CHAPTER 7 SOLAR RADIATION

7.1 Solar Radiation on Inclined Surfaces

The total solar radiation incident on an inclined surface consists of a direct (beam) part, the sky diffuse solar radiation (e.g. reflected from clouds, and scattered radiation) and the ground-reflected radiation, also assumed to be diffuse. All calculations involving solar radiation are based on solar time.

Solar Time is based on the apparent angular motion of the sun across the sky with solar noon the time that the sun crosses the meridian of the observer. Local standard time (LST) is converted to solar time as follows. First, there is a constant correction for the difference in longitude between the location and the meridian on which the local time is based (Eastern, 75 deg W; Central 90 deg W; Mountain 105 deg W; Pacific 120 deg W; Hawaii-Alaska 150 deg W). Note that one degree in longitude is equivalent to 4 minutes (since 360 degrees is one day). Another correction is the equation of time, ET, which takes into account changes in the earth's rotation.

The apparent solar time AST is given by

AST = LST + ET + 4 (LSM - LON)

where ET = Equation of Time, minutes
LST = Local Standard Time
LSM = Local Standard Time Meridian, degrees
LON = Local Longitude, degree
4 = minutes of time required for 1 degree rotation of the earth

$$ET(n) := \left(9.87 \cdot \sin\left(4 \cdot \pi \cdot \frac{n-81}{364}\right) - 7.53 \cdot \cos\left(2 \cdot \pi \cdot \frac{n-81}{364}\right) + 1.5 \cdot \sin\left(2 \cdot \pi \cdot \frac{n-81}{364}\right)\right) \cdot min$$

where n = day of year (1 - 365)

Solar Geometry: The position of the sun and the geometric relationships between a plane and the beam solar radiation incident on it may be described in terms of the following angles:

L, latitude is equal to the angle of the location relative to the equator; North is positive.

δ, declination is equal to the angular position of the sun at solar noon with respect to the equatorial plane (varies from -23.45 to 23.45 degrees).

α, solar altitude is equal to the angle between the sun's rays and the horizontal (between 0 and 90 degrees).

z, zenith angle is equal to the angle between the sun's rays and the vertical.

ϕ, solar azimuth is equal to the angle between the horizontal projection of the sun's rays from due south (positive in the afternoon).

γ, surface solar azimuth is equal to the angle between the projections of the sun's rays and of the normal to the surface on the horizontal plane.

ψ, surface azimuth is equal to the angle between the projection of the normal to the surface on a horizontal plane and due south (east is negative).

β, tilt (slope) angle between the surface and the horizontal (0 - 180 degrees).

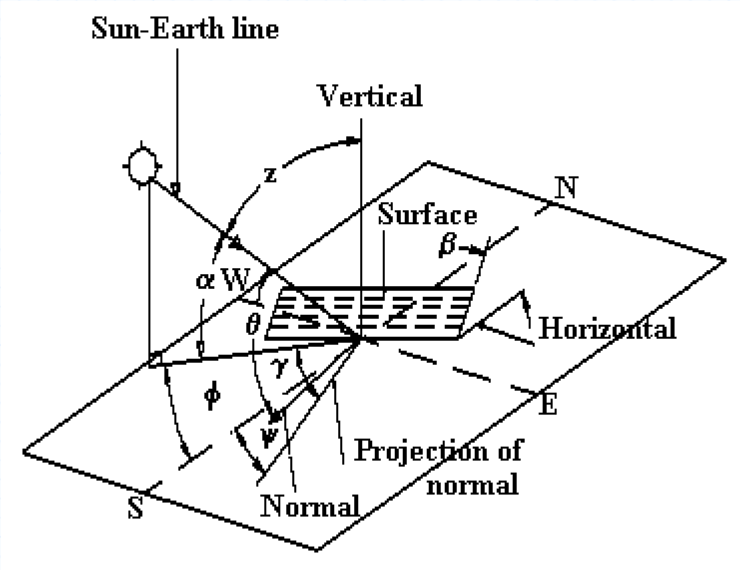
θ, the angle of incidence is the angle between the solar rays and a line normal to the surface.

