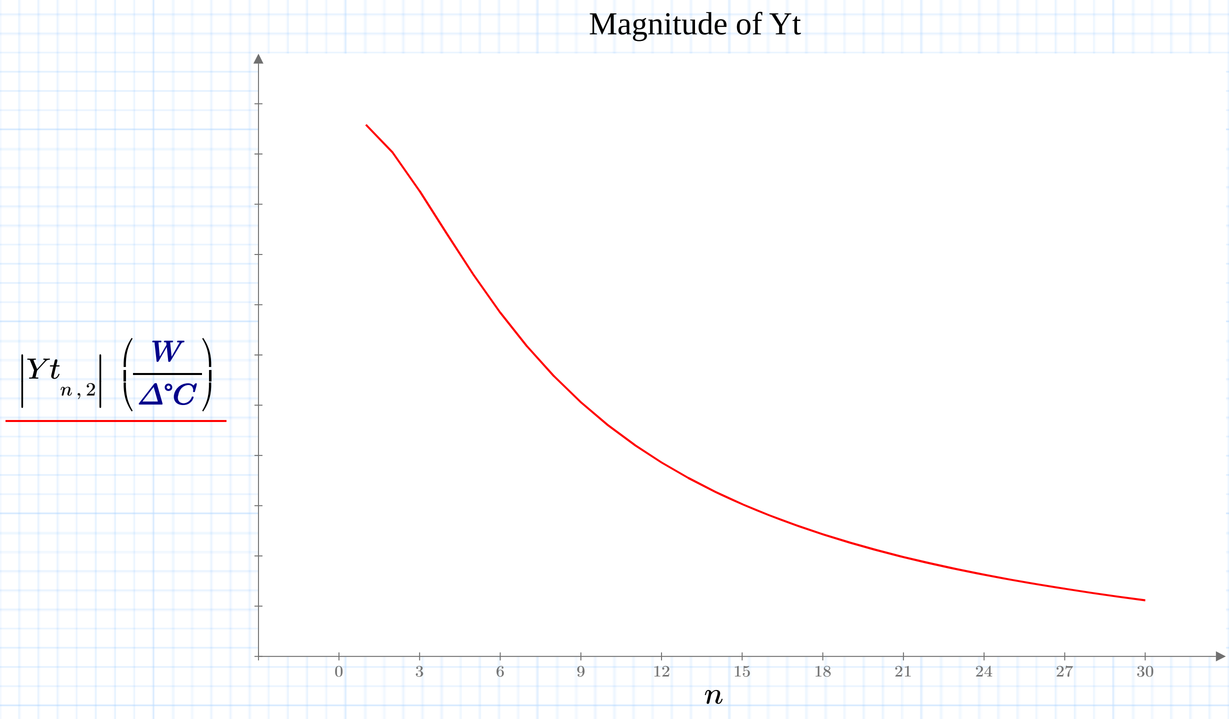
$$Yt_{n,i} := \frac{A}{\frac{\cosh\left(\gamma_n \cdot L_i\right)}{u} + \frac{\sinh\left(\gamma_n \cdot L_i\right)}{k \cdot \gamma_n}}$$

