CHAPTER 7 SOLAR RADIATION

7.3 Solar Radiation Transmitted through Windows

We have seen how solar radiation incident on an inclined surface is calculated and how window transmittance is determined as a function of time. Now, these methods are used together to calculate the instantaneous and daily total solar radiation transmitted through single-glazed and double-glazed windows.

Consider a location with the following data:

$L := 35\ deg$ latitude

$\psi := 0\ deg$ surface azimuth angle

$\beta := 90\ deg$ tilt angle

$n := 21$ day number

$\rho_g := 0.2$ ground reflectance

First Perform Solar Geometry Calculations:

Declination angle:

$$\delta := 23.45\ deg \cdot \sin\left(360 \cdot \frac{284 + n}{365}\ deg\right) = -20.138\ deg$$

Sunset time:

$$t_s := (\mathrm{acos}(-\tan(L) \cdot \tan(\delta))) \cdot \frac{hr}{15\ deg} = 5.008\ hr$$

Determine the sunset time on the surface:

$$t_{ss} := min\left(\left[t_s \quad \mathrm{acos}(-\tan(L - \beta) \cdot \tan(\delta)) \cdot \frac{hr}{15\ deg}\right]\right)$$

Select time interval for calculations: $t_{ss} = 5.008\ hr$ $\quad N := 8 \quad \Delta t := \frac{t_{ss}}{N}$

Time array from noon to sunset and hour angle h:

$$j := 0, 1 .. N \qquad t_j := j \cdot (\Delta t - 1 \cdot min) \qquad h_j := 15\ \frac{deg}{hr} \cdot t_j$$

Solar altitude:

$$\alpha_j := \text{asin}\left(\cos(L) \cdot \cos(\delta) \cdot \cos\left(h_j\right) + \sin(L) \cdot \sin(\delta)\right)$$

Solar azimuth:

$$\phi_j := \begin{cases} \text{if } h_j > 0 \\ \quad \text{return acos}\left(\dfrac{\sin\left(\alpha_j\right) \cdot \sin(L) - \sin(\delta)}{\cos\left(\alpha_j\right) \cdot \cos(L)}\right) \cdot \dfrac{h_j}{\left|h_j\right|} \\ \text{else} \\ \quad \text{return } 0 \end{cases}$$

Angle of incidence:

$$cos\theta_j := \cos\left(\alpha_j\right) \cdot \cos\left(\left|\phi_j - \psi\right|\right) \cdot \sin(\beta) + \sin\left(\alpha_j\right) \cdot \cos(\beta)$$

$$\theta_j := \text{acos}\left(\frac{cos\theta_j + \left|cos\theta_j\right|}{2}\right)$$

Calculate Transmittance of Atmosphere and Glazing:

Beam atmospheric transmittance calculations:

$A := 0.5$ altitude (km)

$$a_o := 1.03 \cdot \left(0.4237 - 0.00821 \cdot (6 - A)^2\right)$$

$$a_1 := 1.01 \cdot \left(0.5055 + (0.00595 \cdot (6.5 - A))^2\right)$$

$$k := 1.0 \cdot \left(0.2711 + (0.01858 \cdot (2.5 - A))^2\right)$$

$$\tau_{b_j} := a_o + a_1 \cdot \exp\left(\frac{-k}{\sin\left(\alpha_j\right)}\right)$$

$$\tau_{b_N} := 0.0$$

Determine the glazing properties as a function of time interval j:

Glass properties:

$kL := 0.03$ extinction coeff. times glazing thickness

$n_g := 1.53$ refractive index

Angle of refraction and component reflectivity:

$$\theta'_j := \operatorname{asin}\left(\frac{\sin\left(\theta_j\right)}{n_g}\right)$$

$$r_j := \frac{1}{2} \cdot \left(\left(\frac{\sin\left(\theta_j - \theta'_j\right)}{\sin\left(\theta_j + \theta'_j\right)}\right)^2 + \left(\frac{\tan\left(\theta_j - \theta'_j\right)}{\tan\left(\theta_j + \theta'_j\right)}\right)^2\right)$$

