CHAPTER 4 PERIODIC HEAT FLOW IN MULTILAYERED WALLS

4.3 Steady-Periodic Heat Transfer in Multilayered Walls

Using the admittance transfer functions described in the previous section, we now determine the response of walls to periodic variations in outside temperature To and solar radiation S. Both weather variables are modeled by discrete Fourier series consisting of the daily average (mean term) and one or more harmonics.

Analysis is typically performed for design days which may represent, depending on the objective of the analysis, either average days or extreme design conditions. Here, we will consider clear days with major objective to optimize the walls of a passive solar building.

First we consider models for the outside temperature and solar radiation.

Outside Temperature

The variation of the ambient temperature To will be modeled by a sinusoid with maximum at 3 p.m. and minimum at 3 a.m. (solar time). Example:

$Tom := -1\ \Delta°C$ daily mean outside temperature

$\Delta To := 10\ \Delta°C$ range of To = (max. - min.)

$n := 1, 2 .. 3$ harmonic index

$i := 0, 1 .. 23$ time index

$w_n := 2 \cdot \dfrac{\pi \cdot n}{24} \dfrac{rad}{hr}$ fundamental frequency (period = 1 day)

$t_i := i \cdot hr$ time

$To_i := Tom + \dfrac{\Delta To}{2} \cdot \cos\left(w_1 \cdot t_i - 5 \cdot \dfrac{\pi}{4}\right)$ outside temperature

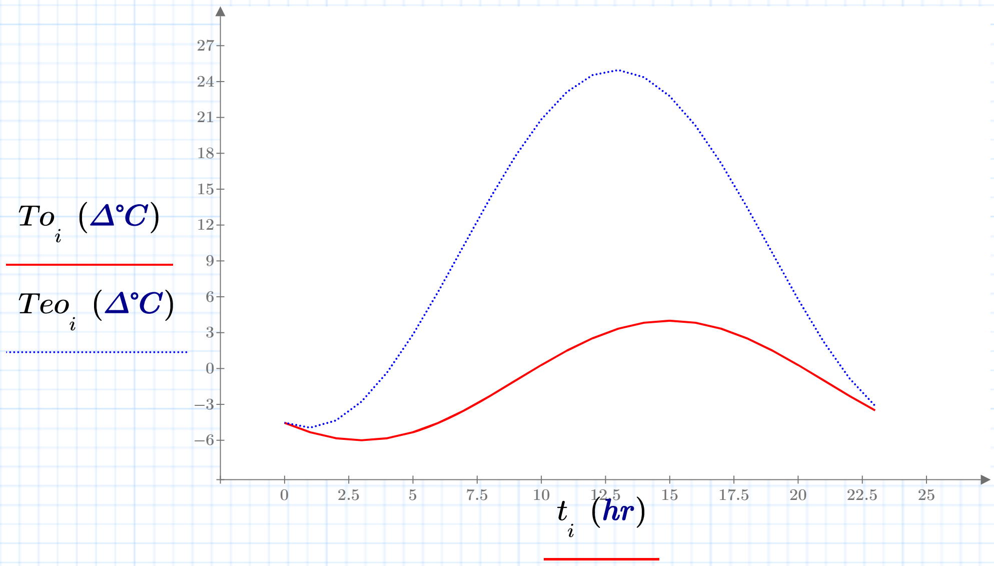
Assume that the sol-air temperature also has one harmonic due to solar radiation with its peak at noon.

$Tem := 10\ \Delta°C$

$Te_1 := 11\ \Delta°C$

$Teo_i := Tem + Te_1 \cdot \cos\left(w_1 \cdot t_i - \pi\right) + \frac{\Delta To}{2} \cdot \cos\left(w_1 \cdot t_i - 5 \cdot \frac{\pi}{4}\right)$

Outside Temperature and Sol-Air Temperature



Solar Radiation

Solar radiation can be modeled by a half-sinusoid from sunrise time $-t_s$ to sunset time t_s.

