CHAPTER 8: Earth Retaining Structures
8.1 Reinforced Concrete Retaining Walls with Level or Surcharged Backfill

Description

Earth retaining walls are used in construction wherever a separation of grades at different levels is required. Retaining walls must be designed to resist the lateral pressures due to the retained soil, and the additional forces due to any surcharge.

This application determines required footing width, footing thickness, wall thickness and reinforcement areas for retaining walls with a level backfill, with or without a surcharge. Lateral earth pressures and the surcharge may be specified by the user.

Intermediate values computed by this application include the lateral forces on the wall due to earth pressure and surcharge, the weight of the wall and the weight of the soil over the toe and heel of the base, the required toe and heel projections of the retaining wall base or footing, bearing length and base soil pressures at the toe and heel under service loads and factored loads, overturning moment and sliding force, resisting moment and force resisting sliding, wall and footing thicknesses required for flexure and shear, and areas of wall and footing reinforcement for walls with a level backfill and any specified surcharge. Surcharge may be specified as zero. The Strength Design Method of ACI 318, is used.

The required input for this application includes the strength of the concrete and the reinforcement, the unit weights of concrete, reinforced concrete and soil, the equivalent horizontal fluid pressure due to the soil, the surcharge weight, the coefficient of friction between footing and soil, the allowable soil bearing pressure at service load, the required safety factor against sliding, total wall height from top of wall to bottom of footing, depth from the lower grade level to the bottom of the footing, estimated wall and footing thicknesses and front and back face wall tapers (if applicable), rounding factor for base width, preferred reinforcement ratio, clear concrete cover of wall, toe, and heel reinforcement, and the estimated reinforcing bar sizes for the wall, toe and heel.

This application uses PTC Mathcad solve blocks to determine toe and heel projections for full bearing on the base with zero heel pressure under service loads (unless a wider base is required to avoid exceeding the permissible soil bearing pressure) to provide an adequate safety factor against sliding, or to obtain a ratio of resisting moment to overturning moment of two. The application uses the estimated values for wall and footing thicknesses unless a larger wall or footing thickness is required for shear or flexure. If it is necessary to revise the estimated values for wall or footing thicknesses, or bar sizes, the application may be rerun with the actual dimensions and bar sizes to confirm the initial results. Walls with known toe and heel dimensions may be checked by entering the toe and heel dimensions where indicated within this document.

A summary of input and calculated values is shown on pages 21-25.

Reference:

- ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)
- Reinforced Concrete Fundamentals, 3rd Edition", by Phil M. Ferguson
**Input**

**Notation**

![Diagram of footing and wall dimensions](image)

**Input Variables**

\[ d_b := [0, 0, 0, 0.375, 0.5, 0.625, 0.75, 0.875, 1.00, 1.128, 1.27, 1.41, 0, 0, 1.693, 0, 0, 2.257] \cdot \text{in} \]

- Total height from top of wall to bottom of footing: \( H := 21 \cdot \text{ft} + 6 \cdot \text{in} \)
- Depth from lower grade level to bottom of footing: \( D := 0 \cdot \text{ft} \)
- Surcharge on backfill: \( q := 300 \cdot \text{psf} \)
- Surcharge weight over the heel: \( q_w := 0 \cdot \text{psf} \)
- Estimated footing thickness: \( h_f := 18 \cdot \text{in} \)
- Estimated wall thickness at top of footing: \( h_{wb} := 18 \cdot \text{in} \)
- Front face wall taper: \( t_f := 0 \text{ in} \)
- Back face wall taper: \( t_b := 6 \text{ in} \)
- Unit weight of soil: \( \gamma := 100 \cdot \text{pcf} \)
- Coefficient of friction between footing and soil: \( c_f := 0.55 \)
- Equivalent horizontal fluid pressure due to soil: \( k_h := 30 \cdot \text{pcf} \)
- Allowable soil bearing pressure: \( p_s := 4 \cdot \text{ksf} \)
- Wall reinforcing bar diameter: \( d_{bw} := d_{b_7} \)
- Toe reinforcing bar diameter: \( d_{bt} := d_{b_6} \)
- Heel reinforcing bar diameter: \( d_{sh} := d_h \)
Heel reinforcing bar diameter: \( d_{bh} := d_{b8} \)

The variable \( d_b \) represents standard reinforcing bars No. 3 to No. 11, No. 14 and No. 18, defined below. The subscript number designates the specific bar size. Bar diameters may also be entered directly.

Initially, the values for wall and footing thicknesses, wall tapers, and bar sizes must be estimated. If the final selected values differ significantly from those initially assumed this application should be repeated using the final values.

Enter \( q_w = 0 \) psf for highway and railroad surcharges and \( q_w = q \) if an actual surcharge load is present over the heel. Surcharge weight over the heel is customarily omitted for highway and railroad surcharges. Omitting surcharge weight over the heel is conservative.

**Computed Variables**
- \( B \): minimum required width of footing
- \( T \): minimum required toe projection from face of wall at top of footing
- \( F \): minimum required heel projection from face of wall at top of footing
- \( P_a \): horizontal force due to soil pressure
- \( P_q \): horizontal force due to surcharge
- \( W_R \): total dead load
- \( M_{ot} \): overturning moment due to active soil pressure and surcharge
- \( M_R \): resisting moment due to dead load
- \( p_{toe} \): soil bearing pressure at toe of footing at service load
- \( p_{heel} \): soil bearing pressure at heel of footing at service loads
- \( L_b \): length of bearing on base at service load
- \( p_{toe,f} \): soil bearing pressure at toe of footing at factored load
- \( p_{heel,f} \): soil bearing pressure at heel of footing at service loads
- \( L_{b,f} \): length of bearing on base at service load
- \( h_f \): minimum required footing thickness
- \( h_{wb} \): minimum required wall thickness at top of footing
- \( t_r \): front face wall taper corresponding to required wall thickness
- \( t_b \): back face wall taper corresponding to required wall thickness
- \( A_{s, toe} \): required area of toe reinforcement
- \( A_{s, heel} \): required area of heel reinforcement
- \( A_{s, wall} \): required areas of wall reinforcement at specified points

**Material Properties and Constants**

Enter \( f_c \), \( f_y \), \( w_c \), \( w_{rc} \), \( k_v \) and \( k_w \) if different from that shown.