The position of the sun may be expressed as a function of solar altitude and the solar azimuth as shown in the figure below. These angles are a function of the local latitude L and the solar declination δ, which is a function of the date and the apparent solar time (AST) expressed as the hour angle h:

h = 0.25·(number of minutes from local solar noon) degrees. (h is positive in the afternoon.)

The declination angle is given by

$$\delta = 23.45 \cdot deg \cdot \sin\left(360 \cdot \frac{284+n}{365} \cdot deg\right)$$



z	zenith angle	θ	incidence angle
α	solar altitude	ϕ	solar azimuth
β	tilt angle	ψ	surface azimuth angle
γ	surface solar azimuth		

$$\alpha = asin\left(cos(L) \cdot cos(\delta) \cdot cos(h) + sin(L) \cdot sin(\delta)\right)$$

$$\phi = acos\left(\frac{sin(\alpha) \cdot sin(L) - sin(\delta)}{cos(\alpha) \cdot cos(L)}\right) \cdot \frac{h}{|h|}$$

As can be seen from the diagram:

$$z = 90 \cdot deg - \alpha \cdot deg$$

$$\gamma = \phi - \psi$$ (note that ψ is negative and ϕ positive in the sketch)

$$\theta = acos\left(cos(\alpha) \cdot cos(|\gamma|) \cdot sin(\beta) + sin(\alpha) \cdot cos(\beta)\right)$$ angle of incidence (if γ is greater than 90 deg or less than 270 deg, then γ=0)

Example: Determine the local solar azimuth and altitude at 8:30 Central Time on October 23 at 32 deg North latitude and 95 deg West longitude. Also, determine the incidence angle for a vertical surface facing southeast:

$n := 273 + 23$ $\qquad$ $LST := 8.5\ hr$

$LSM := 90\ deg$ $\qquad$ $LON := 95\ deg$

$ET(n) = 14.497\ min$ $\qquad$ $L := 32\ deg$

$$AST := LST + ET(n) + 4 \cdot \frac{min}{deg} \cdot (LSM - LON)$$

$AST = 8.408\ hr$

Hour Angle:

$$h := (AST - 12\ hr) \cdot \left(15\ \frac{deg}{hr}\right) = -53.876\ deg$$

$$\delta := 23.45 \cdot deg \cdot \sin\left(360 \cdot \frac{284 + n}{365} \cdot deg\right) = -12.446\ deg$$

$$\alpha := \operatorname{asin}(\cos(L) \cdot \cos(\delta) \cdot \cos(h) + \sin(L) \cdot \sin(\delta)) = 21.963\ deg$$

$$\phi := \operatorname{acos}\left(\frac{\sin(\alpha) \cdot \sin(L) - \sin(\delta)}{\cos(\alpha) \cdot \cos(L)}\right) \cdot \frac{h}{|h|}$$

$\phi = -58.264\ deg$

Note that the solar azimuth is negative because it is east of south.

For a vertical surface facing southeast,

$\beta := 90\ deg$ $\qquad$ $\psi := -45\ deg$ $\qquad$ . . . (east is negative)

$$\gamma := \phi - \psi = -13.264\ deg$$

$$\theta := \operatorname{acos}(\cos(\alpha) \cdot \cos(|\gamma|) \cdot \sin(\beta) + \sin(\alpha) \cdot \cos(\beta)) = 25.487\ deg$$

Estimation of clear sky radiation: A convenient method developed by Hottel (1976) is used here to determine the beam radiation transmitted through a clear atmosphere. The atmospheric transmittance for beam radiation is

$$\tau_b = a_o + a_1 \cdot exp\left(\frac{-k}{cos(z)}\right)$$

where the constants a, k depend on climate and altitude A (km)

$$a_o = r_o \cdot \left(0.4237 - 0.00821 \cdot (6 - A)^2\right)$$

$$a_1 = r_1 \left(0.5055 + 0.00595\ (6.5 - A)^2\right)$$

$$k = r_k \left(0.2711 + 0.01858\ (2.5 - A)^2\right)$$

CLIMATE	ro	r1	k
Tropical	0.95	0.98	1.02
Midlatitudes (summer)	0.97	0.99	1.02
Subarctic	0.99	0.99	1.01
Midlatitudes (winter)	1.03	1.01	1.00