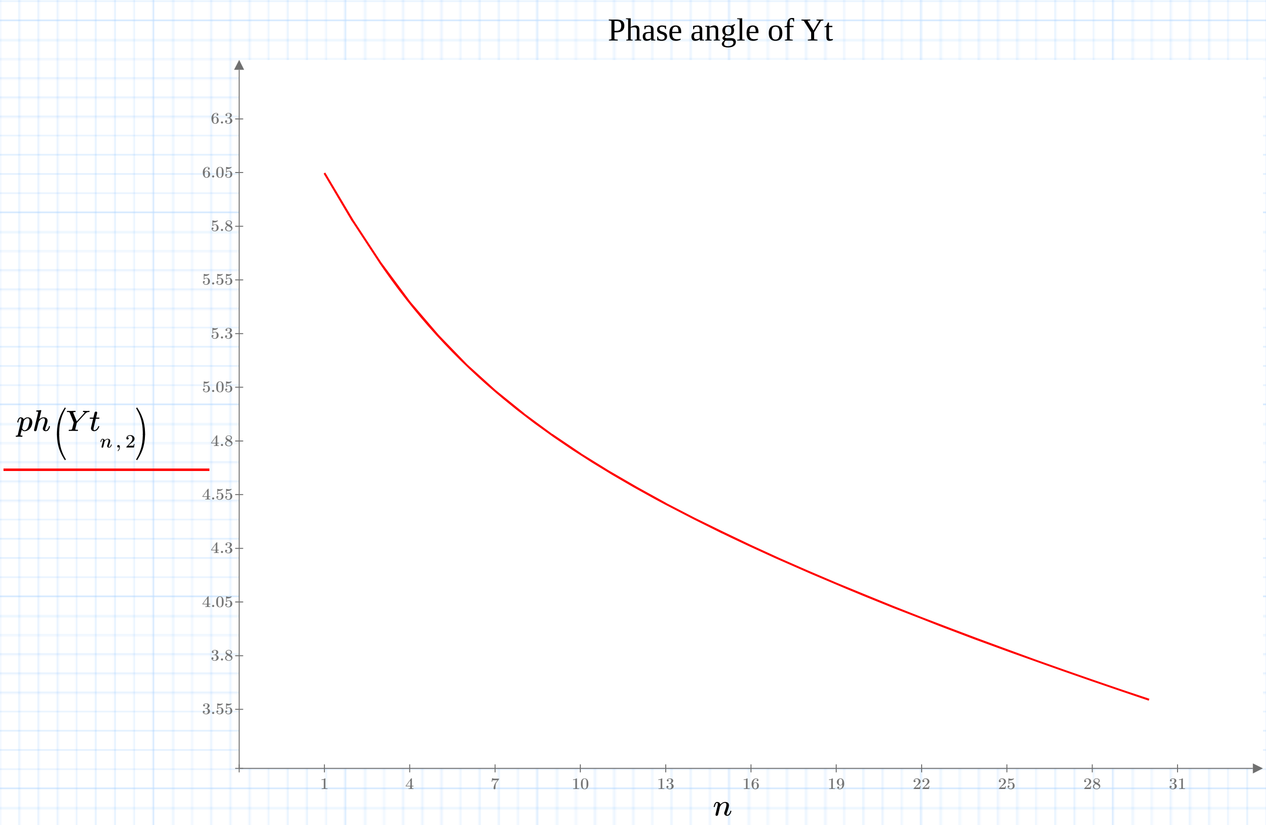
$ph(Ys) := \arg(Ys) + 2 \cdot \pi \cdot (\mathbf{if}(\arg(Ys) < 0, 1, 0))$ phase angle (0-360 degrees) for plots

Substantial insight into wall and building thermal behavior may be gained by studying the magnitude and phase angle of the important transfer functions such as Ys and Yt.


Magnitude of Ys for L = 0.08 m
$\left|Ys_{n,2}\right| \left(\frac{W}{\Delta°C}\right)$
100
90
80
70
60
50
40
30
20
10
0
0
3
6
9
12
15
18
21
24
27
30
n


Phase of Ys
$ph\left(Ys_{n,2}\right)$
1.415
1.355
1.295
1.235
1.175
1.115
1.055
0.995
0.935
0.875
0.815
0.755
0
3
6
9
12
15
18
21
24
27
30
n