Beam transmittance, τ, reflectance, ρ, and absorptance, α, of glazing:

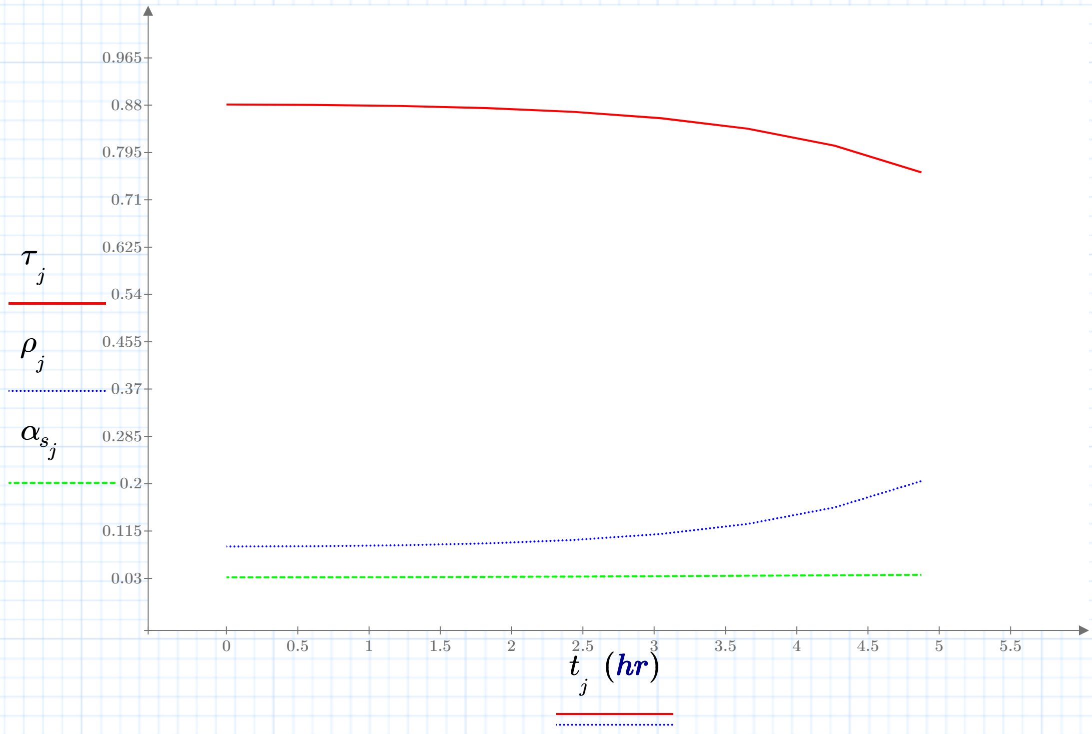
$$a_j := \exp\left(-\frac{kL}{\sqrt{1 - \left(\frac{\sin\left(\theta_j\right)}{n_g}\right)^2}}\right)$$

$$\tau_j := \frac{\left(1 - r_j\right)^2 \cdot a_j}{1 - \left(r_j\right)^2 \cdot \left(a_j\right)^2}$$

$$r_j \cdot \left(1 - r_j\right)^2 \cdot \left(a_j\right)^2$$

$$\rho_j := r_j + \frac{\;_j\,(\quad\;_j)\quad(\;_j)}{1-\left(r_j\right)^2\cdot\left(a_j\right)^2} \qquad\qquad \alpha_{s_j} := 1-\rho_j-\tau_j$$

Solar properties of single glazing:



$$\theta = \begin{bmatrix} 34.862 \\ 35.542 \\ 37.5 \\ 40.533 \\ 44.392 \\ 48.844 \\ 53.693 \\ 58.785 \\ 64 \end{bmatrix} \boldsymbol{deg}$$

Calculate Transmitted Solar Radiation (G) for Single Glazing:

Extraterrestrial normal solar radiation:

$$I_{on_n} := 1353 \frac{W}{m^2} \cdot \left(1 + 0.033 \cdot \cos\left(360 \cdot \frac{n}{365}\,deg\right)\right)$$