For midlatitudes (winter) and an altitude of 0.5 km:

$$A := 0.5$$

$$a_o := 1.03 \cdot \left(0.4237 - 0.00821 \cdot (6 - A)^2\right)$$

$$a_1 := 1.01 \cdot \left(0.5055 + (0.00595 \cdot (6.5 - A))^2\right)$$

$$a_o = 0.181 \qquad a_1 = 0.512$$

$$k := 1.00 \cdot \left(0.2711 + (0.01858 \cdot (2.5 - A))^2\right)$$

$$k = 0.272$$

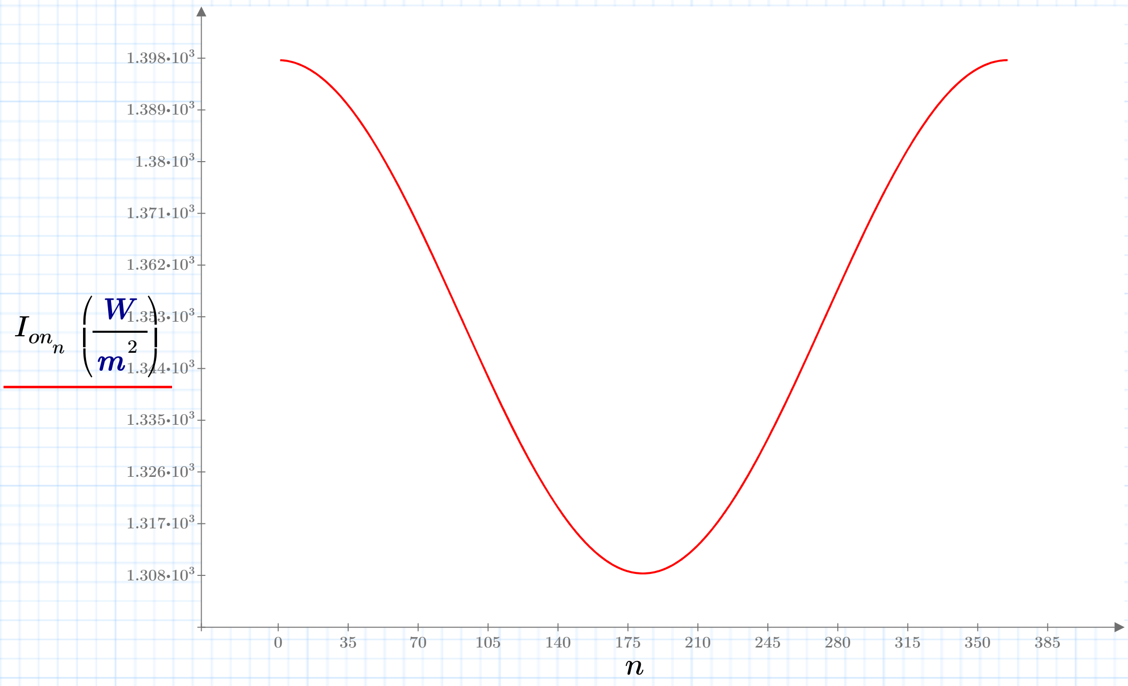
Extraterrestrial solar radiation: The normal solar radiation I_{on} just outside the atmosphere varies due to small changes in the earth-sun distance as follows:

$I_{sc} := 1353 \cdot \dfrac{W}{m^2}$ solar constant

$n := 1, 2 .. 365$ day number

$$I_{on_n} := I_{sc} \cdot \left(1 + 0.033 \cdot \cos\left(360 \cdot \frac{n}{365}\, deg\right)\right)$$

Variation of extraterrestrial solar radiation with time of the year (day number n).



Extraterrestrial solar radiation on horizontal surface (I_{oh}): This is equal to I_{on} times the incidence angle for a horizontal surface, which is equal to the zenith angle z.

