CHAPTER 8: Earth Retaining Structures

8.1 Reinforced Concrete Retaining Walls with Level or Surcharged Backfill

Description

Earth retaining walls are used in construction wherever a separation of grades at different levels is required. Retaining walls must be designed to resist the lateral pressures due to the retained soil, and the additional forces due to any surcharge.

This application determines required footing width, footing thickness, wall thickness and reinforcement areas for retaining walls with a level backfill, with or without a surcharge. Lateral earth pressures and the surcharge may be specified by the user.

Intermediate values computed by this application include the lateral forces on the wall due to earth pressure and surcharge, the weight of the wall and the weight of the soil over the toe and heel of the base, the required toe and heel projections of the retaining wall base or footing, bearing length and base soil pressures at the toe and heel under service loads and factored loads, overturning moment and sliding force, resisting moment and force resisting sliding, wall and footing thicknesses required for flexure and shear, and areas of wall and footing reinforcement for walls with a level backfill and any specified surcharge. Surcharge may be specified as zero. The Strength Design Method of ACI 318, is used.

The required input for this application includes the strength of the concrete and the reinforcement, the unit weights of concrete, reinforced concrete and soil, the equivalent horizontal fluid pressure due to the soil, the surcharge weight, the coefficient of friction between footing and soil, the allowable soil bearing pressure at service load, the required safety factor against sliding, total wall height from top of wall to bottom of footing, depth from the lower grade level to the bottom of the footing, estimated wall and footing thicknesses and front and back face wall tapers (if applicable), rounding factor for base width, preferred reinforcement ratio, clear concrete cover of wall, toe, and heel reinforcement, and the estimated reinforcing bar sizes for the wall, toe and heel.

This application uses PTC Mathcad solve blocks to determine toe and heel projections for full bearing on the base with zero heel pressure under service loads (unless a wider base is required to avoid exceeding the permissible soil bearing pressure) to provide an adequate safety factor against sliding, or to obtain a ratio of resisting moment to overturning moment of two. The application uses the estimated values for wall and footing thicknesses unless a larger wall or footing thickness is required for shear or flexure. If it is necessary to revise the estimated values for wall or footing thicknesses, or bar sizes, the application may be rerun with the actual dimensions and bar sizes to confirm the initial results. Walls with known toe and heel dimensions may be checked by entering the toe and heel dimensions where indicated within this document.

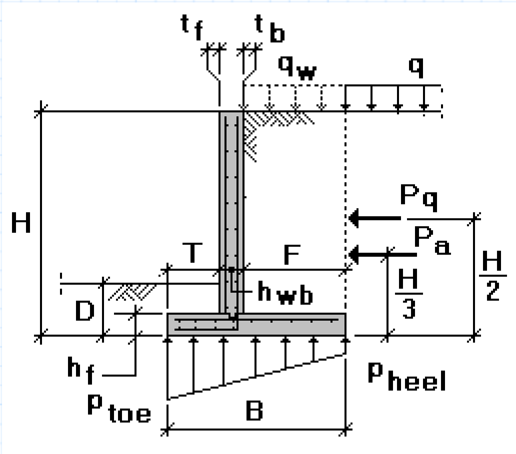
A summary of input and calculated values is shown on pages 21-25.

Reference:

- ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)
- Reinforced Concrete Fundamentals, 3rd Edition", by Phil M. Ferguson

Input

Notation



Input Variables

$$d_b := [0\ \ 0\ \ 0\ \ 0.375\ \ 0.5\ \ 0.625\ \ 0.75\ \ 0.875\ \ 1.00\ \ 1.128\ \ 1.27\ \ 1.41\ \ 0\ \ 0\ \ 1.693\ \ 0\ \ 0\ \ 0\ \ 2.257]^T \cdot in$$