$t_s := 6\ hr$ time from solar noon to sunset

$S_{max} := 500\ \frac{W}{m^2}$ peak solar radiation (at noon)

$f_i := S_{max} \cdot \cos\left(\pi \cdot \frac{t_i - 12\ hr}{2 \cdot t_s}\right)$

$S_i := \frac{f_i + \left|f_i\right|}{2}$ S is positive

S(t) may be modeled with a discrete Fourier series as follows:

$n := 0, 1 .. 3$ . . . harmonics $j := \sqrt{-1}$

$w_n := 2 \cdot \pi \cdot \frac{n}{24\ hr}$ $Sn_n := \sum_i \left(S_i \cdot \frac{\exp\left(-j \cdot w_n \cdot t_i\right)}{24} \right)$

Three term harmonic Fourier series fit for solar radiation:

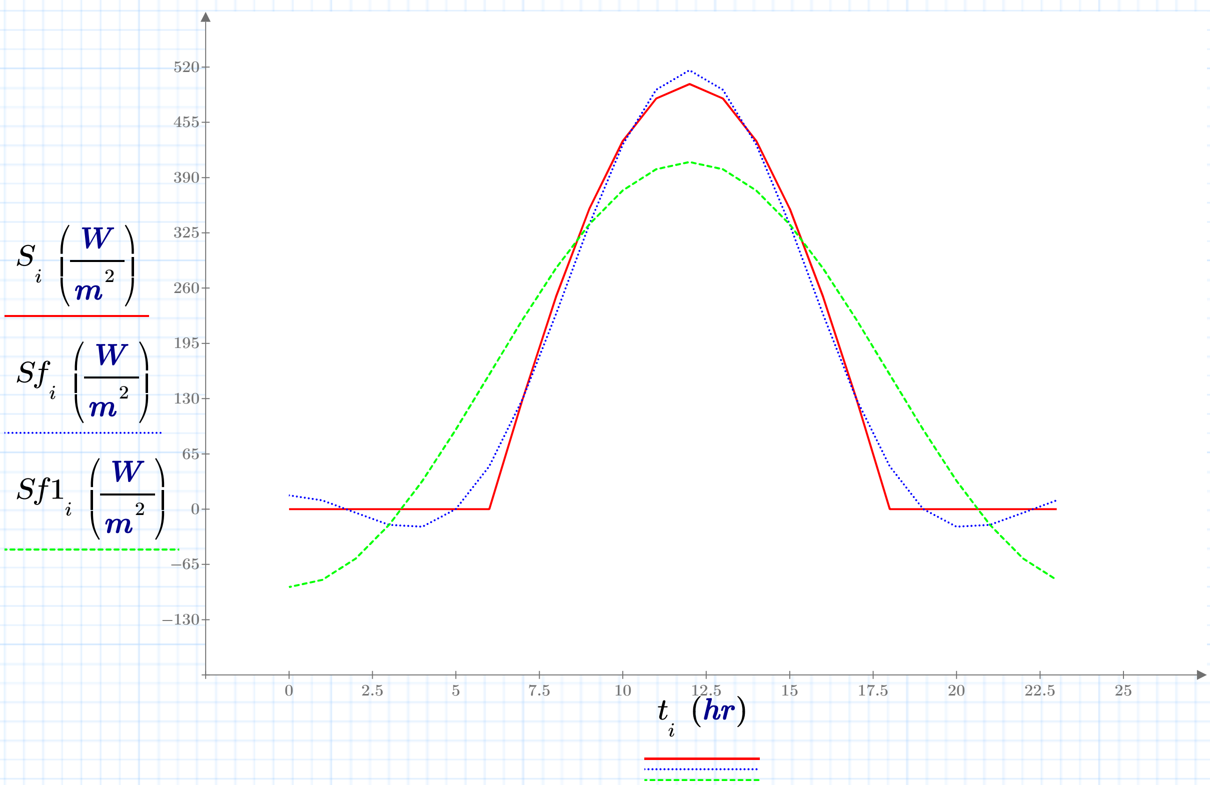
$l := 1 .. 3$

$Sf_i := Sn_0 + 2 \cdot \sum_l \mathrm{Re}\left(Sn_l \cdot \exp\left(j \cdot w_l \cdot t_i\right)\right)$

One term harmonic fit for solar radiation:

$Sf1_i := Sn_0 + 2 \cdot \mathrm{Re}\left(Sn_1 \cdot \exp\left(j \cdot w_1 \cdot t_i\right)\right)$

The graph compares the actual solar radiation S with the one term harmonic (Sf1) and three term harmonic (Sf) Fourier series fits. As can be seen, the three-harmonic fit is a very close approximation to the actual half-sinusoid shape.



