CHAPTER 4: Reinforced Concrete Columns

4.3 Moment Magnification

Description

An ordinary or first order frame analysis does not include either the effects of the lateral sidesway deflections of the column ends, or the effects of the deflections of members along their lengths on the axial loads and moments in a frame. The effects of the deflections of the column ends can be evaluated directly by performing a second order analysis or by using approximate methods.

This application uses the procedure of Section 10.11 of ACI 318 (Approximate evaluation of slenderness effects) to compute moment magnifiers for a square or rectangular column, either subject to sidesway, or braced to prevent appreciable sidesway. Moments computed by an ordinary first order frame analysis are then multiplied by the applicable moment magnifier to obtain the moments to be used for design.

The variables computed by this application include minimum required eccentricity, moment modifier factor, gross moment of inertia, radius of gyration, flexural stiffness, critical column load, moment magnification factors, and the factored moments to be used for design of the compression member under consideration.

The user must enter the strengths of the concrete and the reinforcement, the unit weight of concrete, the summation of factored gravity loads and the summation of critical loads for all columns on the story under consideration for frames that are subject to sidesway, the ratio β_d due to lateral loads (β_{d_s} defined below) and gravity loads (β_{d_g} defined below), factored axial load and end moments, unsupported height and effective length factor, column dimensions and reinforcement.

Summation of critical loads and effective length factors may be calculated using **Section 4.2** of this book.

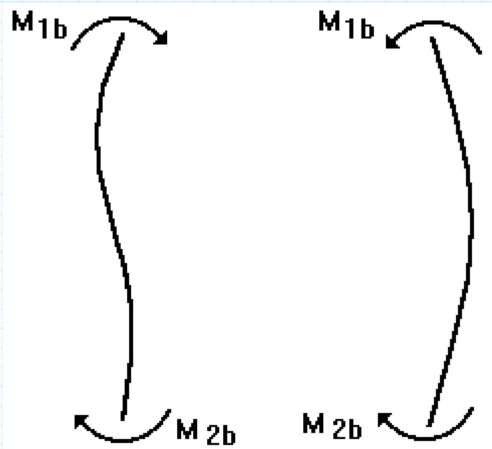
ACI 318 Commentary, R10-11.5 (Moment magnification) provides an excellent summary of this subject.

A summary of input and computed variables is shown on pages 11-15.

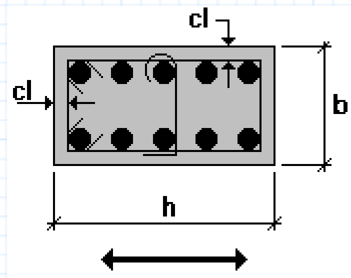
Reference:
ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)

Input

Notation



Double Curvature **Single Curvature**



Direction Under Consideration

Input Variables for Story Under Consideration

If the story is braced to prevent appreciable sidesway enter ΣP_u and ΣP_c as 0 (with units).

Summation of factored gravity loads on the columns of the story under consideration:

$$\Sigma P_u := 5175 \cdot kip$$

Summation of critical loads for all columns on the story, with critical load for each column calculated from Eq. (10-9) of ACI 318:

$$\Sigma P_c := 23303 \cdot kip$$

Ratio of the maximum factored sustained lateral load to the maximum total factored lateral load in that story in the calculation of P_c in Eq. (10-8) of ACI 318:

$$\beta_{d\_s} := 0$$

The β_d factor is 0 for wind or seismic loads, however appreciable sustained lateral load may occur due to an unsymmetrical frame or due to unsymmetrical dead loads.