Total height from top of wall to bottom of footing:	$H := 21 \cdot ft + 6 \cdot in$
Depth from lower grade level to bottom of footing:	$D := 0 \cdot ft$
Surcharge on backfill:	$q := 300 \cdot psf$
Surcharge weight over the heel:	$q_w := 0 \cdot psf$
Estimated footing thickness:	$h_f := 18 \cdot in$
Estimated wall thickness at top of footing:	$h_{wb} := 18 \cdot in$
Front face wall taper:	$t_f := 0\ in$
Back face wall taper:	$t_b := 6\ in$
Unit weight of soil:	$\gamma := 100\ pcf$
Coefficient of friction between footing and soil:	$c_f := 0.55$
Equivalent horizontal fluid pressure due to soil:	$k_h := 30 \cdot pcf$
Allowable soil bearing pressure:	$p_s := 4 \cdot ksf$
Wall reinforcing bar diameter:	$d_{bw} := d_{b_7}$
Toe reinforcing bar diameter:	$d_{bt} := d_{b_6}$
Heel reinforcing bar diameter:	$d_{bh} := d_b$

Heel reinforcing bar diameter: $d_{bh} := d_{b_8}$

The variable d_b represents standard reinforcing bars No. 3 to No. 11, No. 14 and No. 18, defined below. The subscript number designates the specific bar size. Bar diameters may also be entered directly.

Initially, the values for wall and footing thicknesses, wall tapers, and bar sizes must be estimated. If the final selected values differ significantly form those initially assumed this application should be repeated using the final values.

Enter q_w = 0 psf for highway and railroad surcharges and q_w = q if an actual surcharge load is present over the heel. Surcharge weight over the heel is customarily omitted for highway and railroad surcharges. Omitting surcharge weight over the heel is conservative.

Computed Variables

B	minimum required width of footing
T	minimum required toe projection from face of wall at top of footing
F	minimum required heel projection from face of wall at top of footing
P_a	horizontal force due to soil pressure
P_q	horizontal force due to surcharge
W_R	total dead load
M_{ot}	overturning moment due to active soil pressure and surcharge
M_R	resisting moment due to dead load
p_{toe}	soil bearing pressure at toe of footing at service load
p_{heel}	soil bearing pressure at heel of footing at service loads
L_b	length of bearing on base at service load
p_{toe_f}	soil bearing pressure at toe of footing at factored load
p_{heel_f}	soil bearing pressure at heel of footing at service loads
L_{b_f}	length of bearing on base at service load
h_f	minimum required footing thickness
h_{wb}	minimum required wall thickness at top of footing
t_f	front face wall taper corresponding to required wall thickness
t_b	back face wall taper corresponding to required wall thickness
A_{s_toe}	required area of toe reinforcement
A_{s_heel}	required area of heel reinforcement
A_{s_wall}	required areas of wall reinforcement at specified points

Material Properties and Constants

Enter f'_c, f_y, w_c, w_{rc}, k_v and k_w if different from that shown.

Specified compressive strength of concrete: $f'_c := 4 \cdot ksi$

Specified yield strength of reinforcement (f_y may not exceed 60 ksi, ACI 318, 11.5.2): $f_y := 60 \cdot ksi$

Unit weight of concrete: $w_c := 145 \cdot pcf$

Weight of reinforced concrete: $w_{rc} := 150 \cdot pcf$

Shear strength reduction factor for lightweight concrete k_v = 1 $k_v := 1$

for normal weight, 0.75 for all-lightweight and 0.85 for sand-lightweight concrete (ACI 318, 11.2.1.2.):

Weight factor for increasing development and splice lengths kw =1 for normal weight and 1.3 for lightweight aggregate concrete (ACI 318, 12.2.4.2): $k_w := 1$

Modulus of elasticity of reinforcement (ACI 318, 8.5.2): $E_s := 29000 \cdot ksi$

Strain in concrete at compression failure (ACI 318, 10.3.2): $\varepsilon_c := 0.003$

Strength reduction factor for flexure (ACI 318, 9.3.2.1): $\phi_f := 0.9$

Strength reduction factor for shear (ACI 318, 9.3.2.3): $\phi_v := 0.85$

Sizing factor for rounding wall and footing thicknesses: $SzF := 2 \cdot in$

Sizing factor for rounding base width: $SzB := 3 \cdot in$

Concrete cover of bottom toe reinforcement: $cl_t := 3 \cdot in$

Concrete cover of top heel reinforcement: $cl_h := 1.5 \cdot in$

Concrete cover of wall reinforcement: $cl_w := 1.5 \cdot in$

Sliding factor of safety: $SF := 1.5$

The safety factor for the base sliding on soil should not be less than 1.5 unless other means are provided to prevent sliding.