Specified compressive strength of concrete: \( f'_{c} := 4 \cdot ksi \)

Specified yield strength of reinforcement (\( f_y \) may not exceed 60 ksi, ACI 318, 11.5.2): \( f_y := 60 \cdot ksi \)

Unit weight of concrete: \( w_c := 145 \cdot pcf \)

Weight of reinforced concrete: \( w_{rc} := 150 \cdot pcf \)

Shear strength reduction factor for lightweight concrete \( k_v = 1 \) \( k_v := 1 \) 
Specified shrinkage 0.75 for all lightweight and 0.85 for standard
for normal weight, 0.75 for all-lightweight and 0.85 for sand-lightweight concrete (ACI 318, 11.2.1.2):

Weight factor for increasing development and splice lengths $k_w$  
$k_w := 1$ for normal weight and 1.3 for lightweight aggregate concrete

(ACI 318, 12.2.4.2):

Modulus of elasticity of reinforcement 
$E_s := 29000 \cdot ksi$

Strain in concrete at compression failure 
$\varepsilon_c := 0.003$

Strength reduction factor for flexure 
$\phi_f := 0.9$

Strength reduction factor for shear 
$\phi_v := 0.85$

Sizing factor for rounding wall and footing thicknesses: 
$SzF := 2 \cdot in$

Sizing factor for rounding base width: 
$SzB := 3 \cdot in$

Concrete cover of bottom toe reinforcement: 
$cl_t := 3 \cdot in$

Concrete cover of top heel reinforcement: 
$cl_h := 1.5 \cdot in$

Concrete cover of wall reinforcement: 
$cl_w := 1.5 \cdot in$

Sliding factor of safety: 
$SF := 1.5$

The safety factor for the base sliding on soil should not be less than 1.5 unless other means are provided to prevent sliding.

Reinforcing bar number designations, diameters, and areas:

$No := [0 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18]^T$

$d_b := [0 0 0 0.375 0.5 0.625 0.75 0.875 1.00 1.125 1.25 1.41 0 0 1.693 0 0 2.257]^T \cdot in$

$A_b := [0 0 0 0.11 0.20 0.31 0.44 0.60 0.79 1.00 1.27 1.56 0 0 2.25 0 0 4.00]^T \cdot in^2$

Bar numbers, diameters and areas are in the vector rows (or columns in the transposed vectors shown) corresponding to the bar numbers. Individual bar numbers, diameters, areas and development lengths and splices of a specific bar can be referred to by using the vector subscripts as shown in the example below.
Limit the value of $f'_c$ for computing shear and development lengths to 10 ksi by substituting $f'_{c, \text{max}}$ for $f'_c$ in formulas for computing shear (ACI 318, 11.1.2, 12.1.2):

$$f'_{c, \text{max}} := \text{if } (f'_c > 10 \cdot \text{ksi}, 10 \cdot \text{ksi}, f'_c)$$

The following values are computed from the entered material properties.

Nominal "one way" shear strength per unit area in concrete (ACI 318, 11.3.1.1, Eq. (11-3), 11.5.4.3):

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_{c, \text{max}}}{\text{psi}} \cdot \text{psi}} = 126.491 \text{ psi}$$

Strain in reinforcement at yield stress:

$$\varepsilon_y := \frac{f_y}{E_s} = 0.002$$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \text{if } (f'_c \geq 4 \cdot \text{ksi}, (f'_c \leq 8 \cdot \text{ksi}), 0.85 - 0.05 \cdot \frac{f'_c - 4 \cdot \text{ksi}}{\text{ksi}}, \text{if } (f'_c \leq 4 \cdot \text{ksi}, 0.85, 0.65)) = 0.85$$

Reinforcement ratio producing balanced strain conditions (ACI 318, 10.3.2):

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f'_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c - f_y}{E_s \cdot \varepsilon_c + f_y} = 2.851\%$$

Maximum reinforcement ratio (ACI 318, 10.3.3):

$$\rho_{\text{max}} := \frac{3}{4} \cdot \rho_b = 2.138\%$$

Minimum reinforcement ratio for beams (ACI 318, 10.5.1, Eq. (10-3)):

$$\rho_{\text{min}} := \frac{200}{f_y} \cdot \frac{\text{lb}}{\text{in}^2} = 0.333\%$$

Shrinkage and temperature reinforcement ratio (ACI 318, 7.12.2.1):

$$\rho_{\text{temp}} := \text{if } f_y \leq 50 \text{ ksi}$$

$$\left| \begin{array}{c}
\| 0.002 \\
\| \text{else if } f_y \leq 60 \text{ ksi} \\
\| 0.002 - \frac{f_y}{60 \text{ ksi}} \\
\| \text{else if } \frac{0.0018 \cdot (60 \text{ ksi})}{f_y} \geq 0.0014 \\
\| \frac{0.0018 \cdot (60 \text{ ksi})}{f_y} \\
\| \text{else} \\
\end{array} \right| = 0.18\%$$
Preferred reinforcement ratio:

\[ \rho_{\text{pref}} := \frac{1}{2} \cdot \rho_{\text{max}} = 1.069\% \]

Flexural coefficient \( K \), for rectangular beams or slabs, as a function of \( \rho \) (ACI 318, 10.2):
(Moment capacity \( \phi M_n = K(\rho)F \), where \( F = bd^2 \))

\[ K(\rho) := \phi_f \cdot \rho \cdot \left( 1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f'c} \right) \cdot f_y \]
Calculations