Determine beam transmitted solar radiation:

$$I_{b_j} := \left(I_{on_n} \cdot \tau_{b_j} \cdot \cos\left(\theta_j\right)\right) \qquad G_{b_j} := I_{b_j} \cdot \tau_j$$

Determine daily total beam transmitted radiation:

$$MJ := 10^6 \cdot J \qquad l := 0, 1 .. N-1$$

$$Q_{sb} := 2 \cdot \sum_l \left(\frac{G_{b_l} + G_{b_{l+1}}}{2} \cdot \Delta t\right) = 12.434\ MJ \cdot m^{-2}$$

In calculating how much diffuse solar radiation is transmitted, it is assumed that the window diffuse transmittance is equal to the beam transmittance at an angle of incidence equal to 60 degrees. Therefore, in this case:

$$\tau_d := \tau_7$$

$$I_{ds_j} := I_{on_n} \cdot \sin\left(\alpha_j\right) \cdot \left(0.2710 - 0.2939 \cdot \tau_{b_j}\right) \cdot \frac{1 + \cos(\beta)}{2}$$ instantaneous sky diffuse radiation

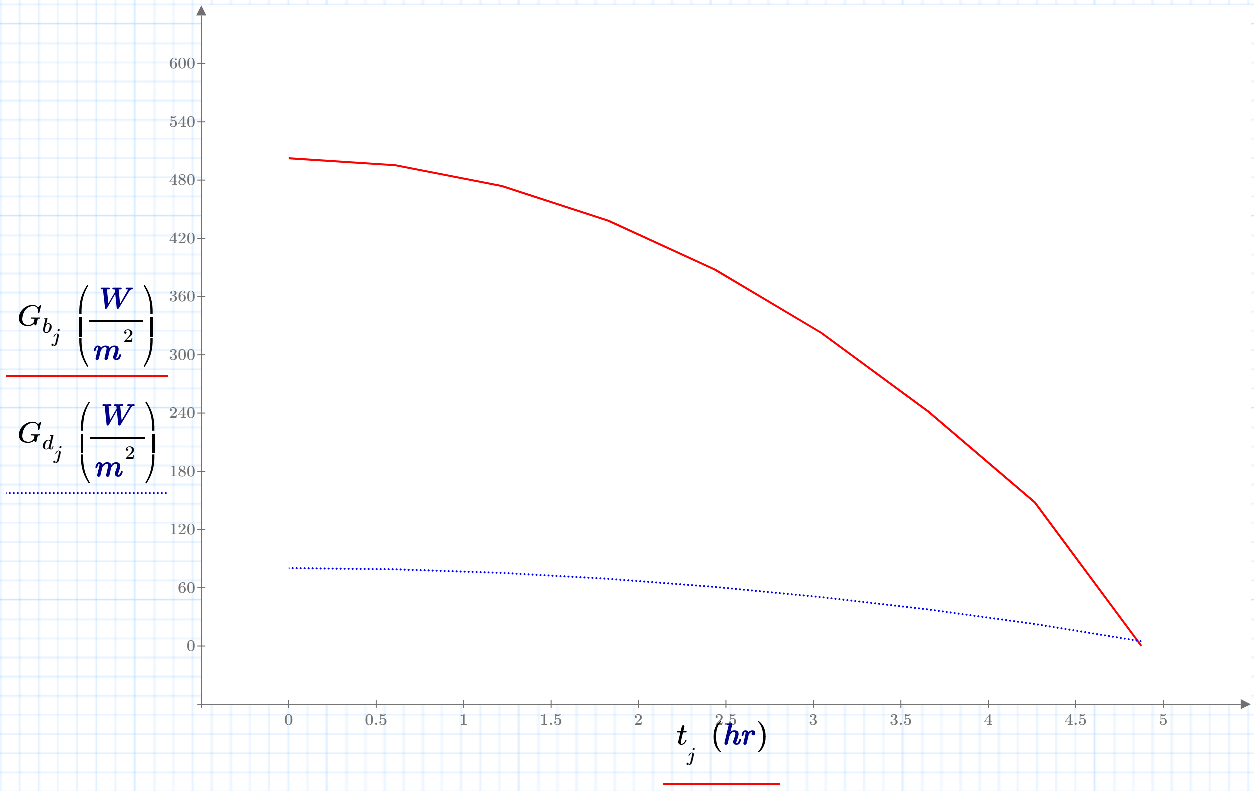
$$I_{dg_j} := \left(I_{on_n} \cdot \sin\left(\alpha_j\right) \cdot \left(0.2710 - 0.2939 \cdot \tau_{b_j} + \tau_{b_j}\right)\right) \cdot \rho_g \cdot \frac{1 - \cos(\beta)}{2}$$ ground reflected

