CHAPTER 1: Analysis of Beams

1.2 Beams with Uniform Load and End Moments

Description

This application computes the maximum positive bending moment, the maximum deflection, and the points of inflection for a beam with a uniformly distributed load and applied end moments.

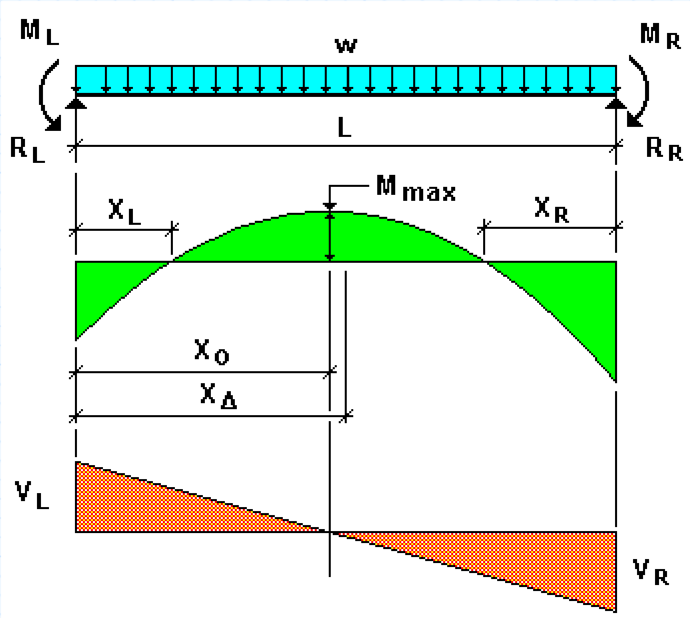
The values calculated include the location of the point of zero shear, the maximum positive bending moment, the rotations at each end of the beam, and the location of the point of zero slope. These computations are made within the **Calculations** section of this document which begins on page 4.

The user must enter the span length, the uniformly distributed load per unit length, the end moments, the modulus of elasticity, and the moment of inertia for the beam.

A summary of input and calculated values is shown on pages 7 and 8. Plots of the moment versus distance across the span and shear versus distance across the span are shown on page 5.

Input

Notation



Input Variables

Span length:	$L := 24 \cdot ft$
Right end moment:	$M_R := 125 \cdot kip \cdot ft$
Uniformly distributed load per unit length:	$w := 2.16 \cdot \frac{kip}{ft}$
Moment of inertia:	$I := 6987 \cdot in^4$
Left end moment:	$M_L := 52 \cdot kip \cdot ft$
Modulus of elasticity:	$E := 3600 \cdot ksi$

Computed Variables

The following variables are computed in this document:

R_L reaction at the left end of the beam

R_R reaction at the right end of the beam

X_o distance from the left end to the point of zero shear and maximum positive bending moment

M_{max} maximum positive bending moment

θ_L slope at the left end of the beam

θ_R slope at the right end of the beam

X_Δ distance from the left reaction to the point of maximum deflection

Δ_{max} maximum deflection

X_L distance from the left end to the nearest point of inflection

X_R distance from right end to the nearest point of inflection

Calculations

Left end reaction: $R_L := \frac{w \cdot L}{2} + \left(\frac{M_L - M_R}{L}\right)$ $R_L = 22.878\ kip$

Right end reaction: $R_R := \frac{w \cdot L}{2} + \left(\frac{M_R - M_L}{L}\right)$ $R_R = 28.962\ kip$

Location of the point of zero shear from the left end: $X_o := \frac{R_L}{w}$ $X_o = 10.592\ ft$

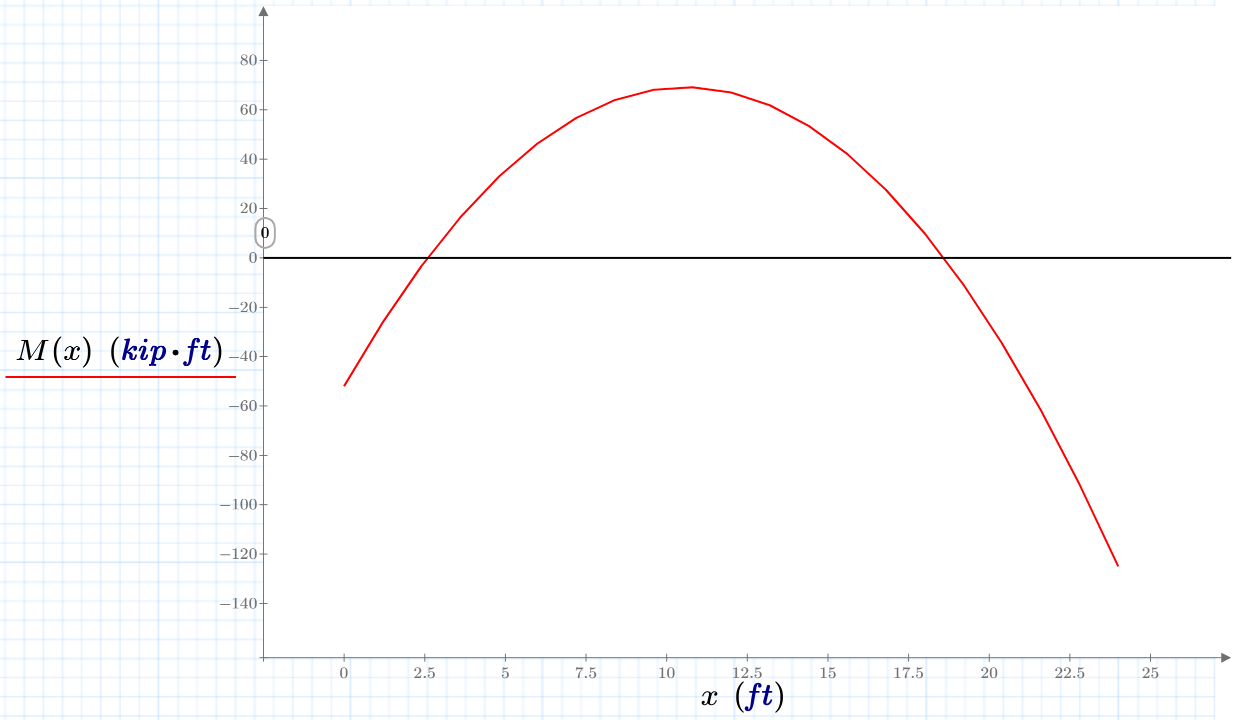
Shear as a function of distance x from the left end: $V(x) := R_L - w \cdot x$