Thickness at top of wall:

\[ h_{wt} := h_{wb} - t_f - t_b = 12 \text{ in} \]

Stem wall height:

\[ C := H - h_f = 20 \text{ ft} \]

Depth of fill over toe:

\[ E := \text{if } ((D - h_f) \geq 0 \cdot ft, D - h_f, 0 \cdot ft) = 0 \text{ ft} \]

Footing width \( B \) as a function of \( F \) and \( T \):

\[ B(F, T) := T + h_{wb} + F \]

Active soil pressure resultant:

\[ P_a := k_h \cdot \frac{H^2}{2} \cdot ft = 6.934 \text{ kip} \]

Surcharge soil pressure resultant:

\[ P_q := k_h \cdot \frac{q}{\gamma} \cdot H \cdot ft = 1.935 \text{ kip} \]

Overturning moment \( M_{ot} \):

\[ M_{ot} := \frac{P_a \cdot H}{3} + \frac{P_q \cdot H}{2} = 70.493 \text{ kip} \cdot \text{ft} \]

Weights of wall segments, soil segments over the wall tapers, and surcharge \( q_w \) over the back face taper per unit length of wall:

\[
\begin{bmatrix}
    h_{wt} \cdot C \cdot w_{rc} \\
    t_f \cdot C \cdot w_{rc} \\
    \frac{t_f}{2} \cdot C \cdot w_{rc} \\
    t_b \cdot C \cdot w_{rc} \\
    \frac{t_b}{2} \cdot C \cdot w_{rc} \\
    t_f \cdot E^2 \\
    2 \cdot C \\
    \frac{t_b}{2} \cdot C \\
    q_w \cdot t_b
\end{bmatrix}
\]

\[
W = \begin{bmatrix}
    3 & 0 & 0.75 & 0 & 0.5 & 0
\end{bmatrix} \begin{bmatrix}
    \text{kip} \\
    \text{ft}
\end{bmatrix}
\]
Distances from the area centroids of segments to the face of the wall at top of footing:

\[ x := \left[ \frac{t_f + h_{wt}}{2} \right] + \frac{3}{2} \cdot t_f + h_{wt} + \frac{t_b}{3} \cdot \left( \frac{E \cdot t_f}{3} \cdot h_{wb} - \frac{t_b}{3} \cdot h_{wb} - \frac{t_b}{2} \right) \]

\[ x^T = [6 \ 0 \ 14 \ 0 \ 16 \ 15] \text{ in} \]

Total dead load as a function of F and T:

\[ w_R(F, T) := \left( \sum w + (B(F, T) \cdot h_f \cdot w_{rc} + ((T \cdot E + F \cdot C) \cdot \gamma + q_w \cdot F)) \right) \cdot \text{ft} \]

Dead load resisting moment as a function of F and T:

\[ M_R(F, T) := \left( \sum (w \cdot (x + T)) + \left( \frac{B(F, T)^2}{2} \cdot h_f \cdot w_{rc} + \left( \frac{T^2}{2} \cdot E \cdot \gamma + (F \cdot C \cdot \gamma + q_w \cdot F) \cdot \left( \frac{F}{2} + T + h_{wb} \right) \right) \right) \right) \cdot \text{ft} \]

Location of dead load resultant from the edge of the toe as a function of F and T:

\[ x_R(F, T) := \frac{M_R(F, T) - M_{ot}}{w_R(F, T)} \]

Eccentricity of the dead load resultant from the footing midpoint as a function of F and T:

\[ e(F, T) := \frac{B(F, T)}{2} - \frac{M_R(F, T) - M_{ot}}{w_R(F, T)} \]

Base contact bearing length as a function of F and T:

\[ L_B(F, T) := \text{if} \left( e(F, T) \leq \frac{B(F, T)}{6}, B(F, T), 3 \cdot x_R(F, T) \right) \]

Concrete volume of the wall per unit length as a function of F and T:

\[ V_{ol}(F, T) := \left( \left( \frac{h_{wt}}{2} + \frac{t_f}{2} + \frac{t_b}{2} \right) \cdot C + B(F, T) \cdot h_f \right) \cdot \text{ft} \]

Soil bearing pressure at toe of base as a function of F and T:

\[ p_{toe}(F, T) := \text{if} \left( e(F, T) \geq \frac{B(F, T)}{6}, \frac{2 \cdot w_R(F, T)}{L_B(F, T)}, \frac{w_R(F, T)}{B(F, T)} + \frac{6 \cdot e(F, T) \cdot w_R(F, T)}{B(F, T)^2} \right) \cdot \frac{1}{\text{ft}} \]

Soil bearing pressure at heel of base as a function of F and T:

\[ p_{heel}(F, T) := \text{if} \left( e(F, T) \geq \frac{B(F, T)}{6}, 0, \frac{k_{ip}}{\text{ft}}, \frac{w_R(F, T)}{B(F, T)} - \frac{6 \cdot e(F, T) \cdot w_R(F, T)}{B(F, T)^2} \right) \cdot \frac{1}{\text{ft}} \]
Calculations to determine required toe and heel dimensions \( T \) and \( F \)

Estimated value of heel projection \( F \) (calculated assuming the toe projection \( T \) is equal to zero, and that the weight of the wall and footing, and the soil and surcharge over the heel is just sufficient to provide the required safety factor against sliding):

\[
F := \frac{SF \cdot (P_a + P_q) - c_f \cdot \left( \sum w + (h_{wb}, h_f, w_{rc}) \right) \cdot ft}{c_f \cdot (h_f, w_{rc}, C \cdot \gamma + q_u) \cdot ft} = 8.809 \ ft
\]