Magnitude of Yt
|Yt n,2| (W/Δ°C)
0
3
6
9
12
15
18
21
24
27
30
n

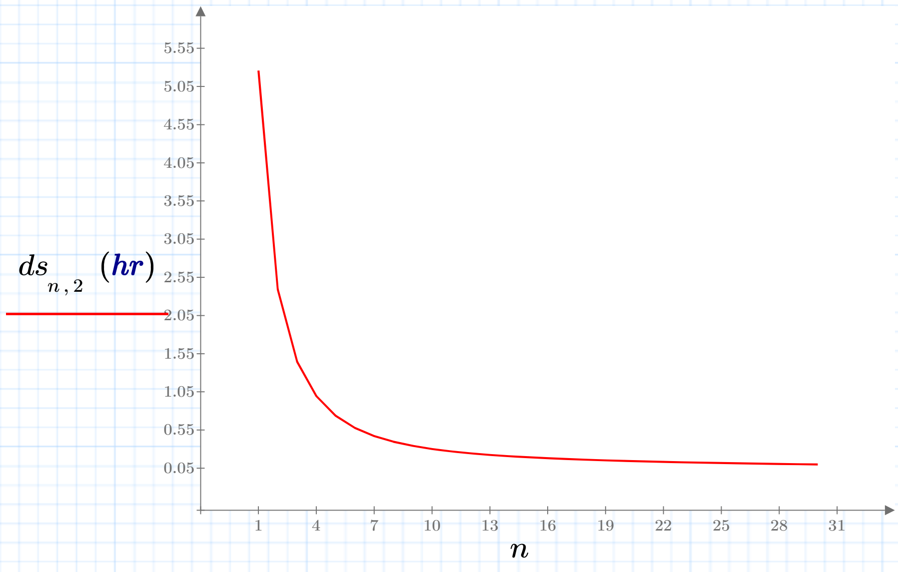

Phase angle of Yt
6.3
6.05
5.8
5.55
5.3
5.05
4.8
4.55
4.3
4.05
3.8
3.55
ph(Yt n,2)
1
4
7
10
13
16
19
22
25
28
31
n

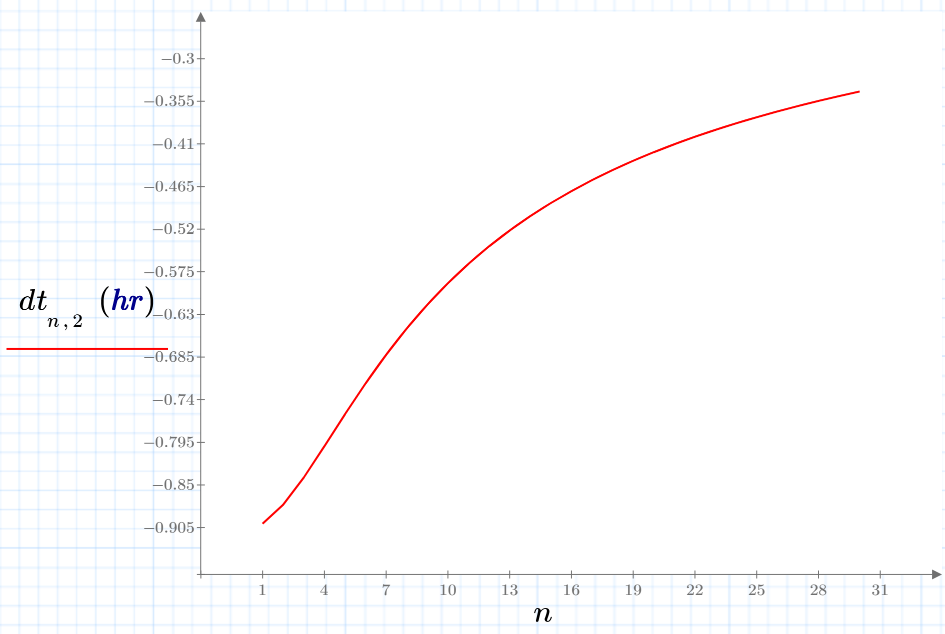
Time lag of Ys for L = 0.08 m:

$$ds_{n,i} := \frac{\arg\left(Ys_{n,i}\right)}{w_n}$$

Time lag of Yt:

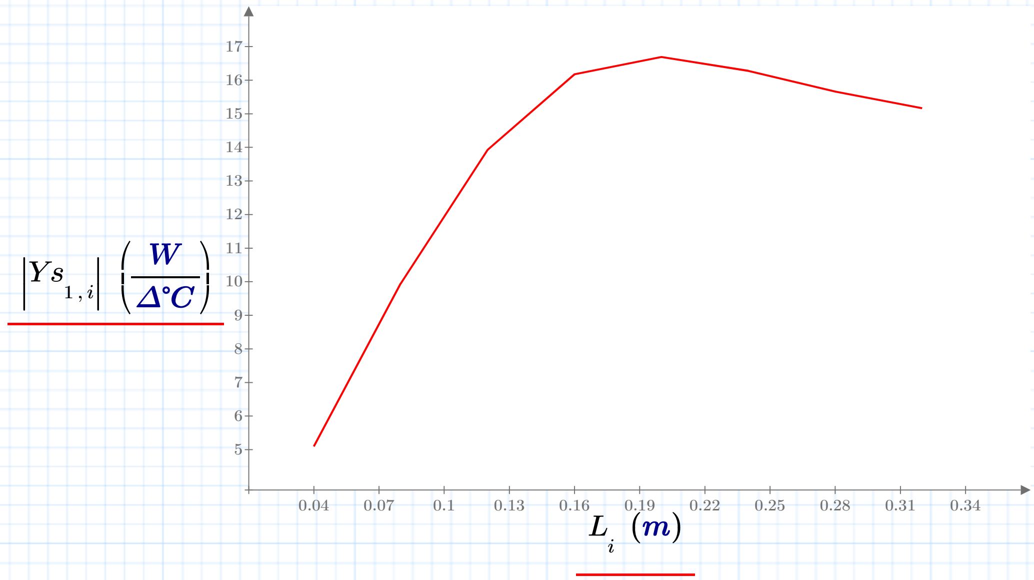
$$dt_{n,i} := \frac{\arg\left(Yt_{n,i}\right)}{w_n}$$