$I_{oh} = I_{on_n} \cdot cos(z)$ or $I_{oh} = I_{on_n} \cdot sin(\alpha)$

$$I_{oh} = I_{on_n} \cdot (sin(L) \cdot sin(\delta) + cos(L) \cdot cos(\delta) \cdot cos(h))$$

Normal terrestrial beam radiation In: $I_n = \tau_b \cdot I_{on_n}$

The following correlation developed by Liu and Jordan (1960) is used to determine the clear sky atmospheric diffuse transmittance (for a horizontal plane):

$$\tau_d = 0.2710 - 0.2939 \cdot \tau_b$$

Total solar radiation incident on an inclined surface: The total solar radiation It incident on a surface is given by the sum of the direct (beam) component Ib, the diffuse sky component Ids and the diffuse solar radiation reflected from the ground Idg. The beam direct component Ib is determined as a function of the incidence angle θ:

$$I_b = I_{on} \cdot \tau_b \cdot \cos(\theta)$$

The diffuse sky radiation incident on the surface is equal to the diffuse horizontal solar radiation transmitted by the sky multiplied by the view factor from the surface to the sky (assuming isotropic diffuse distribution):

$$I_{ds} = I_{on_n} \cdot \sin(\alpha) \cdot \tau_d \cdot \frac{1+\cos(\beta)}{2} = I_{on_n} \cdot \sin(\alpha) \cdot (0.2710 - 0.2939 \cdot \tau_b) \cdot \frac{1+\cos(\beta)}{2}$$

Total horizontal radiation: $I_{th} = I_{on_n} \cdot \sin(\alpha) \cdot (\tau_d + \tau_b)$

Ground-reflected solar radiation incident on a surface is equal to the sum of the diffuse sky radiation and beam radiation falling on the ground multiplied by the view factor from the surface to the ground and the ground reflectivity (Table 1).

$$I_{dg} = \left(I_{on_n} \cdot \sin(\alpha) \cdot (\tau_d + \tau_b)\right) \cdot \rho \cdot \frac{1-\cos(\beta)}{2}$$

(Note: more detailed models are available for diffuse radiation)

Table 1: Solar Reflectances of Various Foreground Surfaces

Foreground Surface	New Concrete	Bitumen and Gravel Roof	Bituminous Parking Lot	Bright Green Grass	Crushed Rock
20	0.31	0.14	0.09	0.21	0.2
30	0.31	0.14	0.09	0.22	0.2
40	0.32	0.14	0.1	0.23	0.2
50	0.32	0.14	0.1	0.25	0.2
60	0.33	0.14	0.11	0.28	0.2
70	0.34	0.14	0.12	0.31	0.2

(from Threlkeld ,1970)

The total instantaneous solar radiation incident on the surface is given by $I_t = I_b + I_{ds} + I_{dg}$

Sunrise and sunset times, and total daily solar radiation: The sunrise and sunset times may be determined by evaluating the times at which the incidence angle (or zenith angle) on a horizontal surface is equal to zero.

$h_s = acos(-tan(L) \cdot tan(\delta))$ sunset hour angle

$-h_s$ is the sunrise hour angle (0 is solar noon)

The corresponding times are determined by

$t_s = h_s \cdot \frac{hr}{15\ deg}$ sunset time and

$-t_s$ sunrise time

The total daily extraterrestrial solar radiation Hoh on a horizontal surface may be determined by integration of the instantaneous radiation between sunset and sunrise times:

$H_{oh} = \int_{-t_s}^{t_s} I_{oh}\, \mathrm{d}t$ Other daily totals are similarly determined.

The sunset and sunrise times for an inclined surface may be smaller than the corresponding times for a horizontal surface. For a south-facing surface:

$$t_{ss} = min\left(\left[h_s \;\; acos\left(-tan\left(L-\beta\right)\cdot tan\left(\delta\right)\right)\right]\right)\cdot\frac{hr}{15\ deg}$$

For surfaces in the southern hemisphere replace -β with +β.