Estimated value of toe projection \( T \) assuming \( F \) equals the value computed above and that the resultant of the gravity loads \( w_R \) is located at \( B/3 \) from the face of the toe:

\[
T := \text{if} \left( x_R(F, 0 \cdot ft) \geq \frac{B(F, 0 \cdot ft)}{3}, 0 \cdot ft, \frac{B(F, 0 \cdot ft)}{3} - x_R(F, 0 \cdot ft) \right) = 1.43 \ ft
\]

\( F \) and \( T \) with dead load equal to \( SF \) x the sum of the horizontal forces, and \( w_R \) at least \( B/3 \) from the face of the toe:

\[
F := 8 \ ft \quad T := 1 \ ft
\]

\[
T > 0
\]

\[
F = \frac{SF \cdot (P_a + P_q) - c_f \cdot \left( \sum w + (h_{wb}, h_f, w_{rc}) + T \cdot (h_f, w_{rc}, E \cdot \gamma) \right) \cdot ft}{c_f \cdot (h_f, w_{rc}, C \cdot \gamma) \cdot ft}
\]

\[
x_R(F, T) \geq \frac{B(F, T)}{3}
\]

\[
\begin{bmatrix} F \\ T \end{bmatrix} = \text{Find}(F, T) = \begin{bmatrix} 8.566 \\ 2.406 \end{bmatrix} \ ft
\]

F and \( T \) with \( w_R \) equal to \( SF \) x the sum of the horizontal forces, the resisting moment greater than \( 2 \times \) the overturning moment, full contact bearing on the soil, the soil bearing pressure at the heel less than or equal to the toe pressure, and toe and heel pressures less than or equal to the allowable \( p_s \):

\[
\frac{M_R(F, T)}{M} \geq 2 \quad \frac{c_f \cdot w_R(F, T)}{P + P} = SF
\]
Constrain:

\[ p_{\text{toe}}(F,T) \leq p_s \quad \text{and} \quad p_{\text{toe}}(F,T) \geq 0 \cdot \text{ksf} \]

Solver:

\[
\begin{bmatrix} F \\ T \end{bmatrix} = \text{Find}(F,T) = \begin{bmatrix} 8.566 \\ 2.406 \end{bmatrix} \text{ ft}
\]

Minimum required footing width \( B_1 \) before rounding:

\[ B_1 := F + T + h_{\text{wb}} \quad B_1 = 12.471 \text{ ft} \]

**Required Base Width, Toe and Heel Projections**

Final footing width \( B(F,T) \) toe projection \( T \), and heel projection \( F \) by rounding \( B_1 \) up to the nearest multiple of \( S_ZF \) and rounding the resulting value of \( T \) up to the nearest inch:

\[
B := \text{ceil} \left( \frac{B_1}{S_ZB} \right) \cdot S_ZB \quad B = 12.5 \text{ ft}
\]

\[
T := \text{if} \left( \text{ceil} \left( \frac{T - (B - B_1)}{S_ZF} \right) \cdot S_ZF > 0 \cdot \text{ft}, \text{ceil} \left( \frac{T - (B - B_1)}{S_ZF} \right) \cdot S_ZF, 0 \cdot \text{ft} \right) \quad T = 2.5 \text{ ft}
\]

\[
F := B - T - h_{\text{wb}} \quad F = 8.5 \text{ ft}
\]

\[ B = 12.5 \text{ ft} \]

**Note** ⇒ Walls may be checked at this point by defining known footing dimensions \( T \) and \( F \) instead of using calculated values.

\[
clear_{\text{sym}}(T) \quad \clear_{\text{sym}}(F) \quad B := T + F + h_{\text{wb}} \quad B = 12.5 \text{ ft}
\]

Values of total dead load, resisting moment, toe and heel pressures, and bearing length with the final selected values of the heel and toe projections:

**Toe projection:** \( T = 2.5 \text{ ft} \)

**Heel projection:** \( F = 8.5 \text{ ft} \)

**Bearing length:** \( L_B := L_B(F,T) \quad L_B = 12.5 \text{ ft} \)

**Resisting moment:** \( M_R := M_R(F,T) \quad M_R = 171.495 \text{ kip\cdot ft} \)

**Soil bearing pressure at toe:** \( p_{\text{toe}} := p_{\text{toe}}(F,T) \quad p_{\text{toe}} = 3.822 \text{ ksf} \)
Soil bearing pressure at heel: \( p_{heel} := p_{heel}(F, T) \) \( p_{heel} = 0.028 \) ksf

Total weight of soil and concrete: \( w_R := w_R(F, T) \) \( w_R = 24.063 \) kip

Concrete volume per unit length of wall: \( Vol := Vol(F, T) \) \( Vol = 43.75 \) ft³

Calculations to determine soil bearing pressures at factored load

Factored dead load \( w_{R,f} \) and resisting moment \( M_{R,f} \) using an 0.9 factor for wall and footing dead loads, and a 1.4 factor for soil and surcharge weights for computing soil bearing pressures at factored loads.

(See: "Reinforced Concrete Fundamentals" by Phil M. Ferguson):

Load factors for weights:

\[
f := \begin{bmatrix} 0.9 & 0.9 & 0.9 & 1.4 & 1.4 & 1.4 \end{bmatrix}^T
\]

\[
w_{R,f} := \left( \sum w \cdot f + 0.9 \cdot \left( B \cdot h_f \cdot w_{re} \right) + 1.4 \cdot \left( (T \cdot E + F \cdot C) \cdot \gamma + q_w \cdot F \right) \right) \cdot ft = 30.406 \text{ kip}
\]

\[
M_{R,f} := \left( \sum \left( w \cdot f \right) \cdot (x + T) + 0.9 \cdot \left( \frac{B^2 \cdot h_f}{2} \cdot w_{re} \right) \right) + 1.4 \cdot \left( \frac{T^2}{2} \cdot E \cdot \gamma + (F \cdot C \cdot \gamma + q_w \cdot F) \cdot \left( \frac{F}{2} + T + h_{wb} \right) \right) \cdot ft
\]