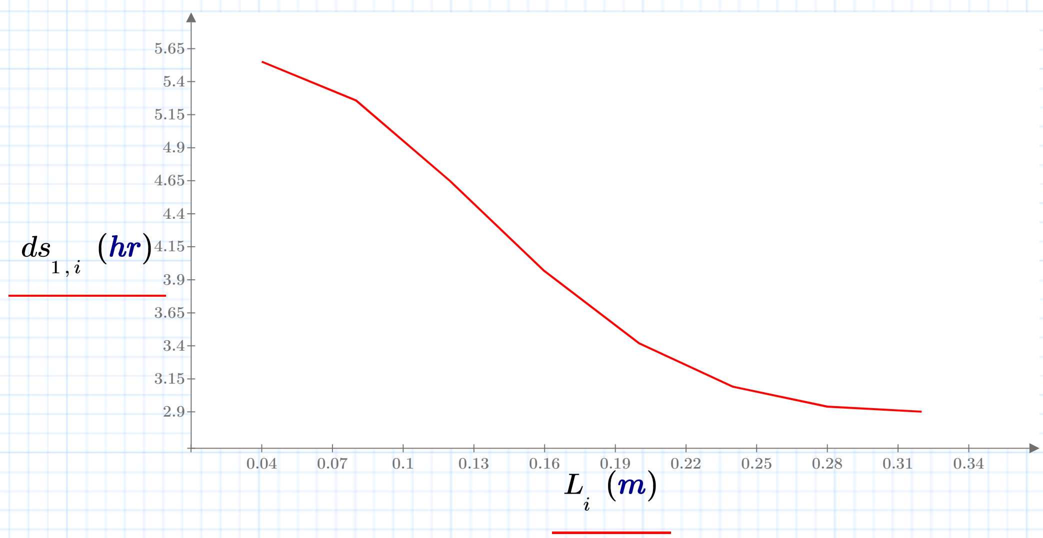


The time delay ds is the time lag between the peak of a sinusoidal input function, such as solar radiation in the case of the room interior surface, and the resulting peak of the interior surface temperature T_i. It is referred to as lag when a temperature is divided by an admittance (otherwise it is a lead).