Wall Response

Now, we determine the response of the wall shown below. The analysis considers a simple room with window area Aw and opaque wall area A. The focus here is on wall analysis. For complete room analysis see **Chapter 9**. The passive response of the wall room surface temperature is to be determined.

$A := 100\ m^2$ $\qquad Aw := 8\ m^2$

$L := 0.05\ m$ mass thickness

Mass (medium density concrete) properties:

$c := 800\ \frac{J}{kg \cdot \Delta °C}$ specific heat

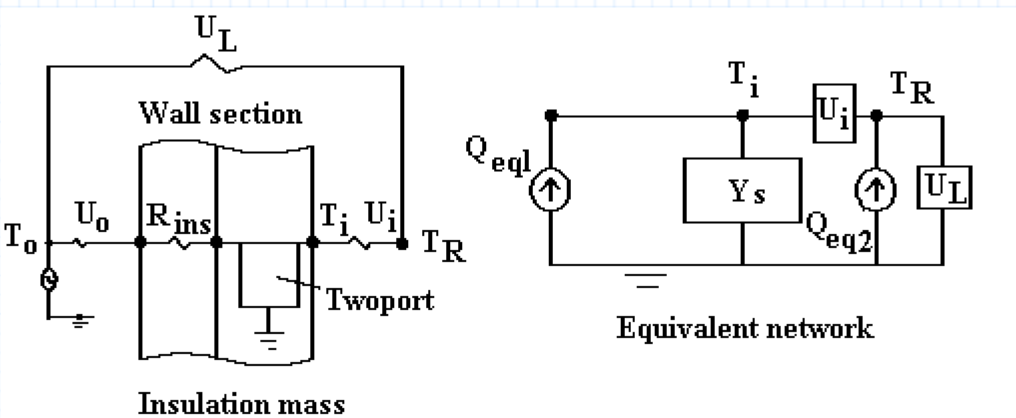
$k := 1.2\ \frac{W}{m \cdot \Delta °C}$ conductivity

$\rho := 1500\ \frac{kg}{m^3}$ density

$\alpha := \frac{k}{\rho \cdot c}$ thermal diffusivity

$h_o := 15\ \frac{W}{m^2 \cdot \Delta °C}$ outside film coefficient

$h_i := 9\ \frac{W}{m^2 \cdot \Delta °C}$ inside film coefficient



Qeq1=Yt·To

Qeq2=UL·To

$$R_{ins} := 2\ m^2 \cdot \frac{\Delta°C}{W}$$

$$u := \frac{1}{R_{ins} + \frac{1}{h_o}}$$

$$Ui := A \cdot h_i$$

$$Uo := h_o \cdot A$$

$$U_{window} := 2\ \frac{W}{m^2 \cdot \Delta°C}$$

$$UL := Aw \cdot U_{window}$$

$$UR := \frac{1}{\frac{1}{UL} + \frac{1}{Ui}}$$

$$Ys_0 := \frac{A}{\frac{L}{k} + \frac{1}{u}}$$

steady state admittance
is equal to wall U-value
(excluding interior film)

$$j := \sqrt{-1}$$

$$Yt_0 := Ys_0$$

$$Ys_0 = 47.431\ \frac{W}{\Delta°C}$$

$$n := 1, 2 .. 3$$

Calculation of admittances:

$$\gamma_n := \sqrt{j \cdot \frac{2 \cdot \pi \cdot n}{\alpha \cdot 86400\ s}}$$

$$Ys_n := A \cdot \frac{u + k \cdot \gamma_n \cdot \tanh\left(\gamma_n \cdot L\right)}{\left(\frac{u}{k \cdot \gamma_n} \cdot \tanh\left(\gamma_n \cdot L\right)\right) + 1}$$

$$Yt_n := \frac{A}{\frac{\cosh\left(\gamma_n \cdot L\right)}{u} + \frac{\sinh\left(\gamma_n \cdot L\right)}{k \cdot \gamma_n}}$$