\( M_{R,f} = 225.429 \text{ kip} \cdot ft \)

Location of factored dead load resultant from toe:

\[
x_f := \frac{M_{R,f} - 1.7 \cdot M_{ol}}{w_{R,f}} = 3.473 \text{ ft}
\]

Eccentricity of dead load resultant from base midpoint at factored load:

\[
e_f := \frac{B}{2} - \frac{M_{R,f} - 1.7 \cdot M_{ol}}{w_{R,f}} = 2.777 \text{ ft}
\]

Bearing length on base at factored load:

\[
L_{B,f} := \text{if} \left( e_f \leq \frac{B}{6}, F, 3 \cdot x_f \right) = 10.418 \text{ ft}
\]

Soil bearing pressures at factored loads:

\[
p_{toe,f} := \frac{1}{f \text{t}} \left( \frac{2 \cdot w_{R,f}}{L_{B,f}} \cdot \left( \frac{w_{R,f}}{F} + \frac{6 \cdot e_f \cdot w_{R,f}}{F^2} \right) \right) = 5.837 \text{ ksf}
\]
\[ p_{heef} := \text{if} \left( \frac{e_f}{6} \geq \frac{B}{k_{ip}} \cdot \left( \frac{w_{R_f} \cdot 6 \cdot e_f \cdot w_{R_f}}{F - F^2} \right) \right) \cdot \frac{1}{ft} = 0 \text{ ksi} \]

**Wall Design**

Shear on stem wall as a function of distance \( y \) above base:

\[ V_{uw} (y) := 1.7 \cdot k_h \cdot (C - y) \cdot \left( \frac{C - y + q}{2} \right) \cdot ft \]

Bending moment on stem wall as a function of distance \( y \) above base:

\[ M_{uw} (y) := 1.7 \cdot k_h \cdot (C - y)^2 \cdot \left( \frac{C - y + q}{6} \right) \cdot ft \]

Minimum required wall thickness for shear:

\[ h_{shear} := \frac{V_{uw} (0 \cdot ft)}{\phi_v \cdot v_c \cdot ft} + c l_w + \frac{1}{2} \cdot d_{bw} = 12.215 \text{ in} \]

Minimum required wall thickness for flexure:

\[ h_{flex} := \sqrt{\frac{M_{uw} (0 \cdot ft)}{K (\rho_{pref}) \cdot ft}} + c l_w + \frac{1}{2} \cdot d_{bw} = 15.67 \text{ in} \]

Minimum required wall thickness, compared to thickness entered:

\[ h_1 := \text{if} \left( h_{flex} \geq h_{shear}, h_{flex}, h_{shear} \right) = 15.67 \text{ in} \]

\[ h_{wb} = 18 \text{ in} \]

**Minimum required wall thickness**

Minimum required wall thickness \( h'w \) at base and corresponding wall tapers (minimum values are indicated by prime marks ' ) rounded up to the nearest even inch:

\[ h'_{wb} := 2 \cdot \text{in} \cdot \text{ceil} \left( \frac{h_1}{2 \cdot \text{in}} \right) = 16 \text{ in} \]

\[ t'_f := t_f - \frac{t_f}{t_f + t_b} \cdot (h_{wb} - h'_{wb}) = 0 \text{ in} \]

\[ t'_b := t_b - \frac{t_b}{t_f + t_b} \cdot (h_{wb} - h'_{wb}) = 4 \text{ in} \]
Wall thickness and effective depth as functions of distance \( y \) above top of footing, using the larger of the entered wall thickness or the calculated wall thickness:

\[
h_w(y) := \begin{cases} h_{wb} > h_{wb'} , h_{wb} , h_{wb'} \to \frac{y}{C} \cdot \begin{cases} h_{wb} > h_{wb'} , (t_f + t_b) , (t_f' + t_b') \end{cases} \\

d_{ew}(y) := h_w(y) - c_{lw} - \frac{d_{bw}}{2}
\]

Wall thickness and effective depth at top of footing and corresponding footing thickness:

\[
h_w(0 \text{ ft}) = 18 \text{ in} \quad d_{ew}(0 \text{ ft}) = 16.063 \text{ in} \quad h_f = 18 \text{ in}
\]

Distance \( y \) from top of footing at 1 ft intervals and at top of wall:

\[
i1 := 0 \ldots \frac{C}{ft} \quad y_i := i1 \cdot ft \quad N := \text{ceil} \left( \frac{C}{ft} \right) \quad y_N := C \quad i := 0 \ldots N
\]

Theoretical calculated reinforcement ratio required for flexure as a function of \( y \):

\[
\rho(y) := \left[ 1 - \left( 1 - \frac{2 \cdot M_{ew}(y)}{\phi_f \cdot ft \cdot d_{ew}(y)^2 \cdot 0.85 \cdot f_c'^2} \right) \right] \cdot \frac{0.85 \cdot f_c'}{f_y}
\]

Minimum required reinforcement ratio: \( \rho_{min} = 0.333\% \)

Preferred maximum reinforcement ratio: \( \rho_{pref} = 1.069\% \)

Maximum reinforcement ratio: \( \max(\rho(y)) = 0.008 \)

Maximum permissible reinforcement ratio: \( \rho_{max} = 2.138\% \)

Required reinforcement area, the larger of the theoretical calculated reinforcement or the minimum required reinforcement:

\[
A_{s,wall} := \begin{cases} \rho_y \geq \rho_{min} \to \rho_y \cdot ft \cdot d_{ew}(y) \cdot \rho_{min} \cdot ft \cdot d_{ew}(y) \end{cases}
\]

Maximum and minimum reinforcement areas:

\[
\max(A_{s,wall}) = 1.462 \text{ in}^2 \quad \min(A_{s,wall}) = 0.403 \text{ in}^2
\]

All calculated values of \( y_i \) and \( A_{s,wall} \) are displayed in graphical and tabular form in the Summary.
Footing Design