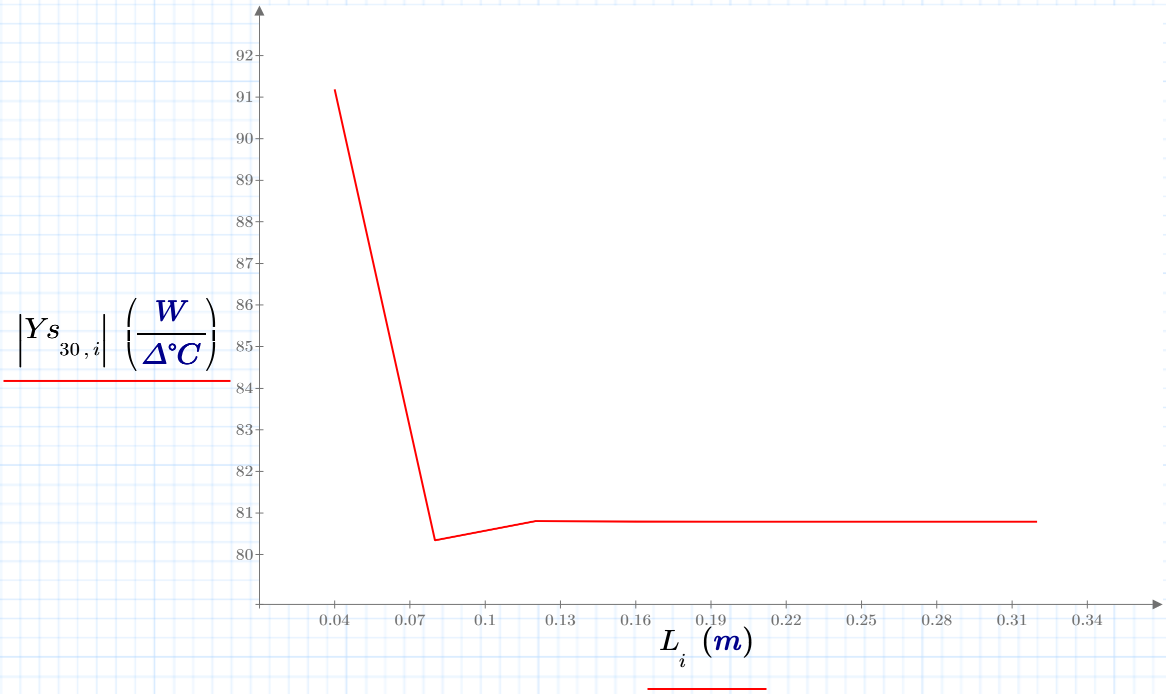
Now, we consider the variation of wall thermal admittance with thermal mass thickness L for the fundamental frequency (one cycle per day -- n = 1) and for a high frequency (n = 48). Note that the diurnal (n = 1) frequency is important in the analysis of variables with a dominant diurnal harmonic such as solar radiation. The high frequencies are important in analyzing the effect of varying heat inputs such as those due to on/off cycling of a furnace.



This graph shows an extremely important result in steady-periodic analysis of building thermal response; there is an optimum thermal mass thickness for passive solar design (in this case for L = 0.2 m) corresponding to the maximum admittance. Therefore, this wall mass thickness will reduce room temperature fluctuations the most.

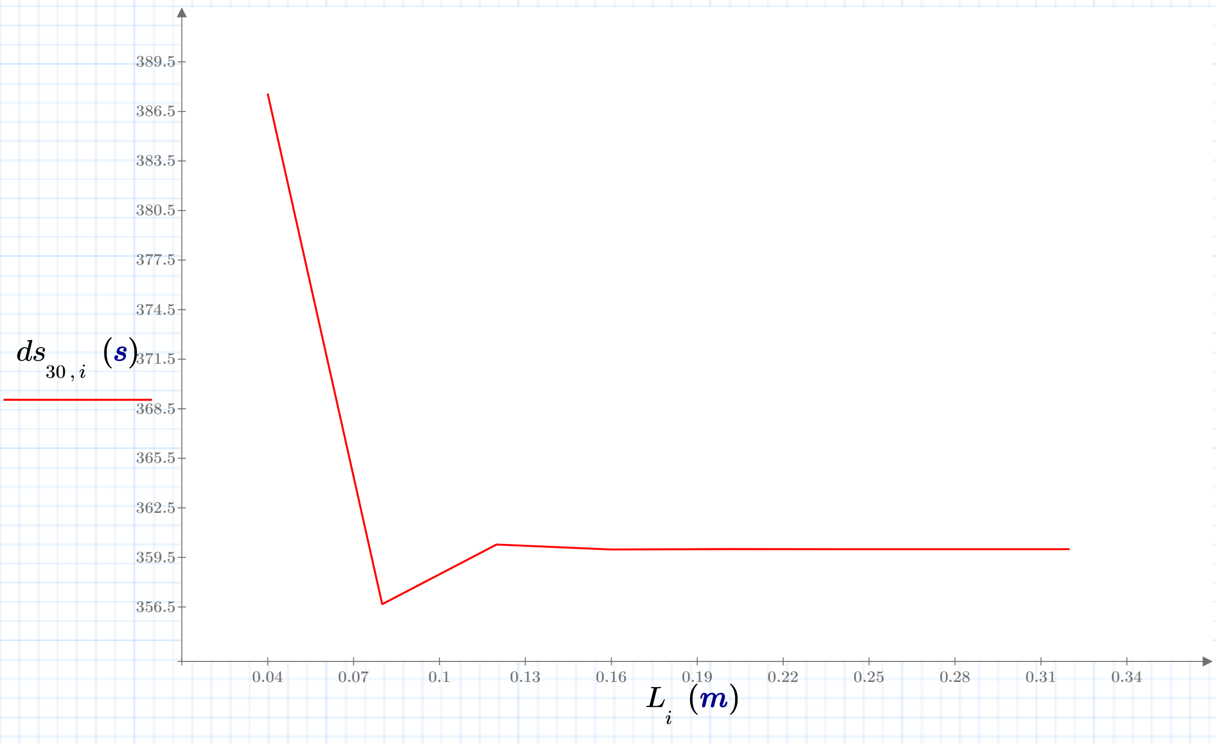


This graph shows the time delay of the wall self-admittance corresponding to the above figure.