Input Variables for Column Under Consideration

Ratio of maximum factored axial dead load to maximum total factored axial load, where the load is due to gravity effects only in the calculation of Pc in Eq. (10-7) of ACI 318: $\beta_{d\_g} := 0.735$

Factored axial load at given eccentricity: $P_u := 247\ kip$

Unsupported length of compression member taken as the clear distance between floor slabs, beams or other members capable of providing lateral support: $L_u := 10\ ft$

Effective length factor for compression member subject to appreciable sidesway: $k_s := 2.187$

Effective length factor for compression member not subject to appreciable sidesway: $k_b := 0.927$

Column dimension in direction that slenderness effects are being considered: $h := 24\ in$

Column dimension transverse to the direction that slenderness effects are being considered: $b := 12\ in$

Bar size number: $x := 8$

Number of bars on "h" face: $N_h := 3$

Number of bars on "b" face: $N_b := 2$

Clear cover of longitudinal reinforcement : $cl := 2\ in$

The smaller factored end moment on compression member due to loads that result in no appreciable sidesway, calculated using conventional elastic analysis, positive if bent in single curvature, negative if bent in double curvature: $M_{1b} := -38\ kip \cdot ft$

The larger factored end moment on compression member due to loads that result in no appreciable sidesway (calculated using conventional elastic analysis): $M_{2b} := 52\ kip \cdot ft$

The larger factored end moment on compression member due to loads that result in appreciable sidesway (calculated by $M_{2s} := 25 \cdot kip \cdot ft$

sidesway:

In most practical cases, the M_{1b} and M_{2b} moments are due to gravity loads and the M_{2s} moment is due to wind or seismic lateral loads.

By entering a value of M_{2s} greater than 0 the user signifies that the frame is not braced against appreciable sidesway.

Computed Variables

e	minimum required eccentricity for design (ACI 318, 10.11.5.4)
C_m	moment modifier factor relating actual moment diagram to an equivalent moment diagram (ACI 318, Eq. (10-12))
I_g	moment of inertia of gross concrete section about centroidal axis of member neglecting reinforcement
r	radius of gyration
SR	slenderness ratio, equal to kL_u/r
SR_{max}	maximum slenderness ratio for which slenderness effects may be neglected (ACI 318, 10.11.4.1 and 10.11.4.2)
N_{total}	total number of reinforcing bars
A_{st}	total area of longitudinal reinforcement
ρ	ratio of the reinforcement area to the gross area of concrete
I_{se}	moment of inertia of reinforcement about the centroidal axis of member cross section
M_{min}	minimum required design moment, factored axial load at minimum eccentricity (ACI 318, 10.11.5.4)
EI	flexural stiffness computed by Eqs. (10-10) and (10-11) of ACI 318

P_c	critical column load, Eq. (10-9) ACI 318
δ_b	moment magnification factor for frames braced against sidesway, to reflect effects of member curvature between ends of compression member (ACI 318, Eq. (10-7))
δ_s	moment magnification factor for frames not braced against sidesway, to reflect drift resulting from gravity and lateral loads (ACI 318, Eq. (10-8))
M_c	factored moment to be used for design of compression member

Material Properties and Constants

The user may enter any material strength or unit weight of concrete acceptable under the requirements of ACI 318.

Compressive strength of concrete: $f'_c := 4 \cdot ksi$

Yield strength of reinforcement: $f_y := 60 \cdot ksi$

Unit weight of concrete: $w_c := 145 \cdot pcf$

Modulus of elasticity of reinforcement (ACI 318, 8.5.2): $E_s := 29000 \cdot ksi$

Strength reduction factor (ACI 318, 9.3): $\phi := 0.7$

Columns subject to significant slenderness effects have relatively high axial load to moment ratios and are not likely to qualify for a strength reduction factor higher than 0.7.

Reinforcing bar number designations, diameters and areas:

$$No := [0\ 1\ 2\ 3\ 4\ 5\ 6\ 7\ 8\ 9\ 10\ 11\ 12\ 13\ 14\ 15\ 16\ 17\ 18]^T$$

$$d_b := [0\ 0\ 0\ 0.375\ 0.5\ 0.625\ 0.75\ 0.875\ 1.00\ 1.128\ 1.27\ 1.41\ 0\ 0\ 1.693\ 0\ 0\ 0\ 2.257]^T \cdot in$$

$$A_b := [0\ 0\ 0\ 0.11\ 0.20\ 0.31\ 0.44\ 0.60\ 0.79\ 1.00\ 1.27\ 1.56\ 0\ 0\ 2.25\ 0\ 0\ 0\ 4.00]^T \cdot in^2$$

$A_b := \lfloor 0\ \ 0\ \ 0\ \ 0.11\ \ 0.20\ \ 0.31\ \ 0.44\ \ 0.60\ \ 0.79\ \ 1.00\ \ 1.27\ \ 1.56\ \ 0\ \ 0\ \ 2.25\ \ 0\ \ 0\ \ 0\ \ 4.00 \rfloor \cdot in^2$

Bar numbers, diameters and areas are in the vector rows (columns in the transposed vectors) corresponding to the bar numbers. Individual bar numbers, diameters, and areas of a specific bar can be referred to by using the vector subscripts as shown in the example below.