$$n := 0, 1 .. 3$$

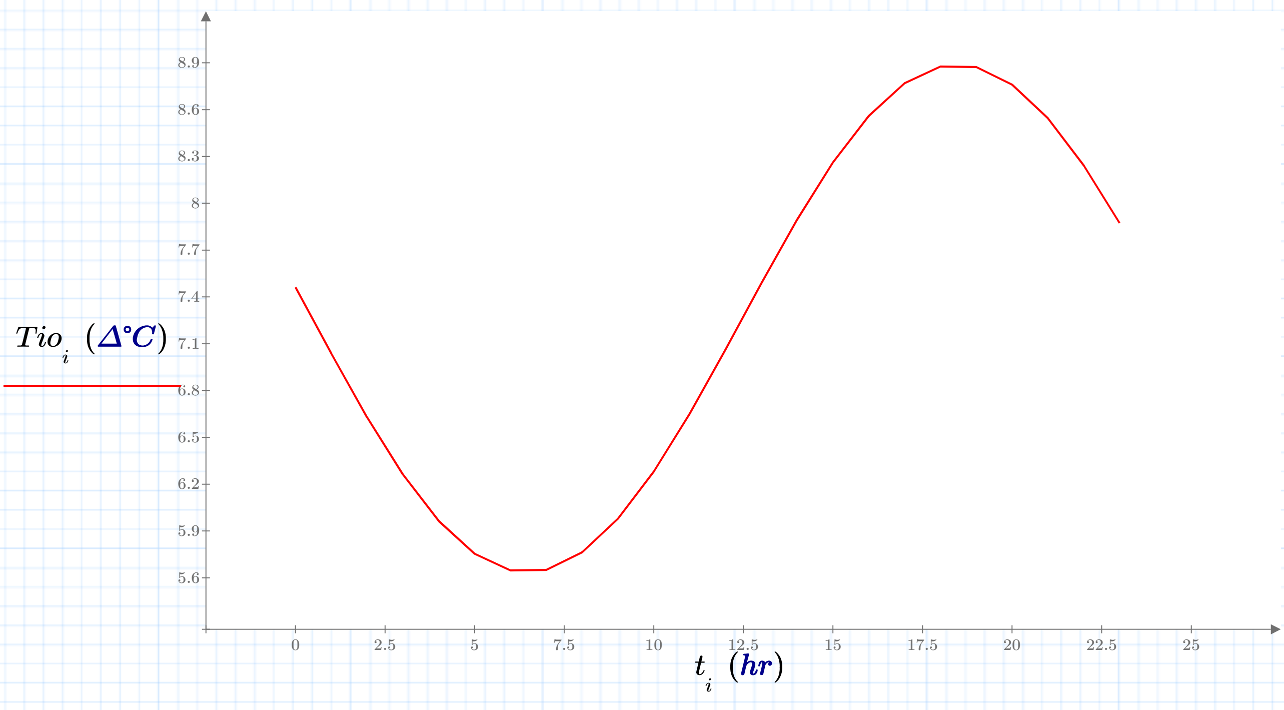
$Y_n := Ys_n + UR$ room total admittance as seen from wall room surface

Note that, for the steady-state term, both Yt and Ys are simply equal to the U-value of the wall (excluding the interior film).

The effect of Teo on Ti is equal to Yt·To/(Ys+UR). In the absence of solar radiation, the total response of Ti consists of the mean term and harmonics due to Teo and To, and the harmonics due to Teo. This response (Tio) is determined as follows:

$fo := \arg\left(Y_1\right) - \arg\left(Yt_1\right)$ . . . phase delay angle

$$Tio_i := \frac{Yt_0 \cdot Tem + UR \cdot Tom}{Ys_0 + UR} + \frac{\left|Yt_1\right| \cdot \left(\Delta To \cdot \frac{\cos\left(w_1 \cdot t_i - 5 \cdot \frac{\pi}{4} - fo\right)}{2} + Te_1 \cdot \cos\left(w_1 \cdot t_i - \pi - fo\right)\right)}{\left|Y_1\right|}$$