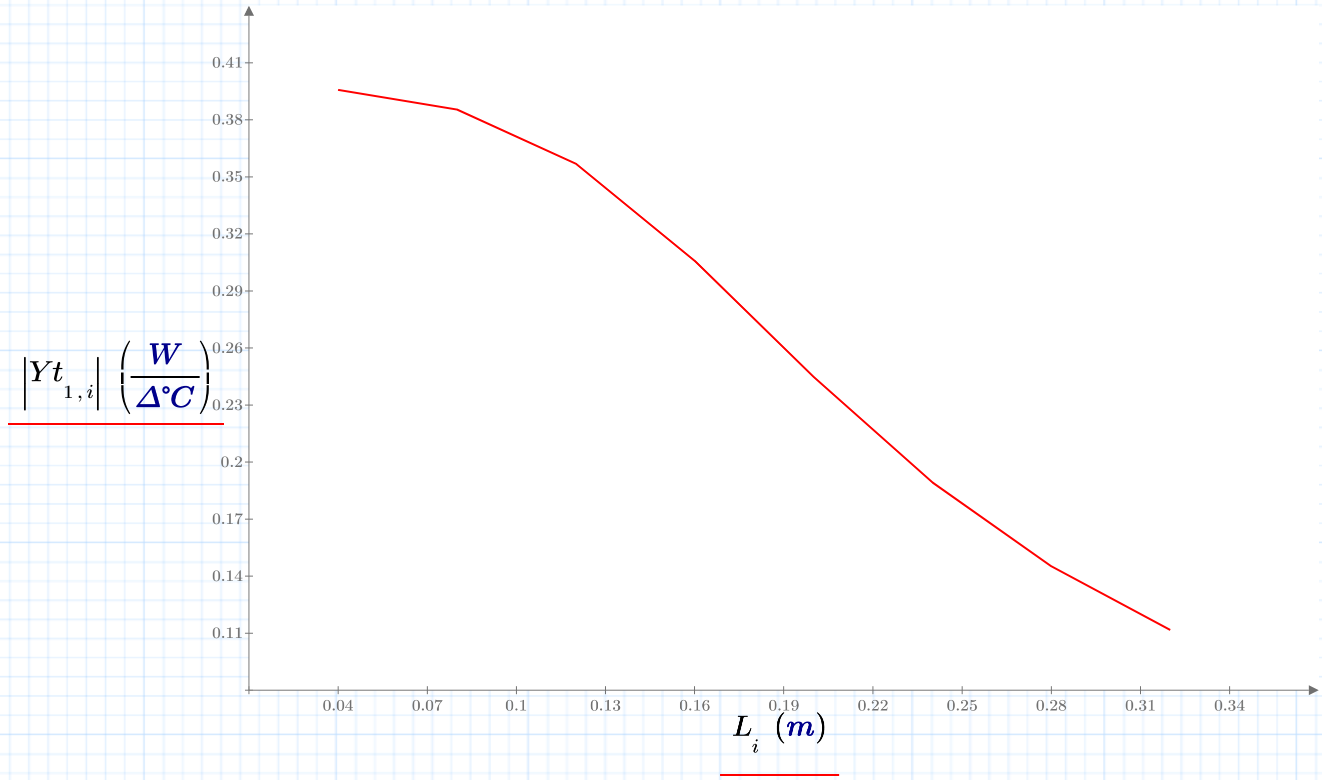
For n = 30 (period equal to half-hour), the wall behaves like a semi-infinite solid for thicknesses greater than 0.08 m: the magnitude of the self-admittance Ys is approximately constant for L > 0.07 m.