Example: $No_5 = 5$ $\quad d_{b_5} = 0.625\ in$ $\quad A_{b_5} = 0.31\ in^2$

Modulus of elasticity of column concrete (ACI 318, 8.5.1):

$$E_c := \left(\frac{w_c}{pcf}\right)^{1.5} \cdot 33 \cdot \sqrt{\frac{f'_c}{psi}} \cdot psi \qquad E_c = 3644\ ksi$$

Calculations

The calculations in this section determine the magnified moments to be used to design the column under consideration.

Moment of inertia of gross concrete section about centroidal axis of member neglecting reinforcement:

$I_g := \frac{1}{12} \cdot b \cdot h^3$ $\qquad$ $I_g = 13824\ in^4$

Radius of gyration:

$r := 0.3 \cdot h$ $\qquad$ $r = 7.2\ in$

Slenderness ratio for braced frames (or for loads which do not produce appreciable sidesway):

$SR_b := \frac{k_b \cdot L_u}{r}$ $\qquad$ $SR_b = 15.45$

Maximum slenderness ratio for braced frames (or for loads which do not produce appreciable sidesway) before effects of slenderness must be considered (ACI 318, 10.11.4.1):

$SR_{max\_b} := 34 - 12 \cdot \frac{M_{1b}}{M_{2b}}$ $\qquad$ $SR_{max\_b} = 42.769$

Slenderness ratio for frames subject to appreciable sidesway:

$SR_s := \frac{k_s \cdot L_u}{r}$ $\qquad$ $SR_s = 36.45$

If the larger calculated slenderness ratio is greater than 100, this application is not applicable, and an analysis as defined in ACI 318, 10.10.1 must be made.

Maximum slenderness ratio for frames subject to appreciable sidesway before effects of slenderness must be considered (ACI 318, 10.11.4.2):

$SR_{max\_s} := 22$

Total number of reinforcing bars:

$N_{total} := 2 \cdot \left(N_h + N_b - 2\right)$ $\qquad$ $N_{total} = 6$

Total area of reinforcement:

$$A_{st} := A_{b_x} \cdot N_{total} \qquad A_{st} = 4.74\ in^2$$

Reinforcement ratio:

$$\rho := \frac{A_{st}}{b \cdot h} \qquad \rho = 1.646\%$$

Spacing of reinforcing bars along the "h" face from the centroidal axis:

$$u := N_h - 1 \qquad i := 0..u$$

$$Sp_i := \frac{h - 2 \cdot cl - d_{b_x}}{2 \cdot (N_h - 1)} \cdot i \qquad Sp^T = [0\ \ 4.75\ \ 9.5]\ in$$

Moment of inertia of reinforcement about the centroidal axis of member cross section:

$$I_{se} := \sum \overrightarrow{2 \cdot A_{b_x} \cdot \left(Sp^2 + (N_b - 2) \cdot \left(Sp_u\right)^2\right)} \qquad I_{se} = 178.244\ in^4$$

Minimum required eccentricity for design (ACI 318, 10.11.5.4):

$$e := 0.6\ in + 0.03 \cdot h = 1.32\ in$$

Minimum required design moment:

$$M_{min} := P_u \cdot e = 27.17\ kip \cdot ft$$

If M_{min} is larger than M_{2b} it is substituted for M_{2b} in ACI 318 Eq. (10-6) on page 9 of this document.