Reinforcing bar number designations, diameters, and areas:

$$No := \begin{bmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 & 12 & 13 & 14 & 15 & 16 & 17 & 18 \end{bmatrix}^{T}$$

$$d_b := \begin{bmatrix} 0 & 0 & 0 & 0.375 & 0.5 & 0.625 & 0.75 & 0.875 & 1.00 & 1.128 & 1.27 & 1.41 & 0 & 0 & 1.693 & 0 & 0 & 0 & 2.257 \end{bmatrix}^{T} \cdot in$$

$$A_b := \begin{bmatrix} 0 & 0 & 0 & 0.11 & 0.20 & 0.31 & 0.44 & 0.60 & 0.79 & 1.00 & 1.27 & 1.56 & 0 & 0 & 2.25 & 0 & 0 & 0 & 4.00 \end{bmatrix}^{T} \cdot in^2$$

Bar numbers, diameters and areas are in the vector rows (or columns in the transposed vectors shown) corresponding to the bar numbers. Individual bar numbers, diameters, areas and development lengths and splices of a specific bar can be referred to by using the vector subscripts as shown in the example below.

Example: $No_5 = 5$ $d_{b_5} = 0.625\ in$ $A_{b_5} = 0.31\ in^2$

Limit the value of f'c for computing shear and development lengths to 10 ksi by substituting f'c_max for f'c in formulas for computing shear (ACI 318, 11.1.2, 12.1.2):

$$f'_{c\_max} := \textbf{if}\left(f'_c > 10 \cdot \boldsymbol{ksi}, 10 \cdot \boldsymbol{ksi}, f'_c\right)$$

The following values are computed from the entered material properties.

Nominal "one way" shear strength per unit area in concrete (ACI 318, 11.3.1.1, Eq. (11-3), 11.5.4.3):

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_{c\_max}}{\boldsymbol{psi}}} \cdot \boldsymbol{psi} = 126.491\ \boldsymbol{psi}$$

Strain in reinforcement at yield stress:

$$\varepsilon_y := \frac{f_y}{E_s} = 0.002$$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \textbf{if}\left(\left(f'_c \geq 4 \cdot \boldsymbol{ksi}\right) \cdot \left(f'_c \leq 8 \cdot \boldsymbol{ksi}\right), 0.85 - 0.05 \cdot \frac{f'_c - 4 \cdot \boldsymbol{ksi}}{\boldsymbol{ksi}}, \textbf{if}\left(\left(f'_c \leq 4 \cdot \boldsymbol{ksi}\right), 0.85, 0.65\right)\right) = 0.85$$

einforcement ratio producing balanced strain conditions (ACI 318, 10.3.2):

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f'_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c}{E_s \cdot \varepsilon_c + f_y} = 2.851\%$$

Maximum reinforcement ratio (ACI 318, 10.3.3):

$$\rho_{max} := \frac{3}{4} \cdot \rho_b = 2.138\%$$

Minimum reinforcement ratio for beams (ACI 318, 10.5.1, Eq. (10-3)):

$$\rho_{min} := \frac{200}{f_y} \cdot \frac{\boldsymbol{lbf}}{\boldsymbol{in}^2} = 0.333\%$$

Shrinkage and temperature reinforcement ratio (ACI 318, 7.12.2.1):

$$\rho_{temp} := \begin{array}{l} \text{if } f_y \leq 50\ \boldsymbol{ksi} \\ \quad 0.002 \\ \text{else if } f_y \leq 60\ \boldsymbol{ksi} \\ \quad 0.002 - \dfrac{f_y}{60\ \boldsymbol{ksi}} \cdot 0.0002 \\ \text{else if } \dfrac{0.0018 \cdot (60\ \boldsymbol{ksi})}{f_y} \geq 0.0014 \\ \quad \dfrac{0.0018 \cdot (60\ \boldsymbol{ksi})}{f_y} \\ \text{else} \end{array} = 0.18\%$$