Difference between factored toe and heel soil bearing pressures, and the effective wall depth $d_w$ to the centroid of the wall reinforcement at top of footing:

$$\Delta p_f := p_{toe,f} - p_{heel,f} = 0.041 \text{ ksi}$$

$$d_w := d_{ew} (0 \text{ ft}) = 16.063 \text{ in}$$

Intermediate values used in calculating soil bearing pressures on footing:

$$X1 := \frac{L_{B,f} - T}{L_{B,f}} = 0.76$$

$$X2 := \frac{B - T}{B} = 0.8$$

$$X3 := \frac{L_{B,f} - T - d_w}{L_{B,f}} = 0.632$$

$$X4 := \frac{B - T - d_w}{B} = 0.693$$

Location of critical section for shear on the toe from front face of wall as a function of required toe thickness for shear:

$$X5(h1) := T - \left(h1 - c_{t} - \frac{1}{2} \cdot d_{bt}\right)$$

Ratio of the distance from the point of minimum bearing pressure on the footing to the critical section for shear on the toe to the bearing length on the footing as a function of required toe thickness for shear:

$$X6(h1) := \frac{L_{B,f} - X5(h1)}{L_{B,f}}$$

Ratio of the distance from the heel to the critical section for shear on the toe to the bearing length on the footing, as a function of required toe thickness for shear:

$$X7(h1) := \frac{B - X5(h1)}{B}$$

Soil bearing pressures due to factored loads at distance from front face of wall to the critical section for shear on the toe, as a function of required toe thickness for shear:

$$p_{d,f}(h1) := \begin{cases} X6(h1) \cdot p_{toe,f} & \text{if } (L_{B,f} \geq X5(h1)) \lor (L_{B,f} \leq B) \\
0 \cdot \text{ksf} & \text{else if } L_{B,f} \leq X5(h1) \\
p_{heel,f} + X7(h1) \cdot (\Delta p_f) & \text{else} \end{cases}$$

Soil bearing pressures on footing at front and back face of wall:

$$p_{front,f} := \begin{cases} (L_{B,f} \geq T) + (L_{B,f} \leq B) \cdot X1 \cdot p_{toe,f} & \text{if } (L_{B,f} \leq T, 0 \cdot \text{ksf}, p_{heel,f} + X2 \cdot (\Delta p_f)) \end{cases}$$
\[ p_{\text{back},f} := \text{if} \left( \langle L_{B,f} \leq T + d_w \rangle + \langle L_{B,f} \leq B \rangle, X3 \cdot p_{\text{toe},f}, \text{if} \left( \langle L_{B,f} \leq T - d_w \rangle, 0 \cdot ksf, p_{\text{heel},f} + X4 \cdot (\Delta p_f) \right) \right) \]

Shear on toe as a function of footing thickness, using 0.9 factor for opposing dead load:

\[ V_{u,\text{toe}}(h1) := \left( \frac{p_{\text{toe},f} + p_{d,f}(h1)}{2} - 0.9 \cdot (h_f \cdot w_{rc} + E \cdot \gamma) \right) \cdot X5(h1) \cdot ft \]

Solve for footing thickness \( h1 \) required for shear on toe:

Guess value of \( h1 = h_f \), the initial assumed footing thickness:

\[ h1 := h_f = 18 \text{ in} \]

\[ f1(h1) := \phi_v \cdot v_c \cdot ft \cdot \left( h1 - c_l + \frac{1}{2} \cdot d_{bt} \right) - V_{u,\text{toe}}(h1) \]

\[ h1 := \text{root}(f1(h1), h1) \]

\[ h1 = 0.275 \text{ m} \]

\[ V_{u,\text{toe}}(h1) = (4.271 \cdot 10^4) \frac{kg \cdot m}{s^2} \]

Bending moment on toe using 0.9 factor for opposing dead load:

\[ M_{u,\text{toe}} := \left( \left( \frac{p_{\text{toe},f}}{3} + \frac{p_{\text{front},f}}{6} \right) \cdot T^2 - 0.9 \cdot (h_f \cdot w_{rc} + (D - h_f) \cdot \gamma) \cdot \frac{T^2}{2} \right) \cdot ft \]

\[ M_{u,\text{toe}} = (2.247 \cdot 10^4) \frac{kg \cdot m^2}{s^2} \]

Toe thickness required for flexure:

\[ h2 := \sqrt{\frac{M_{u,\text{toe}}}{K(p_{\text{pref}}) \cdot ft}} + c_l + \frac{1}{2} \cdot d_{bt} \]

\[ h2 = 0.229 \text{ m} \]

The larger toe thickness required for shear or flexure:

\[ h_{\text{toe}} := \text{if} \left( h1 \geq h2, h1, h2 \right) \]

\[ h_{\text{toe}} = 0.275 \text{ m} \]

Intermediate values used to calculate shear and bending moment on heel:

\[ Y1 := c_l + \frac{d_{pw}}{2} = 1.938 \text{ in} \]

\[ Y2 := L_{B,f} - T - d_w = 6.579 \text{ ft} \]
Shear $V_u$ and bending moment $M_u$ on heel using a 1.7 load factor for surcharge load $q_w$:

$$V_{u,}\text{heel} := \left(1.4 \cdot (C \cdot \gamma + h_f \cdot w_{rc}) + 1.7 \cdot q_w\right) \cdot F - \frac{P_{\text{back},f} + P_{\text{heel},f}}{2} \cdot (Y2) \cdot ft = 14.35 \text{ kip}$$

$$M_{u,}\text{heel} := \left(1.4 \cdot (C \cdot \gamma + h_f \cdot w_{rc}) + 1.7 \cdot q_w\right) \cdot F \cdot (Y3) - \left(\frac{P_{\text{back},f} + P_{\text{heel},f}}{6} + \frac{P_{\text{heel},f}}{3}\right) \cdot Y2^2 \cdot ft = 90.207 \text{ kip} \cdot ft$$