Actual eccentricity for smaller end moment:

$$e_1 := \frac{M_{1b}}{P_u} = -1.846\ in$$

Actual eccentricity for larger end moment:

$$e_2 := \frac{M_{2b}}{P_u} = 2.526\ in$$

Ratio of the smaller to the larger end moment for use in ACI 318 Eq. (10-12), not less than 1 if entered end moments are 0:

$$R := \mathbf{if}\left(\left(M_{1b} = 0\ kip \cdot ft\right) \cdot \left(M_{2b} = 0\ kip \cdot ft\right), 1, \frac{M_{1b}}{M_{2b}}\right) = -0.731$$

Sum of the larger end moments before magnification:

$$\Sigma M_2 := M_{2b} + M_{2s} \qquad \Sigma M_2 = 77\ kip \cdot ft$$

Moment modifier factor relating actual moment diagram to an equivalent moment diagram for members braced against sidesway and without transverse loads between supports for use in Eq. (10-12) of ACI 318, not less than 0.4 nor more than 1.0:

$$C_m := 0.6 + \mathbf{if}\left(R < -0.2, -0.2, \mathbf{if}\left(R > 0.4, 0.4, R\right)\right) \qquad C_m = 0.4$$

Note: If there are transverse loads between support the user may define C_m = 1.0 at this point in accordance with ACI 318, 10.11.5.3.

Flexural stiffness of column computed by Eq. (10-11) of ACI 318:

$$EI_{10\_11} := \frac{E_c \cdot I_g}{2.5} \cdot \frac{1}{1 + \beta_{d\_g}}$$

$$EI_{10\_11} = \left(1.161 \cdot 10^7\right)\ kip \cdot in^2$$

Flexural stiffness of column computed by Eq. (10-10) of ACI 318:

$$EI_{10\_10} := \left(\frac{E_c \cdot I_g}{5} + I_{se} \cdot E_s\right) \cdot \frac{1}{1 + \beta_{d\_g}}$$

$$EI_{10\_10} = \left(2.522 \cdot 10^7\right)\ \frac{kg \cdot m^3}{s^2}$$

Flexural stiffness of column computed by Eq. (10-11) of ACI 318:

$$EI_{10\_11} := \left(\frac{E_c \cdot I_g}{2.5}\right) \cdot \frac{1}{1 + \beta_{d\_g}}$$

$$EI_{10\_11} = \left(1.161 \cdot 10^7\right)\ kip \cdot in^2$$

Larger flexural stiffness computed by ACI Eqs. (10-10) or (10-11):

$$EI := \text{if}\left(EI_{10\_10} > EI_{10\_11}, EI_{10\_10}, EI_{10\_11}\right)$$

$$EI = \left(1.161 \cdot 10^{7}\right)\ kip \cdot in^{2}$$

Critical column load for braced frames (or for loads which do not produce appreciable sidesway) (ACI 318, Eq. (10-9)):

$$P_c := \frac{\pi^2 \cdot EI}{\left(k_b \cdot L_u\right)^2} \qquad P_c = 9263\ kip$$

Moment magnification factor for braced frames (or for loads which do not produce appreciable sidesway), to reflect effects of member curvature between ends of compression member (ACI 318, Eq. (10-7)):

$$\delta_b := \text{if}\left(\frac{C_m}{1 - \frac{P_u}{\phi \cdot P_c}} > 1.0, \frac{C_m}{1 - \frac{P_u}{\phi \cdot P_c}}, 1.0\right) \qquad \delta_b = 1$$

Larger factored end moment due to loads not producing sidesway, amplified for slenderness. M_{min} is substituted for M_{2b} when M_{min} is larger than M_{2b} (ACI 318, 10.11.5.1, Eq. (10-6), 10.11.5.4):

$$M_{cb} := \delta_b \cdot \left(\text{if}\left(M_{2b} > M_{min}, M_{2b}, M_{min}\right)\right) \qquad M_{cb} = 52\ kip \cdot ft$$

Moment magnification factor for frames not braced against sidesway, to reflect drift resulting from gravity and lateral loads (ACI 318, Eq. (10-8)):

$$\delta_s := \text{if}\left(\frac{1}{1 - \frac{\Sigma P_u}{\phi \cdot \Sigma P_c}} \ge 1.0, \frac{1}{1 - \frac{\Sigma P_u}{\phi \cdot \Sigma P_c}}, 1.0\right) \qquad \delta_s = 1.465$$

Larger factored end moment due to loads producing sidesway, amplified for slenderness:

$$M_{cs} := \delta_s \cdot M_{2s} \qquad M_{cs} = 36.617\ kip \cdot ft$$

Factored moment to be used for design of compression member.