$\|\ 0.0014$

Preferred reinforcement ratio:

$$\rho_{pref} := \frac{1}{2} \cdot \rho_{max} = 1.069\%$$

Flexural coefficient K, for rectangular beams or slabs, as a function of ρ (ACI 318, 10.2):
(Moment capacity ϕM_n = K(ρ)F, where F = bd^2)

$$K(\rho) := \phi_f \cdot \rho \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y$$

Calculations

Thickness at top of wall:

$$h_{wt} := h_{wb} - t_f - t_b = 12\ \boldsymbol{in}$$

Stem wall height:

$$C := H - h_f = 20\ \boldsymbol{ft}$$

Depth of fill over toe:

$$E := \textbf{if}\left(\left(D - h_f\right) \geq 0 \cdot \boldsymbol{ft}, D - h_f, 0 \cdot \boldsymbol{ft}\right) = 0\ \boldsymbol{ft}$$

Footing width B as a function of F and T:

$$B(F, T) := T + h_{wb} + F$$

Active soil pressure resultant:

$$P_a := k_h \cdot \frac{H^2}{2} \cdot \boldsymbol{ft} = 6.934\ \boldsymbol{kip}$$

Surcharge soil pressure resultant:

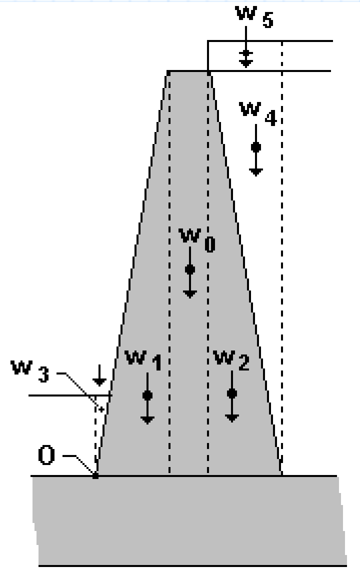
$$P_q := k_h \cdot \frac{q}{\gamma} \cdot H \cdot \boldsymbol{ft} = 1.935\ \boldsymbol{kip}$$

Overturning moment Mot:

$$M_{ot} := P_a \cdot \frac{H}{3} + P_q \cdot \frac{H}{2} = 70.493\ \boldsymbol{kip} \cdot \boldsymbol{ft}$$

Weights of wall segments, soil segments over the wall tapers, and surcharge q_w over the back face taper per unit length of wall:

$$w := \begin{bmatrix} h_{wt} \cdot C \cdot w_{rc} \\ \frac{t_f}{2} \cdot C \cdot w_{rc} \\ \frac{t_b}{2} \cdot C \cdot w_{rc} \\ \frac{t_f \cdot E^2}{2 \cdot C} \cdot \gamma \\ \frac{t_b \cdot C}{2} \cdot \gamma \\ q_w \cdot t_b \end{bmatrix}$$



$$w^{\mathrm{T}} = [3\ \ 0\ \ 0.75\ \ 0\ \ 0.5\ \ 0]\ \frac{kip}{ft}$$

Distances from the area centroids of segments to the face of the wall at top of footing:

$$x := \left[t_f + \frac{h_{wt}}{2} \;\; \frac{2 \cdot t_f}{3} \;\; t_f + h_{wt} + \frac{t_b}{3} \;\; \frac{E \cdot t_f}{3 \cdot C} \;\; h_{wb} - \frac{t_b}{3} \;\; h_{wb} - \frac{t_b}{2} \right]^{\mathrm{T}}$$

$$x^{\mathrm{T}} = [6 \;\; 0 \;\; 14 \;\; 0 \;\; 16 \;\; 15] \; in$$

Total dead load as a function of F and T:

$$w_R(F,T) := \left(\sum w + \left(B(F,T) \cdot h_f \cdot w_{rc} + \left((T \cdot E + F \cdot C) \cdot \gamma + q_w \cdot F \right) \right) \right) \cdot ft$$

Dead load resisting moment as a function of F and T:

$$M_R(F,T) := \left(\sum \left(\overrightarrow{w \cdot (x+T)} \right) + \left(\frac{B(F,T)^2 \cdot h_f}{2} \cdot w_{rc} + \left(\frac{T^2}{2} \cdot E \cdot \gamma + \left(F \cdot C \cdot \gamma + q_w \cdot F \right) \cdot \left(\frac{F}{2} + T + h_{wb} \right) \right) \right) \right) \cdot ft$$