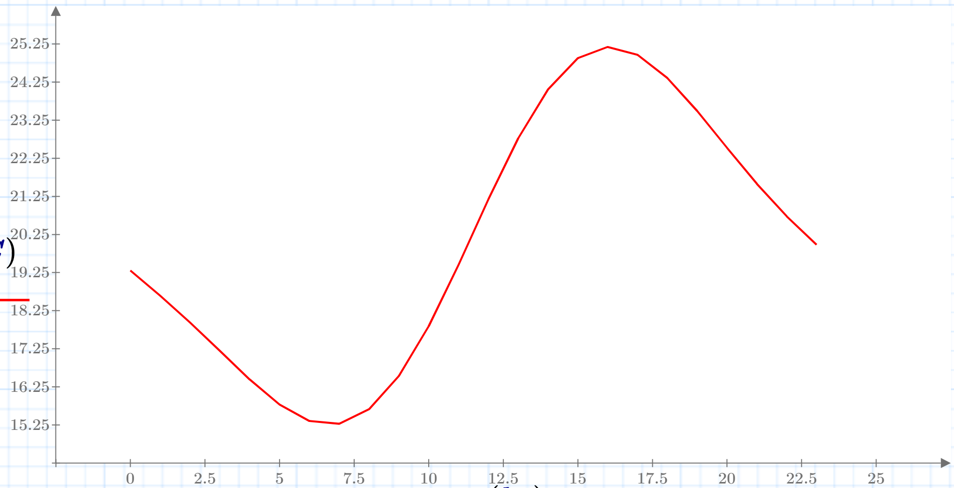


The response (Tis) of wall room surface temperature Ti to transmitted solar radiation S absorbed at the same surface is equal to Sn/Yn for each harmonic.

$$Tis_i := \left(\frac{Sn_0}{Y_0} + 2 \cdot \sum_l \mathrm{Re}\left(\frac{Sn_l}{Y_l} \cdot \exp\left(j \cdot w_l \cdot t_i\right)\right)\right) \cdot Aw$$

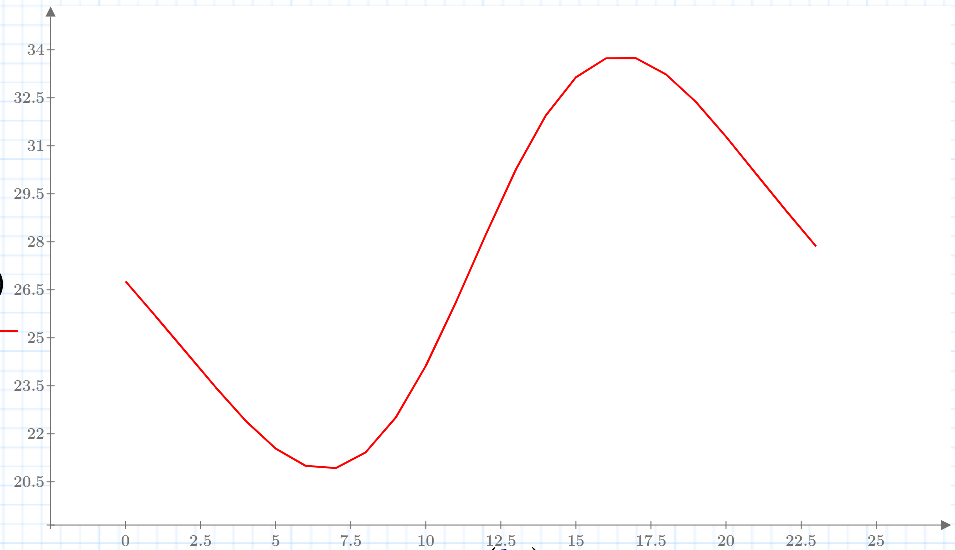
Total response:

$$Ti_i := Tio_i + Tis_i$$



$Tis_i \; (\Delta°C)$

$t_i \; (hr)$



$Ti_i \; (\Delta°C)$

$t_i \; (hr)$

References

Athienitis, A. K., M. Stylianou and J. Shou. 1990. "A Methodology for Building Thermal Dynamics Studies and Control Applications," *ASHRAE Transactions*, Vol. 96, Pt. 2, pp. 839-48.

Athienitis, A. K., H. F. Sullivan and K. G. T. Hollands. 1986. "Analytical Model, Sensitivity Analysis, and Temperature Swings in Direct Gain Rooms," *Solar Energy*, Vol.36, pp. 303-12.

ASHRAE. 1989. Handbook-Fundamentals, Atlanta, GA.