$$G_{d_j} := \tau_d \cdot \left(I_{ds_j} + I_{dg_j}\right)$$ transmitted diffuse irradiation (instantaneous)

Daily diffuse transmitted solar radiation:

$$Q_{sd} := 2 \cdot \sum_l \left(\frac{G_{d_l} + G_{d_{l+1}}}{2} \cdot \Delta t\right) = 1.969\ MJ \cdot m^{-2}$$

Instantaneous beam and diffuse transmitted solar radiation for single-glazing:



Daily total transmitted solar radiation:

$$Q_s := Q_{sb} + Q_{sd} \qquad Q_s = 14.403\ MJ \cdot m^{-2}$$

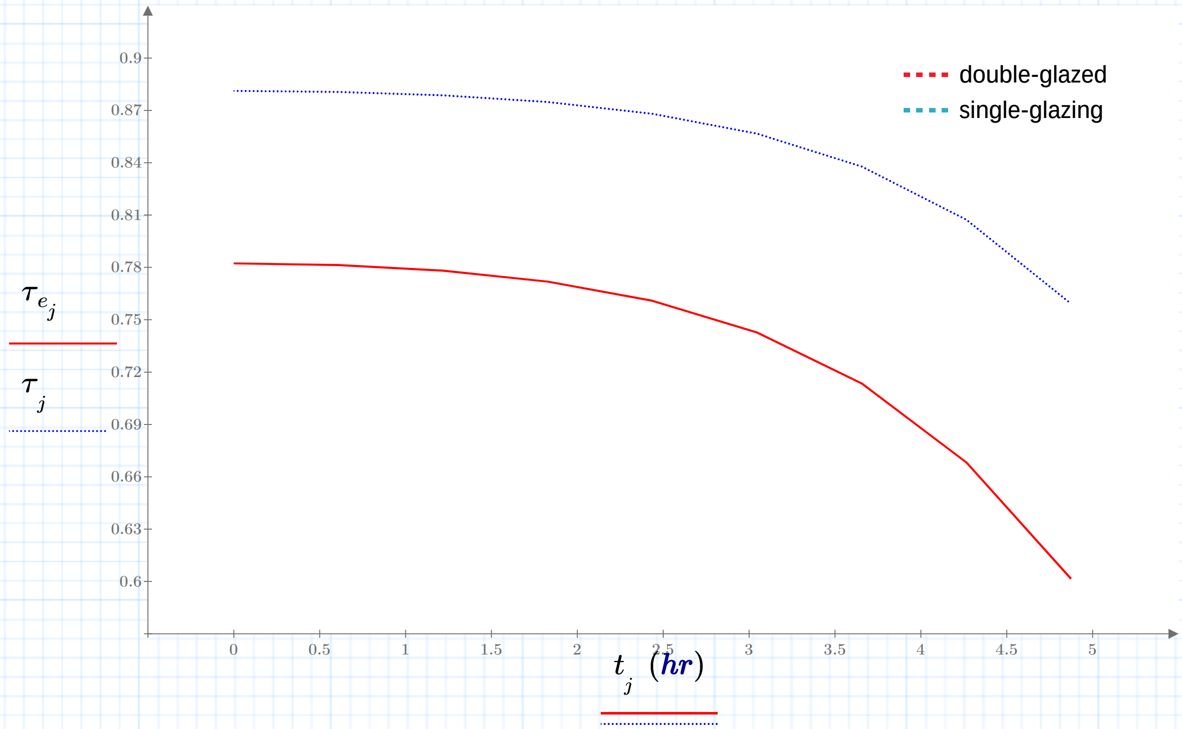
Calculate Transmitted Solar Radiation for a Double-Glazed Window:

Example: Consider a window consisting of two glazings each identical to the one considered above. Determine the effective transmittance of the window and the effective absorptance of the indoor and outdoor glazings as a function of time t_j.

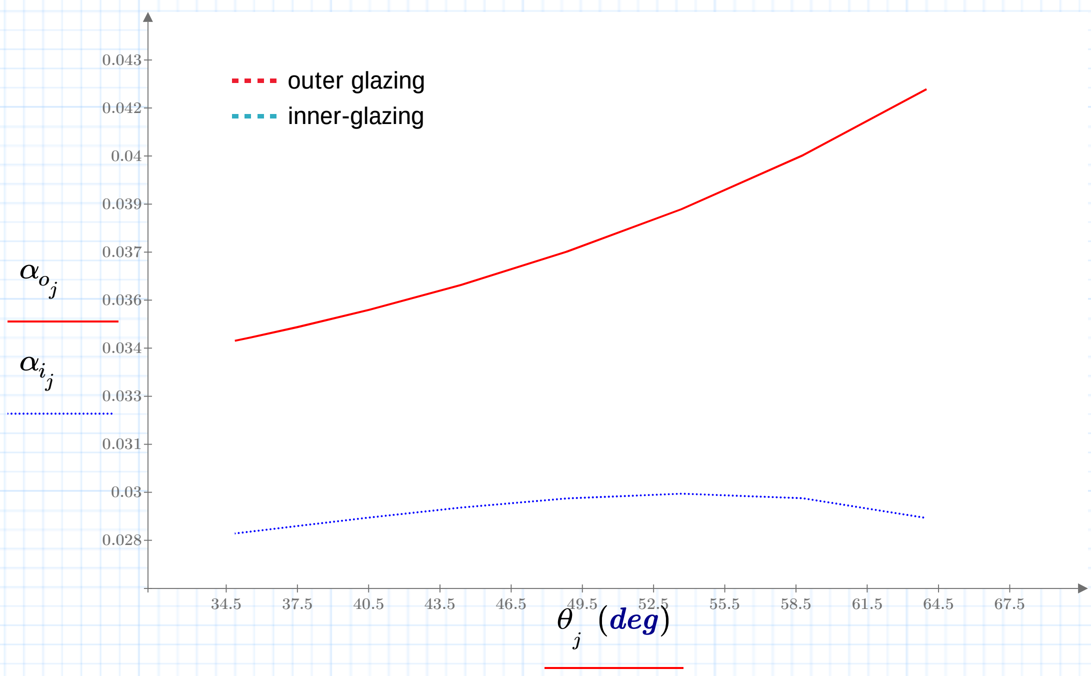
$$\tau_{e_j} := \frac{\left(\tau_j\right)^2}{1-\left(\rho_j\right)^2} \qquad \alpha_{i_j} := \alpha_{s_j} \cdot \frac{\tau_j}{1-\left(\rho_j\right)^2}$$

$$\alpha_{o_j} := \alpha_{s_j} + \alpha_{s_j} \cdot \frac{\tau_j \cdot \rho_j}{1-\left(\rho_j\right)^2}$$

Transmittance as a function of time:



Absorptance as a function of angle:



Determine beam transmitted solar radiation:

$$G_{2b_j} := I_{on_n} \cdot \tau_{b_j} \cdot \cos\left(\theta_j\right) \cdot \tau_{e_j}$$

Determine daily total beam transmitted radiation:

$$l := 0, 1 .. N - 1$$

$$Q_{2sb} := 2 \cdot \sum_{l} \left(\frac{G_{2b_l} + G_{2b_{l+1}}}{2} \cdot \Delta t \right) = 10.894 \, MJ \cdot m^{-2}$$