Location of dead load resultant from the edge of the toe as a function of F an T:

$$x_R(F,T) := \frac{M_R(F,T) - M_{ot}}{w_R(F,T)}$$

Eccentricity of the dead load resultant from the footing midpoint as a function of F and T:

$$e(F,T) := \frac{B(F,T)}{2} - \frac{M_R(F,T) - M_{ot}}{w_R(F,T)}$$

Base contact bearing length as a function of F and T:

$$L_B(F,T) := \mathbf{if} \left(e(F,T) \le \frac{B(F,T)}{6}, B(F,T), 3 \cdot x_R(F,T) \right)$$

Concrete volume of the wall per unit length as a function of F and T:

$$Vol(F,T) := \left(\left(h_{wt} + \frac{t_f}{2} + \frac{t_b}{2} \right) \cdot C + B(F,T) \cdot h_f \right) \cdot ft$$

Soil bearing pressure at toe of base as a function of F and T:

$$p_{toe}(F,T) := \mathbf{if} \left(e(F,T) \ge \frac{B(F,T)}{6}, \frac{2 \cdot w_R(F,T)}{L_B(F,T)}, \frac{w_R(F,T)}{B(F,T)} + \frac{6 \cdot e(F,T) \cdot (w_R(F,T))}{B(F,T)^2} \right) \cdot \frac{1}{ft}$$

Soil bearing pressure at heel of base as a function of F and T:

$$p_{heel}(F,T) := \mathbf{if} \left(e(F,T) \ge \frac{B(F,T)}{6}, 0 \cdot \frac{kip}{ft}, \frac{w_R(F,T)}{B(F,T)} - \frac{6 \cdot e(F,T) \cdot w_R(F,T)}{B(F,T)^2} \right) \cdot \frac{1}{ft}$$

Calculations to determine required toe and heel dimensions T and F

Estimated value of heel projection F (calculated assuming the toe projection T is equal to zero, and that the weight of the wall and footing, and the soil and surcharge over the heel is just sufficient to provide the required safety factor against sliding):

$$F := \frac{SF \cdot \left(P_a + P_q\right) - c_f \cdot \left(\sum w + \left(h_{wb} \cdot h_f \cdot w_{rc}\right)\right) \cdot ft}{c_f \cdot \left(h_f \cdot w_{rc} + C \cdot \gamma + q_w\right) \cdot ft} = 8.809\ ft$$

Estimated value of toe projection T assuming F equals the value computed above and that the resultant of the gravity loads w_R is located at B/3 from the face of the toe:

$$T := \mathbf{if}\left(x_R(F, 0 \cdot ft) \geq \frac{B(F, 0 \cdot ft)}{3}, 0 \cdot ft, \frac{B(F, 0 \cdot ft)}{3} - x_R(F, 0 \cdot ft)\right) = 1.43\ ft$$

F and T with dead load equal to SF x the sum of the horizontal forces, and w_R at least B/3 from the face of the toe:

Guess Values

$F := 8\ ft \qquad T := 1\ ft$

Constraints

$T > 0$

$$F = \frac{SF \cdot \left(P_a + P_q\right) - c_f \cdot \left(\sum w + h_{wb} \cdot h_f \cdot w_{rc} + T \cdot \left(h_f \cdot w_{rc} + E \cdot \gamma\right)\right) \cdot ft}{c_f \cdot \left(h_f \cdot w_{rc} + C \cdot \gamma\right) \cdot ft}$$

$$x_R(F, T) \geq \frac{B(F, T)}{3}$$

Solver

$$\begin{bmatrix} F \\ T \end{bmatrix} := \mathbf{Find}(F, T) = \begin{bmatrix} 8.566 \\ 2.406 \end{bmatrix} ft$$

F and T with w_R equal to SF x the sum of the horizontal forces, the resisting moment greater than 2 x the overturning moment, full contact bearing on the soil, the soil bearing pressure at the heel less than or equal to the toe pressure, and toe and heel pressures less than or equal to the allowable p_s:

Guess Values

$$\frac{M_R(F, T)}{M_{...}} \geq 2 \qquad \frac{c_f \cdot w_R(F, T)}{P_a + P_{...}} = SF$$

Constrain

$p_{toe}(F,T) \le p_s$ $\quad$ $p_{toe}(F,T) \ge 0 \cdot ksf$

$L_B(F,T) = B(F,T)$ $\quad$ $p_{heel}(F,T) \le p_{toe}(F,T)$

Solver

$$\begin{bmatrix} F \\ T \end{bmatrix} := \mathbf{Find}(F, T) = \begin{bmatrix} 8.566 \\ 2.406 \end{bmatrix} ft$$

Minimum required footing width B1 before rounding:

$B_1 := F + T + h_{wb}$ $\quad$ $B_1 = 12.471\ ft$

Required Base Width, Toe and Heel Projections

Final footing width B(F,T) toe projection T, and heel projection F by rounding B_1 up to the nearest multiple of SzF and rounding the resulting value of T up to the nearest inch:

$B := \text{ceil}\left(\frac{B_1}{SzB}\right) \cdot SzB$ $\quad$ $B = 12.5\ ft$

$$T := \mathbf{if}\left(\text{ceil}\left(\frac{T-(B-B_1)}{SzF}\right) \cdot SzF > 0 \cdot ft, \text{ceil}\left(\frac{T-(B-B_1)}{SzF}\right) \cdot SzF, 0 \cdot ft\right)$$

$F := B - T - h_{wb}$ $\quad$ $T = 2.5\ ft$

$B = 12.5\ ft$ $\quad$ $F = 8.5\ ft$

Note ⇒ Walls may be checked at this point by defining known footing dimensions T and F instead of using calculated values.

$\mathbf{clear_{sym}}(T)$ $\quad$ $\mathbf{clear_{sym}}(F)$ $\quad$ $B := T + F + h_{wb}$ $\quad$ $B = 12.5\ ft$

Values of total dead load, resisting moment, toe and heel pressures, and bearing length with the final selected values of the heel and toe projections:

Toe projection:	$T = 2.5\ ft$	
Heel projection:	$F = 8.5\ ft$	
Bearing length:	$L_B := L_B(F,T)$	$L_B = 12.5\ ft$
Resisting moment:	$M_R := M_R(F,T)$	$M_R = 171.495\ kip \cdot ft$
Soil bearing pressure at toe:	$p_{toe} := p_{toe}(F,T)$	$p_{toe} = 3.822\ ksf$

Soil bearing pressure at heel: $p_{heel} := p_{heel}(F, T)$ $p_{heel} = 0.028\ ksf$

Total weight of soil and concrete: $w_R := w_R(F, T)$ $w_R = 24.063\ kip$

Concrete volume per unit length of wall: $Vol := Vol(F, T)$ $Vol = 43.75\ ft^3$

Calculations to determine soil bearing pressures at factored load

Factored dead load w_{R_f} and resisting moment M_{R_f} using an 0.9 factor for wall and footing dead loads, and a 1.4 factor for soil and surcharge weights for computing soil bearing pressures at factored loads.
(See: "Reinforced Concrete Fundamentals" by Phil M. Ferguson):

Load factors for weights:

$$f := \begin{bmatrix} 0.9 & 0.9 & 0.9 & 1.4 & 1.4 & 1.4 \end{bmatrix}^{T}$$

$$w_{R\_f} := \left(\sum \overrightarrow{w \cdot f} + 0.9 \cdot \left(B \cdot h_f \cdot w_{rc} \right) + 1.4 \cdot \left((T \cdot E + F \cdot C) \cdot \gamma + q_w \cdot F \right) \right) \cdot ft = 30.406\ kip$$

$$M_{R\_f} := \left(\left(\sum \overrightarrow{(w \cdot f) \cdot (x + T)} + 0.9 \cdot \left(\frac{B^2 \cdot h_f}{2} \cdot w_{rc} \right) \right) + 1.4 \cdot \left(\frac{T^2}{2} \cdot E \cdot \gamma + \left(F \cdot C \cdot \gamma + q_w \cdot F \right) \cdot \left(\frac{F}{2} + T + h_{wb} \right) \right) \right) \cdot ft$$

$$M_{R\_f} = 225.429\ kip \cdot ft$$

Location of factored dead load resultant from toe:

$$x_f := \frac{M_{R\_f} - 1.7 \cdot M_{ot}}{w_{R\_f}} = 3.473\ ft$$

Eccentricity of dead load resultant from base midpoint at factored load:

$$e_f := \frac{B}{2} - \frac{M_{R\_f} - 1.7 \cdot M_{ot}}{w_{R\_f}} = 2.777\ ft$$