Heel thickness required for shear:

$$h_3 := \frac{V_{u,}\text{heel}}{\phi_c \cdot v_c \cdot ft} + cl_h + \frac{1}{2} \cdot d_{bh} = 13.122 \text{ in}$$

Heel thickness required for flexure:

$$h_4 := \sqrt[3]{\frac{M_{u,}\text{heel}}{K \cdot (\rho_{\text{pref}}) \cdot ft}} + cl_h + \frac{1}{2} \cdot d_{bh} = 15.136 \text{ in}$$

The larger heel thickness required for shear or flexure:

$$h_{\text{heel}} := \text{if}(h_3 \geq h_4, h_3, h_4) = 15.136 \text{ in}$$

Minimum required footing thickness as the larger thickness required for the toe or heel:

$$h_{f1} := \text{if}(h_{\text{toe}} \geq h_{\text{heel}}, h_{\text{toe}}, h_{\text{heel}}) = 15.136 \text{ in}$$

**Minimum required footing thickness**

Minimum footing thickness rounded up to the nearest multiple of SzF:

$$h'_f := SzF \cdot \text{ceil} \left(\frac{h_{f1}}{SzF}\right) = 16 \text{ in}$$

Effective footing depths to centroids of reinforcement using the larger of the entered thickness or the calculated footing thickness:

$$d_{e,\text{toe}} := \text{if}(h_f > h'_f, h_f, h'_f) - cl_t - \frac{1}{2} \cdot d_{bt} = 14.625 \text{ in}$$

$$d_{e,\text{heel}} := \text{if}(h_f > h'_f, h_f, h'_f) - \text{if}(M_{u,}\text{heel} < 0 \cdot \text{kip} \cdot ft, cl_r, cl_h) - \frac{1}{2} \cdot d_{bh} = 16 \text{ in}$$
Minimum required toe and heel reinforcement

Toe reinforcement:

\[
\rho_{\text{toe}} := \left( \frac{1}{1 - \left( 1 - \frac{2 \cdot M_{u,\text{toe}}}{\phi_f \cdot f_t \cdot d_{e,\text{toe}}^2 \cdot 0.85 \cdot f'_c} \right) \cdot 0.85 \cdot f'_c} \right) \cdot \frac{f_y}{f_y}
\]

\[
\rho_{\text{toe}} = 0.001
\]

\[
A_{s,\text{toe}} := \frac{1}{\rho_{\text{toe}} \cdot \rho_{\text{min}}} \cdot \frac{\rho_{\text{toe}} \cdot f_t \cdot d_{e,\text{toe}} \cdot \rho_{\text{min}} \cdot f_t \cdot d_{e,\text{toe}}}{0.85 \cdot f'_c}
\]

\[
A_{s,\text{toe}} = \left(3.774 \cdot 10^{-4}\right) \text{ m}^2
\]

Heel reinforcement:

\[
\rho_{\text{heel}} := \left( \frac{1}{1 - \left( 1 - \frac{2 \cdot M_{u,\text{heel}}}{\phi_f \cdot f_t \cdot d_{e,\text{heel}}^2 \cdot 0.85 \cdot f'_c} \right) \cdot 0.85 \cdot f'_c} \right) \cdot \frac{f_y}{f_y}
\]

\[
\rho_{\text{heel}} = 0.007
\]

\[
A_{s,\text{heel}} := \frac{1}{\rho_{\text{heel}} \cdot \rho_{\text{min}}} \cdot \frac{\rho_{\text{heel}} \cdot f_t \cdot d_{e,\text{heel}} \cdot \rho_{\text{min}} \cdot f_t \cdot d_{e,\text{heel}}}{0.85 \cdot f'_c}
\]

\[
A_{s,\text{heel}} = 1.335 \text{ in}^2
\]

Compare factored shear and moment on toe and heel with useable capacities:

\[
\phi V_{n,\text{toe}} := \phi_v \cdot v_c \cdot f_t \cdot d_{e,\text{toe}}
\]

\[
\phi V_{n,\text{toe}} = \left(8.393 \cdot 10^4\right) \frac{kg \cdot m}{s^2} \quad V_{u,\text{toe}} (h_f) = \left(3.007 \cdot 10^4\right) \frac{kg \cdot m}{s^2}
\]

\[
\phi M_{n,\text{toe}} := \left( \phi_f \cdot \rho_{\text{toe}} \cdot \left(1 - \frac{\rho_{\text{toe}} \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y \right) \cdot f_t \cdot d_{e,\text{toe}}^2
\]

\[
\phi M_{n,\text{toe}} = \left(2.247 \cdot 10^4\right) \frac{kg \cdot m^2}{s^2} \quad M_{u,\text{toe}} = \left(2.247 \cdot 10^4\right) \frac{kg \cdot m^2}{s^2}
\]

\[
\phi V_{n,\text{heel}} := \phi_v \cdot v_c \cdot f_t \cdot d_{e,\text{heel}}
\]

\[
\phi V_{n,\text{heel}} = \left(9.183 \cdot 10^4\right) \frac{kg \cdot m}{s^2} \quad V_{u,\text{heel}} = \left(6.383 \cdot 10^4\right) \frac{kg \cdot m}{s^2}
\]

\[
\phi M_{n,\text{heel}} := \phi_f \cdot \rho_{\text{heel}} \cdot \left(1 - \frac{\rho_{\text{heel}} \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y \right) \cdot f_t \cdot d_{e,\text{heel}}^2
\]

\[
\phi M_{n,\text{heel}} = \left(2.247 \cdot 10^4\right) \frac{kg \cdot m^2}{s^2}
\]
\[
\phi M_{n, \text{heel}} = \left( 1.223 \cdot 10^5 \right) \frac{kg \cdot m^2}{s^2}
\]

\[
M_{u, \text{heel}} = \left( 1.223 \cdot 10^5 \right) \frac{kg \cdot m^2}{s^2}
\]
Summary