Diffuse transmittance for double-glazing:

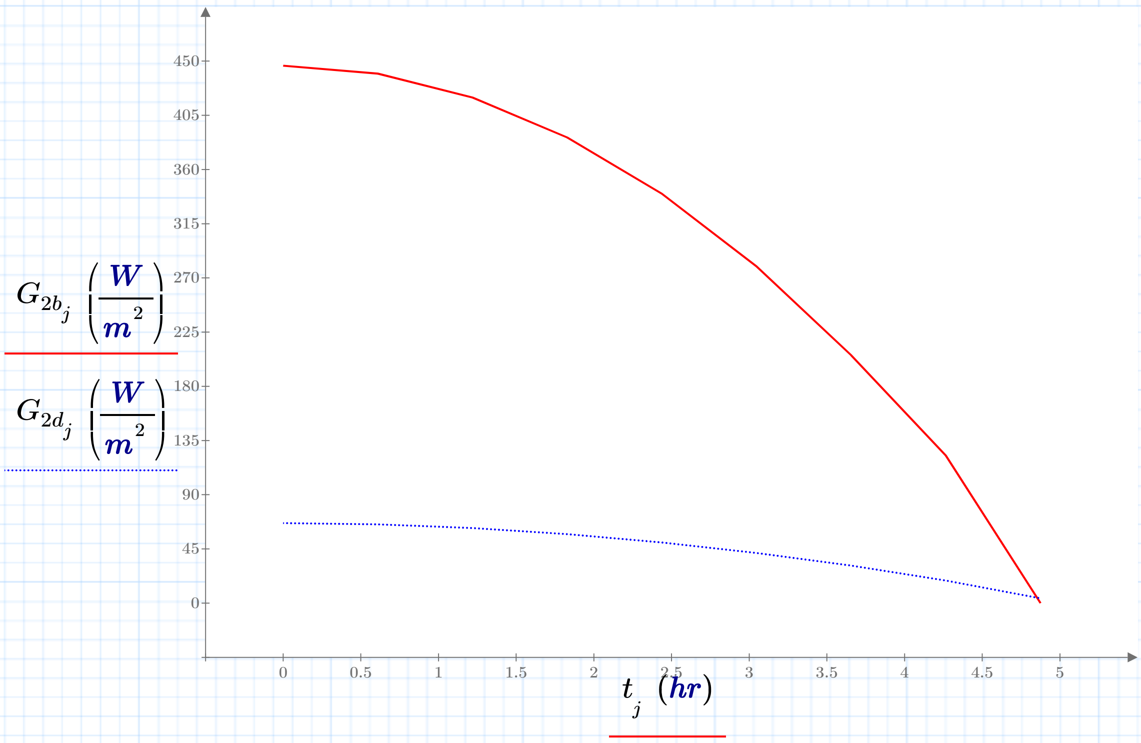
$$\tau_{ed} := \tau_{e_7}$$

$$G_{2d_j} := \tau_{ed} \cdot \left(I_{ds_j} + I_{dg_j} \right)$$ transmitted diffuse irradiation (instantaneous)

Daily diffuse transmitted solar radiation:

$$Q_{2sd} := 2 \cdot \sum_{l} \left(\frac{G_{2d_l} + G_{2d_{l+1}}}{2} \cdot \Delta t \right) = 1.63 \, MJ \cdot m^{-2}$$

Instantaneous beam and diffuse transmitted solar radiation for double-glazing.