Bearing length on base at factored load:

$$L_{B\_f} := \mathbf{if} \left(e_f \le \frac{B}{6}, F, 3 \cdot x_f \right) = 10.418\ ft$$

Soil bearing pressures at factored loads:

$$p_{toe\_f} := \mathbf{if} \left(e_f \ge \frac{B}{6}, \frac{2 \cdot w_{R\_f}}{L_{B\_f}}, \left(\frac{w_{R\_f}}{F} + \frac{6 \cdot e_f \cdot w_{R\_f}}{F^2} \right) \right) \cdot \frac{1}{ft} = 5.837\ ksf$$

$$p_{heel\_f} := \text{if}\left(e_f \ge \frac{B}{6}, 0 \cdot \frac{kip}{ft}, \left(\frac{w_{R\_f}}{F} - \frac{6 \cdot e_f \cdot w_{R\_f}}{F^2}\right)\right) \cdot \frac{1}{ft} = 0\ ksi$$

Wall Design

Shear on stem wall as a function of distance y above base:

$$V_{uw}(y) := 1.7 \cdot k_h \cdot (C - y) \cdot \left(\frac{C - y}{2} + \frac{q}{\gamma}\right) \cdot ft$$

Bending moment on stem wall as a function of distance y above base:

$$M_{uw}(y) := 1.7 \cdot k_h \cdot (C - y)^2 \cdot \left(\frac{C - y}{6} + \frac{q}{2 \cdot \gamma}\right) \cdot ft$$

Minimum required wall thickness for shear:

$$h_{shear} := \frac{V_{uw}(0 \cdot ft)}{\phi_v \cdot v_c \cdot ft} + cl_w + \frac{1}{2} \cdot d_{bw} = 12.215\ in$$

Minimum required wall thickness for flexure:

$$h_{flex} := \sqrt{\frac{M_{uw}(0 \cdot ft)}{K(\rho_{pref}) \cdot ft}} + cl_w + \frac{1}{2} \cdot d_{bw} = 15.67\ in$$

Minimum required wall thickness, compared to thickness entered:

$$h_1 := \text{if}(h_{flex} \ge h_{shear}, h_{flex}, h_{shear}) = 15.67\ in$$

$$h_{wb} = 18\ in$$

Minimum required wall thickness

Minimum required wall thickness h'w at base and corresponding wall tapers (minimum values are indicated by prime marks ') rounded up to the nearest even inch:

$$h'_{wb} := 2 \cdot in \cdot \text{ceil}\left(\frac{h_1}{2 \cdot in}\right) = 16\ in$$

$$t'_f := t_f - \frac{t_f}{t_f + t_b} \cdot (h_{wb} - h'_{wb}) = 0\ in$$

$$t'_b := t_b - \frac{t_b}{t_f + t_b} \cdot (h_{wb} - h'_{wb}) = 4\ in$$

Wall thickness and effective depth as functions of distance y above top of footing, using the larger of the entered wall thickness or the calculated wall thickness:

$$h_w(y) := \text{if}\left(h_{wb} > h'_{wb}, h_{wb}, h'_{wb}\right) - \frac{y}{C} \cdot \text{if}\left(h_{wb} > h'_{wb}, \left(t_f + t_b\right), \left(t'_f + t'_b\right)\right)$$

$$d_{ew}(y) := h_w(y) - cl_w - \frac{d_{bw}}{2}$$

Wall thickness and effective depth at top of footing and corresponding footing thickness:

$$h_w(0 \cdot ft) = 18\ in \qquad d_{ew}(0 \cdot ft) = 16.063\ in \qquad h_f = 18\ in$$