$M_c := M_{cb} + M_{cs}$ $\qquad$ $M_c = 88.617\ kip \cdot ft$

$M_c := \delta_b \cdot \left(\textbf{if}\left(M_{2b} > M_{min}, M_{2b}, M_{min}\right)\right) + \delta_s \cdot M_{2s}$

$M_c = 88.617\ kip \cdot ft$

Summary

Input

Strength reduction factor:	$\phi = 0.7$
Unit weight of concrete:	$w_c = 145$ ***pcf***
Compressive strength of concrete:	$f'_c = 4$ ***ksi***
Yield strength of reinforcement:	$f_y = 60$ ***ksi***
Summation of factored gravity loads on the story:	$\Sigma P_u = 5175$ ***kip***
Summation of critical loads on the story (story free to sway):	$\Sigma P_c = 23303$ ***kip***
Ratio β_{d_g} for gravity loads:	$\beta_{d\_g} = 0.735$
Ratio β_{d_s} for lateral loads:	$\beta_{d\_s} = 0$
Effective length factor for braced frame:	$k_b = 0.927$
Effective length factor for unbraced frame:	$k_s = 2.187$
Factored axial load at given eccentricity:	$P_u = 247$ ***kip***
Unsupported length of compression member:	$L_u = 10$ ***ft***
Column dimension in direction stability is being considered:	$h = 24$ ***in***
Column dimension transverse to h or 0 inches for round column:	$b = 12$ ***in***
Number of bars on "h" face:	$N_h = 3$
Number of bars on "b" face:	$N_b = 2$
Bar size number:	$x = 8$
Clear cover of reinforcement:	$cl = 2$ ***in***
Smaller factored end moment due to loads that do not result in appreciable sidesway:	$M_{1b} = -38\ kip \cdot ft$
Larger factored end moment due to loads that do not result in appreciable sidesway:	$M_{2b} = 52\ kip \cdot ft$
Larger factored end moment due to loads that result in appreciable sidesway:	$M_{2s} = 25\ kip \cdot ft$

Computed Variables

Minimum required eccentricity:	$e = 1.32\ in$
Moment modifier:	$C_m = 0.4$
Moment of inertia:	$I_g = 13824\ in^4$
Radius of gyration:	$r = 7.2\ in$
Total number of reinforcing bars:	$N_{total} = 6$
Total area of reinforcement:	$A_{st} = 4.74\ in^2$
Reinforcement ratio:	$\rho = 1.646\%$
Moment of inertia of reinforcement:	$I_{se} = 178.244\ in^4$
Flexural stiffness:	$EI = \left(1.16 \cdot 10^7\right)\ kip \cdot in^2$
Critical column load (braced frame):	$P_c = 9263.3\ kip$
Minimum required moment:	$M_{min} = 27.17\ kip \cdot ft$
Slenderness ratio for frame subject to appreciable sidesway:	$SR_s = 36.45$
Maximum slenderness ratio for frame subject to appreciable sidesway for which slenderness effects may be neglected:	$SR_{max\_s} = 22$
Slenderness ratio for braced frame:	$SR_b = 15.5$
Maximum slenderness ratio for braced frame for which slenderness effects may be neglected:	$SR_{max\_b} = 42.8$
Moment magnification factor for braced frames:	$\delta_b = 1$
Moment magnification factor for unbraced frames:	$\delta_s = 1.465$
Larger factored end moment due to loads not producing sidesway, amplified for slenderness:	$M_{cb} = 52\ kip \cdot ft$
Larger factored end moment due to loads producing sidesway, amplified for slenderness:	$M_{cs} = 36.6\ kip \cdot ft$

For the loading case of gravity plus wind load the sum of the amplified moments should be multiplied by a factor of 0.75, as given in ACI 318 Section 9.2.2.