Note that the solar radiation incident on a south facing surface is symmetric about solar noon. Therefore, the total daily solar radiation incident on a south-facing surface is given by

$$H_t = 2\cdot\int_0^{t_s} I_t \, dt$$

Solar Collector

Example: Determine the collector tilt angle for which the total daily solar radiation incident on a south-facing collector located at a latitude of 35 deg N is maximum on January 21st.

$$L := 35\ deg \qquad \psi := 0\ deg$$

$$\rho := 0.2 \qquad n := 21$$

$$\delta := 23.45\ deg\cdot\sin\left(360\cdot\frac{284+n}{365}\ deg\right) = -20.138\ deg$$

We will consider tilt angles between 0 and 90 degrees:

$$i := 0, 1 .. 9 \qquad \beta_i := i\cdot 10\ deg$$

$$t_s := \left(\mathrm{acos}\left(-\tan\left(L\right)\cdot\tan\left(\delta\right)\right)\right)\cdot\frac{hr}{15\ deg} = 5.008\ hr$$

Determine the sunset time on the surface:

$$t_{ss_i} := min\left(\begin{bmatrix} t_s \\ \mathrm{acos}\left(-\tan\left(L-\beta_i\right)\cdot\tan\left(\delta\right)\right)\cdot\frac{hr}{15\ deg}\end{bmatrix}\right)$$

In the winter, the sunset time on a horizontal surface is smaller than the sunset time for an inclined surface. Try varying the day number n to see the change (e.g. n = 180) of this parameter.

$$t_{ss_4} = 5.008\ hr$$

$$\Delta t_i := \frac{t_{ss_i}}{8} \qquad j := 0, 1 .. 8$$

$$t_{i,j} := j \cdot \Delta t_i$$ time intervals from noon to sunset

$$h_{i,j} := 15 \cdot \frac{deg}{hr} \cdot t_{i,j}$$ hour angle

Note that the surface receives diffuse radiation throughout the daytime.

$$\alpha_{i,j} := \operatorname{asin}\left(\cos(L) \cdot \cos(\delta) \cdot \cos\left(h_{i,j}\right) + \sin(L) \cdot \sin(\delta)\right)$$

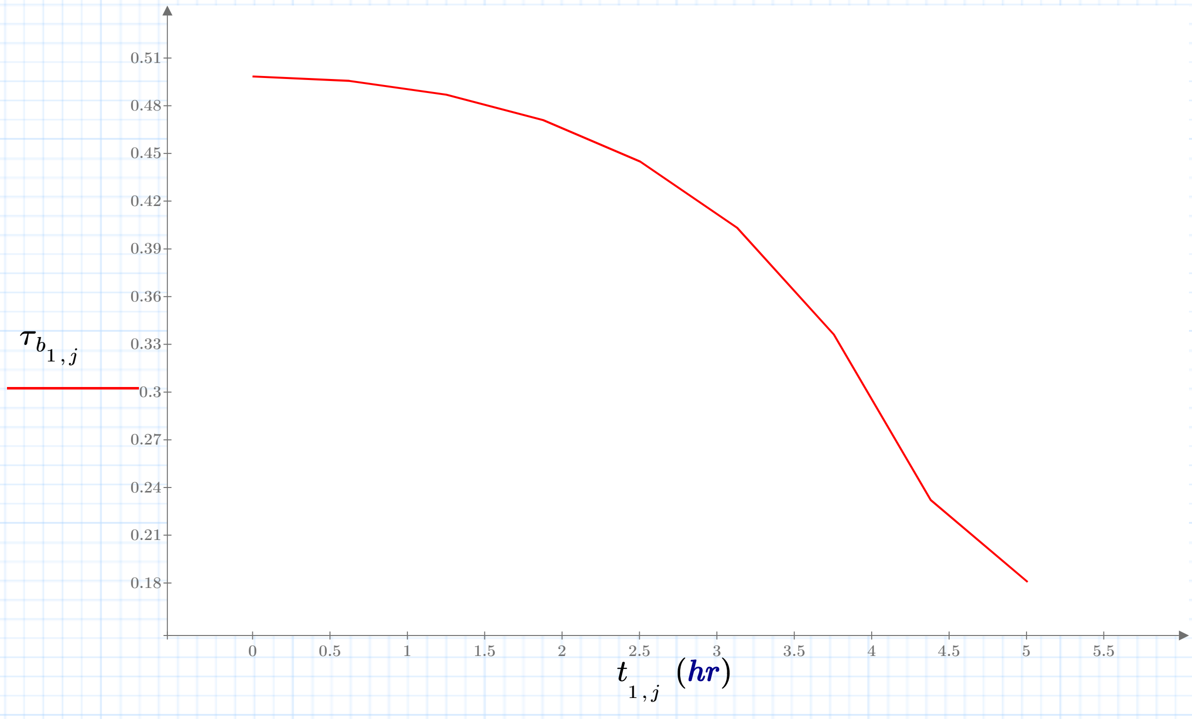
$$\phi_{i,j} := \begin{array}{l} \text{if } h_{i,j} > 0 \\ \quad \text{return } \operatorname{acos}\left(\dfrac{\sin\left(\alpha_{i,j}\right) \cdot \sin(L) - \sin(\delta)}{\cos\left(\alpha_{i,j}\right) \cdot \cos(L)}\right) \cdot \dfrac{h_{i,j}}{\left|h_{i,j}\right|} \\ \text{else} \\ \quad \text{return } 0 \end{array}$$