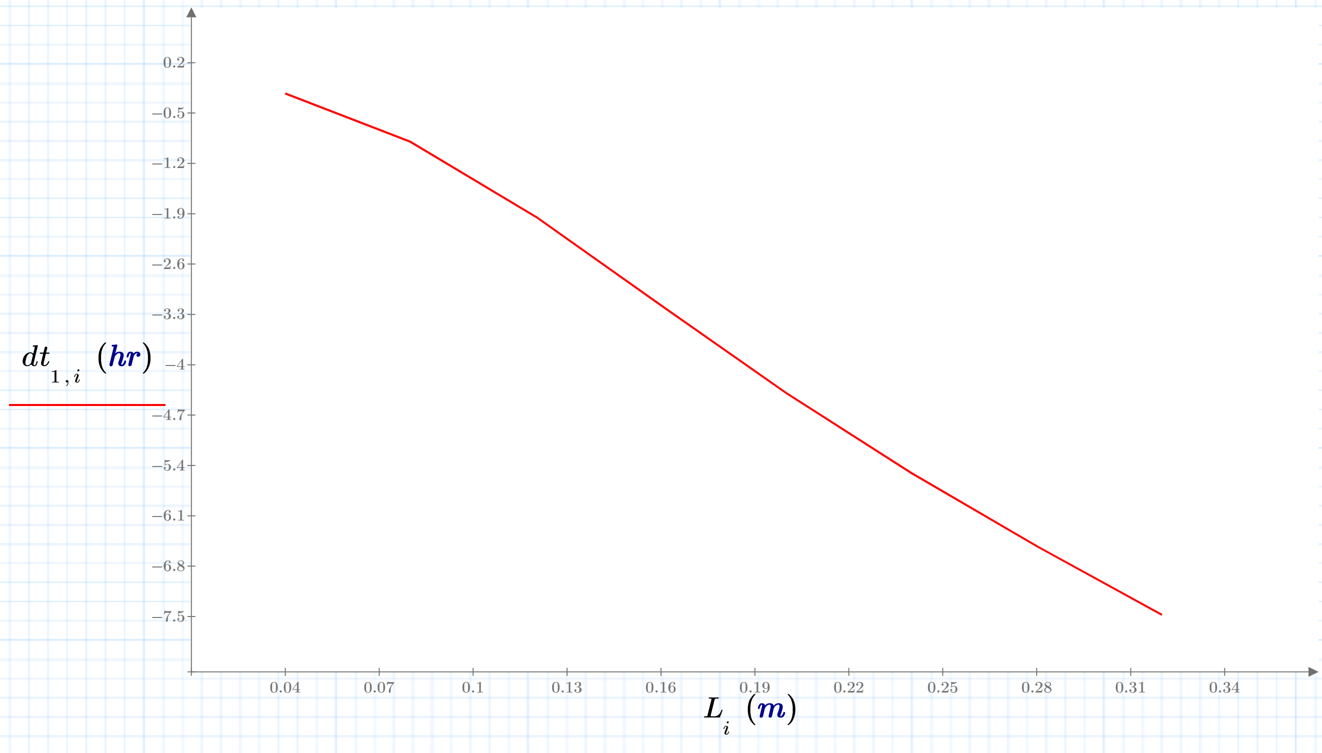
The time delay corresponding to Ys for n=30 varies little with wall thickness.



The room admittance is determined in **Section 9.2** from the individual wall admittances and other room conductances (including that due to infiltration).

Now, we consider the effect of wall interior mass thickness on transfer admittance Yt:





The above two graphs show the magnitude of the transfer-admittance for the first harmonic and the corresponding time delay $dt_{1,i}$ as a function of wall thickness. As expected, Yt decreases with increasing wall thickness and its time lag increases (in absolute magnitude) to about 8 hours for L = 0.3 m. The next section demonstrates how the admittance transfer functions are used to determine wall periodic response to solar radiation and ambient temperature.

References

Athienitis, A. K., M. Stylianou and J. Shou. 1990. "A methodology for building thermal dynamics studies and control applications." *ASHRAE Transactions* Vol. 96, Pt. 2, pp. 839-48.

Athienitis, A. K., H. F. Sullivan and K. G. T. Hollands. 1986. "Analytical model, sensitivity analysis, and temperature swings in direct gain rooms." *Solar Energy*, Vol. 36, pp. 303-12.

ASHRAE. 1989. Handbook-Fundamentals, Atlanta, GA.