Material Properties and Constants

Sizing factor for rounding base width: \( SzB = 0.076 \text{ m} \)

Sizing factor for rounding toe projection and wall and footing thicknesses: \( SzF = 0.051 \text{ m} \)

Factor of safety against sliding: \( SF = 1.5 \)

Specified compressive strength of concrete: \( f'_c = 4 \text{ ksi} \)

Specified yield strength of reinforcement: \( f_y = 60 \text{ ksi} \)

Unit weight of concrete: \( w_c = 0.084 \frac{\text{lbf}}{\text{ft}^3} \)

Concrete cover of wall reinforcement: \( c_{lw} = 0.038 \text{ m} \)

Concrete cover of bottom toe reinforcement: \( c_{lt} = 0.076 \text{ m} \)

Concrete cover of top heel reinforcement: \( c_{lh} = 0.038 \text{ m} \)

Preferred reinforcement ratio: \( \rho_{\text{pref}} = 0.011 \)

Shear Strength reduction factor for lightweight concrete: \( k_v = 1 \)

Unit weight of reinforced concrete: \( w_{rc} = 0.087 \frac{\text{psi}}{\text{in}} \)

Input

Total height from top of wall to bottom of footing: \( H = 6.553 \text{ m} \)

Depth from lower grade level to bottom of footing: \( D = 0 \text{ m} \)

Surcharge on backfill: \( q = 2.083 \text{ psi} \)

Surcharge weight over the heel: \( q_w = 0 \text{ psi} \)

Estimated footing thickness: \( h_f = 0.457 \text{ m} \)

Estimated wall thickness at top of footing: \( h_{wb} = 0.457 \text{ m} \)

Front face wall taper: \( t_r = 0 \text{ m} \)
Front face wall taper: \[ t_f = 0 \ m \]
Back face wall taper: \[ t_b = 0.152 \ m \]
Unit weight of soil: \[ \gamma = 0.694 \ \text{psi/ft} \]
Equivalent horizontal fluid pressure due to soil: \[ k_h = (1.736 \times 10^{-5}) \ \text{ksi/in} \]
Coefficient of friction between footing and soil: \[ c_f = 0.55 \]
Allowable soil bearing pressure: \[ p_s = 0.028 \ \text{ksi} \]
Wall reinforcing bar diameter: \[ d_{bw} = 0.875 \ \text{in} \]
Toe reinforcing bar diameter: \[ d_{bt} = 0.75 \ \text{in} \]
Heel reinforcing bar diameter: \[ d_{bh} = 1 \ \text{in} \]

**Wall Dimensions**

Toe projection: \[ T = 0.762 \ m \]
Footing width: \[ B = 3.81 \ m \]
Heel Projection: \[ F = 2.591 \ m \]
Minimum required wall thickness: \[ h'_{wb} = 0.406 \ m \]
Minimum required footing thickness: \[ h'_f = 0.406 \ m \]
Front wall taper corresponding to minimum required wall thickness: \[ t'_f = 0 \ m \]
Back wall taper corresponding to minimum required wall thickness: \[ t'_b = 0.102 \ m \]
The larger of the entered or calculated wall and footing thicknesses are used for calculating shear and required reinforcement. If the calculated values for wall and footing thickness are larger than those entered, the application may be re-run with the calculated values as a final check.

**Overturning and Resisting Forces and Moments**

Horizontal force due to active soil pressure: \( P_a = 6.934 \text{kip} \)

Horizontal force due to surcharge: \( P_q = 1.935 \text{kip} \)

Overturning Moment: \( M_{ot} = 70.493 \text{kip} \cdot \text{ft} \)

Total dead load: \( w_R = 24.063 \text{kip} \)

Ratio of frictional sliding resistance to horizontal sliding forces:
\[
\frac{c_f \cdot w_R}{P_a + P_q} = 1.492
\]

Dead load resisting moment: \( M_R = 171.495 \text{kip} \cdot \text{ft} \)

Ratios of resisting dead load moment to overturning moment:
\[
\frac{M_R}{M_{ot}} = 2.433
\]

**Service and Factored Load Bearing Pressures on Base**

Service load toe pressure: \( p_{toe} = 0.027 \text{ksi} \)

Service load heel pressure: \( p_{heel} = (1.977 \cdot 10^{-4}) \text{ksi} \)

Base bearing length at service load: \( L_B = 12.5 \text{ft} \)

Factored load toe pressure: \( p_{toe,f} = 0.041 \text{ksi} \)

Factored load heel pressure: \( p_{heel,f} = \frac{kg}{m \cdot s^2} \)

Base bearing length at factored load: \( L_{B,f} = 3.175 \text{m} \)

**Required Toe and Heel Reinforcement**

Required toe reinforcement per unit length of wall: \( A_{s,toe} = (3.774 \cdot 10^{-4}) \text{m}^2 \)

Required toe reinforcement per unit length of wall: \( A_{s,heel} = (8.611 \cdot 10^{-3}) \text{m}^2 \)
Concrete volume per lineal foot of wall: \[ Vol = 1.239 \text{ m}^3 \]

Calculated and minimum required reinforcement area for stem wall versus height \( y \) from top of footing to top of wall:

\[
\begin{array}{ccccccccccc}
0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\
1.462 & 1.289 & 1.129 & 0.983 & 0.849 & 0.727 & 0.617 & 0.559 & 0.547 & 0.535 & 0.523 \\
\end{array}
\]

\[ A_{s\_wall_i} = \rho(y) \cdot ft \cdot d_{ew}(y) \text{ (in}^2\text{)} \]

\[ y_i \text{ (ft)} \]

\[ A_{s\_wall_i} \text{ (in}^2\text{)} \]

- Minimum
- Calculated