$$cos\theta_{i,j} := \cos\left(\alpha_{i,j}\right) \cdot \cos\left(\left|\phi_{i,j} - \psi\right|\right) \cdot \sin\left(\beta_i\right) + \sin\left(\alpha_{i,j}\right) \cdot \cos\left(\beta_i\right)$$

$$\theta_{i,j} := \operatorname{acos}\left(\frac{cos\theta_{i,j} + \left|cos\theta_{i,j}\right|}{2}\right)$$ is used to ensure physically possible angles of incidence (0-90 degrees).

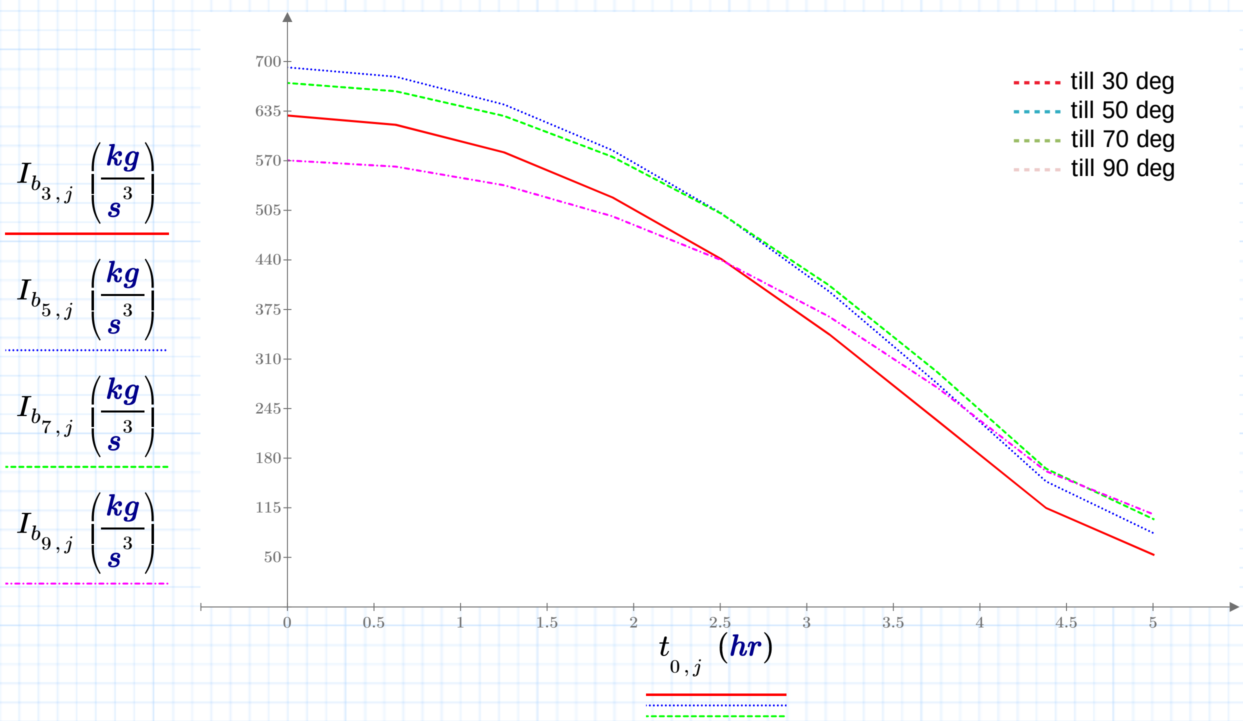
$$\tau_{b_{i,j}} := a_o + a_1 \cdot \exp\left(\frac{-k}{\sin\left(\alpha_{i,j} + 0.0000001\right)}\right)$$