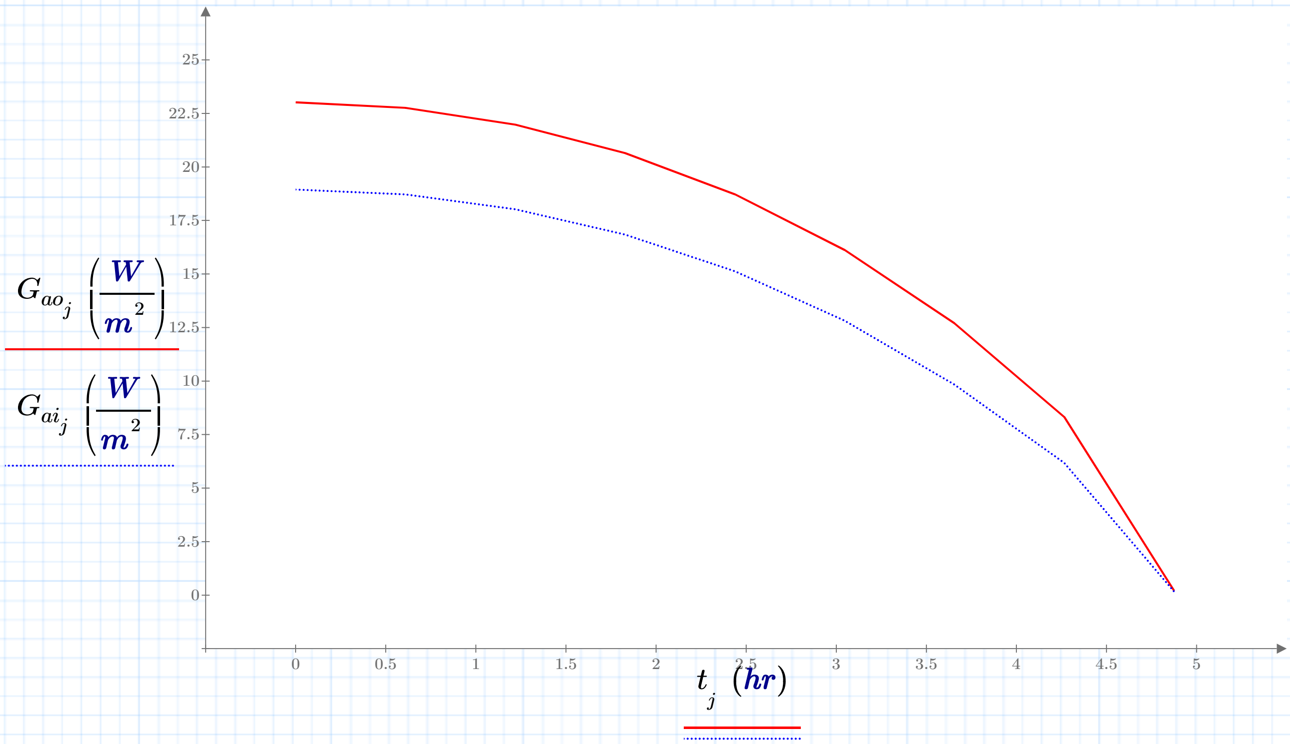
Daily total transmitted solar radiation:

$$Q_{2s} := Q_{2sb} + Q_{2sd} = 12.523\ MJ \cdot m^{-2}$$

Radiation absorbed in outer and inner glazing:

$$G_{ao_j} := \alpha_{o_j} \cdot I_{b_j} + \alpha_{o_3} \cdot \left(I_{ds_j} + I_{dg_j}\right) \qquad G_{ai_j} := \alpha_{i_j} \cdot I_{b_j} + \alpha_{i_3} \cdot \left(I_{ds_j} + I_{dg_j}\right)$$

Solar irradiation absorbed (watts/square meter):



$$Q_{ao} := 2 \cdot \sum_{l} \left(\frac{G_{ao_l} + G_{ao_{l+1}}}{2} \cdot \Delta t \right) = 0.599\ MJ \cdot m^{-2}$$

total daily amounts absorbed in each glazing

$$Q_{ai} := 2 \cdot \sum_{l} \left(\frac{G_{ai_l} + G_{ai_{l+1}}}{2} \cdot \Delta t \right) = 0.483\ MJ \cdot m^{-2}$$

Note: For nonsouth facing surfaces, the solar irradiation is not symmetric about noon. Therefore, it must be determined and integrated from sunrise to sunset.