Moment as a function of distance x from the left end: $M(x) := -M_L + R_L \cdot x - \frac{1}{2} \cdot w \cdot x^2$

Maximum positive (or least negative) moment at distance x from the left end: $M_{max} := M(X_o)$ $M_{max} = 69.162\ kip \cdot ft$

Plot of Moment M(x) versus x for N Points Across the Span

$N := 20$ $\qquad x := 0 \cdot ft, \frac{L}{N} .. L$



Plot of Shear V(x) versus x for N Points Across the Span

The following computation locates the distance X_L from the left end to the nearest point of inflection:

Guess value of X_L:

$$X_L := 0 \cdot ft$$

$$X_L := \mathbf{root}\left(M\left(X_L\right), X_L\right)$$

$$X_L = 2.589\ ft$$

The following computation locates the distance X_R from the right end to the nearest point of inflection:

Guess value of X_R:

$$X_R := L$$

$$X_R := L - \mathbf{root}\left(M\left(X_R\right), X_R\right)$$

$$X_R = 5.406\ ft$$

Beam rotation θ_L at left end:

$$\theta_L := \frac{w \cdot L^3 - 8 \cdot M_L \cdot L - 4 \cdot M_R \cdot L}{24 \cdot E \cdot I} \qquad \theta_L = 0.0019$$

Beam rotation θ_R at right end:

$$\theta_R := \frac{w \cdot L^3 - 8 \cdot M_R \cdot L - 4 \cdot M_L \cdot L}{24 \cdot E \cdot I} \qquad \theta_R = 0.0002$$

Slope $\theta(x)$ along the length of the beam expressed as a function of distance x from the left end:

$$\theta(x) := \theta_L - \frac{1}{E \cdot I} \cdot \left(\left(\frac{w \cdot L \cdot x^2}{4} - \frac{w \cdot x^3}{4}\right) + \frac{w \cdot x^3}{12} - \frac{M_L \cdot x}{2} - \left(\frac{L - x}{L}\right) \cdot \frac{M_L \cdot x}{2} - \frac{M_R \cdot x^2}{2 \cdot L}\right)$$

Distance X_Δ from the left reaction to the point of zero slope and maximum deflection:

Guess value of X_Δ:

$$X_\Delta := \frac{L}{2}$$

$$X_\Delta := \mathbf{root}\left(\theta\left(X_\Delta\right), X_\Delta\right)$$

$$X_\Delta = 10.93\ ft$$

Beam deflection δ(x) expressed as a function of distance x from the left end reaction:

$$\delta(x) := \theta_L \cdot x - \frac{1}{E \cdot I} \cdot \left(\left(\frac{w \cdot L \cdot x^2}{4} - \frac{w \cdot x^3}{4}\right) \cdot \frac{x}{3} + \frac{w \cdot x^4}{24} - \frac{M_L \cdot x^2}{3} - \left(\frac{L - x}{L}\right) \cdot \frac{M_L \cdot x^2}{6} - \frac{M_R \cdot x^3}{6 \cdot L}\right)$$

Maximum deflection Δ_{max} at distance X_Δ from left end reaction:

$$X_\Delta = 10.93\ ft$$

$$\Delta_{max} := \delta\left(X_\Delta\right)$$

$$\Delta_{max} = 0.206\ in$$

Summary

Input

Span length: $L = 24\ ft$

Left end moment: $M_L = 52\ kip \cdot ft$

Right end moment: $M_R = 125\ kip \cdot ft$

Modulus of elasticity: $E = 3600\ ksi$

Uniformly distributed load per unit length: $w = 2.16\ \dfrac{kip}{ft}$

Moment of inertia: $I = 6987\ in^4$

Computed Variables

Left end reaction: $R_L = 22.878\ kip$

Right end reaction: $R_R = 28.962\ kip$

Distance from the left end to the point of zero shear and maximum positive moment: $X_o = 10.592\ ft$

Maximum positive moment: $M_{max} = 69.162\ kip \cdot ft$

Slope at the left end of the beam: $\theta_L = 1.879 \cdot 10^{-3}$

Slope at the right end of the beam: $\theta_R = 2.07 \cdot 10^{-4}$

Distance from the left end to the point of maximum deflection: $X_\Delta = 10.93\ ft$

Maximum deflection: $\Delta_{max} = 0.206\ in$

Distance from the left end to the closer point of inflection: $X_L = 2.589\ ft$

Distance from the right end to the closer point of inflection: $X_R = 5.406\ ft$