(Note that cos(z) = sin(a) and 0.0000001 was added to avoid overflow.)



$$I_{b_{i,j}} := I_{on_n} \cdot \tau_{b_{i,j}} \cdot \cos\left(\theta_{i,j}\right)$$

Beam solar radiation incident on surfaces of different tilt angle from noon to sunset:



Total daily beam solar radiation:

$$l := 0, 1..7$$

$$H_{b_i} := 2 \cdot \sum_{l} \left(\frac{I_{b_{i,l}} + I_{b_{i,l+1}}}{2} \cdot \Delta t_i \right)$$

$$H_{b_9} = 14313247.66 \ \frac{J}{m^2}$$

daily beam solar radiation on a vertical surface (south-facing) determined by numerical integration of I in the above diagram

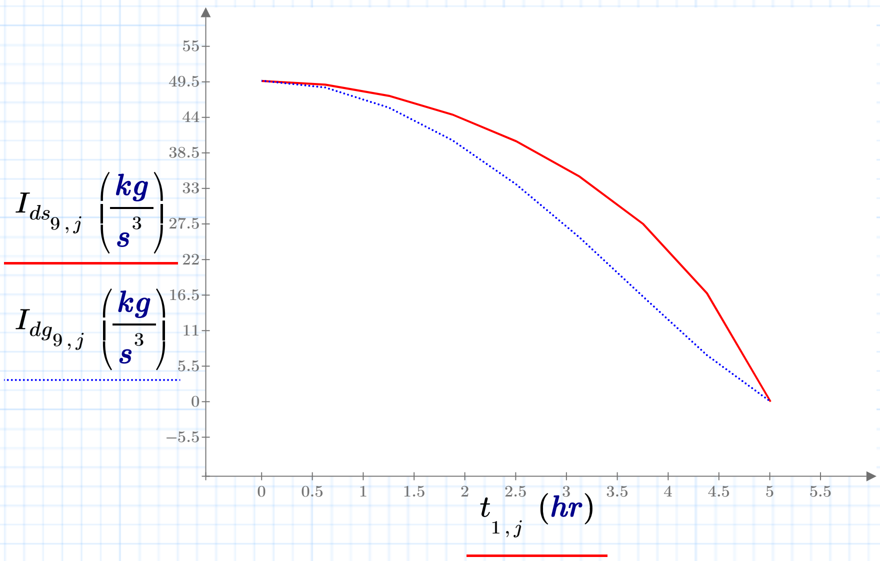
$$I_{ds_{i,j}} := I_{on_n} \cdot \sin\left(\alpha_{i,j}\right) \cdot \left(0.2710 - 0.2939 \cdot \tau_{b_{i,j}}\right) \cdot \frac{1 + \cos\left(\beta_i\right)}{2}$$

$$I_{dg_{i,j}} := \left(I_{on_n} \cdot \sin\left(\alpha_{i,j}\right) \cdot \left(0.2710 - 0.2939 \cdot \tau_{b_{i,j}} + \tau_{b_{i,j}}\right)\right) \cdot \rho \cdot \frac{1 - \cos\left(\beta_i\right)}{2}$$

$$H_{ds_i} := 2 \cdot \sum_{l} \left(\frac{I_{ds_{i,l}} + I_{ds_{i,l+1}}}{2} \cdot \Delta t_i \right)$$

$$H_{dg_i} := 2 \cdot \sum_{l} \left(\frac{I_{dg_{i,l}} + I_{dg_{i,l+1}}}{2} \cdot \Delta t_i \right)$$

Sky diffuse radiation and ground-reflected radiation incident on a vertical south-facing surface:

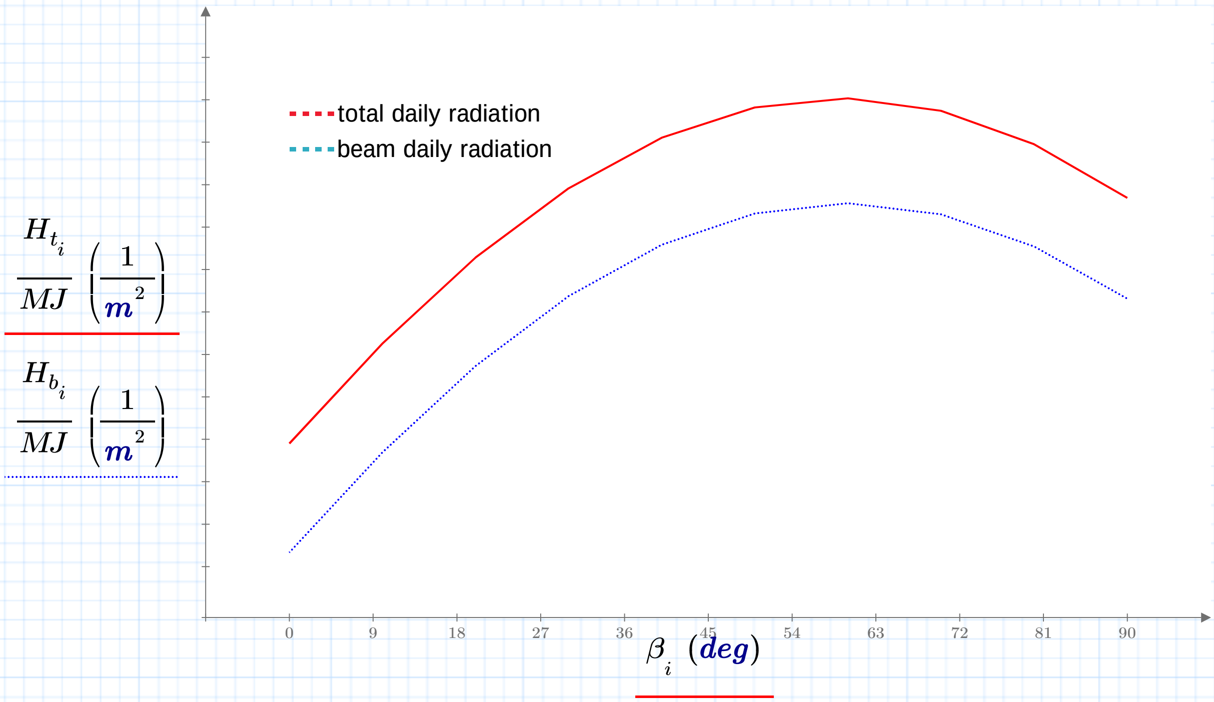


$MJ := J \cdot 10^6$

$H_{t_i} := H_{b_i} + H_{ds_i} + H_{dg_i}$

The graph below shows the daily beam radiation and the total daily radiation (MJ/sq.m.) as a function of tilt angle.

$$\max\left(\frac{H_t}{\frac{MJ}{m^2}}\right) = 19.033$$



Note that the solar radiation is symmetrical about noon in this example because the surface is south-facing.

As we can see, the optimum tilt angle is approximately 60 degrees for this particular date of the year. The optimum tilt angle is higher for winter since the solar altitude decreases the closer we get to December 21st (northern hemisphere).

References

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