Distance y from top of footing at 1 ft intervals and at top of wall:

$$i1 := 0 .. \frac{C}{ft} \qquad y_{i1} := i1 \cdot ft \qquad N := \text{ceil}\left(\frac{C}{ft}\right) \qquad y_N := C \qquad i := 0 .. N$$

Theoretical calculated reinforcement ratio required for flexure as a function of y:

$$\rho(y) := \overrightarrow{\left(1 - \left(\sqrt{1 - \frac{2 \cdot M_{uw}(y)}{\phi_f \cdot ft \cdot d_{ew}(y)^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y}}$$

Minimum required reinforcement ratio:	$\rho_{min} = 0.333\%$
Preferred maximum reinforcement ratio:	$\rho_{pref} = 1.069\%$
Maximum reinforcement ratio:	$\max(\rho(y)) = 0.008$
Maximum permissible reinforcement ratio:	$\rho_{max} = 2.138\%$

Required reinforcement area, the larger of the theoretical calculated reinforcement or the minimum required reinforcement:

$$A_{s\_wall_i} := \text{if}\left(\rho\left(y_i\right) \ge \rho_{min}, \rho\left(y_i\right) \cdot ft \cdot d_{ew}\left(y_i\right), \rho_{min} \cdot ft \cdot d_{ew}\left(y_i\right)\right)$$

Maximum and minimum reinforcement areas:

$$\max\left(A_{s\_wall}\right) = 1.462\ in^2 \qquad min\left(A_{s\_wall}\right) = 0.403\ in^2$$

All calculated values of y, and A_{s_wall} are displayed in graphical and tabular form in the **Summary**.

Footing Design

Difference between factored toe and heel soil bearing pressures, and the effective wall depth dw to the centroid of the wall reinforcement at top of footing:

$$\Delta p\_f := p_{toe\_f} - p_{heel\_f} = 0.041\ ksi$$

$$d_w := d_{ew}(0\ ft) = 16.063\ in$$

Intermediate values used in calculating soil bearing pressures on footing:

$$X1 := \frac{L_{B\_f} - T}{L_{B\_f}} = 0.76 \qquad X3 := \frac{L_{B\_f} - T - d_w}{L_{B\_f}} = 0.632$$

$$X2 := \frac{B - T}{B} = 0.8 \qquad X4 := \frac{B - T - d_w}{B} = 0.693$$

Location of critical section for shear on the toe from front face of wall as a function of required toe thickness for shear:

$$X5(h1) := T - \left(h1 - cl_t - \frac{1}{2} \cdot d_{bt}\right)$$

Ratio of the distance from the point of minimum bearing pressure on the footing to the critical section for shear on the toe to the bearing length on the footing as a function of required toe thickness for shear:

$$X6(h1) := \frac{L_{B\_f} - X5(h1)}{L_{B\_f}}$$

Ratio of the distance from the heel to the critical section for shear on the toe to the bearing length on the footing, as a function of required toe thickness for shear:

$$X7(h1) := \frac{B - X5(h1)}{B}$$

Soil bearing pressures due to factored loads at distance from front face of wall to the critical section for shear on the toe, as a function of required toe thickness for shear:

$$p_{d\_f}(h1) := \begin{array}{|l} \text{if } \left(L_{B\_f} \ge X5(h1)\right) \vee \left(L_{B\_f} \le B\right) \\ \quad \| X6(h1) \cdot p_{toe\_f} \\ \text{else if } L_{B\_f} \le X5(h1) \\ \quad \| 0 \cdot ksf \\ \text{else} \\ \quad \| p_{heel\_f} + X7(h1) \cdot (\Delta p\_f) \end{array}$$

Soil bearing pressures on footing at front and back face of wall:

$$p_{front\_f} := \textbf{if}\left(\left(L_{B\_f} \ge T\right) + \left(L_{B\_f} \le B\right), X1 \cdot p_{toe\_f}, \textbf{if}\left(L_{B\_f} \le T, 0 \cdot ksf, p_{heel\_f} + X2 \cdot (\Delta p\_f)\right)\right)$$

$$p_{back\_f} := \mathbf{if}\left(\left(L_{B\_f} \geq T + d_w\right) + \left(L_{B\_f} \leq B\right), X3 \cdot p_{toe\_f}, \mathbf{if}\left(L_{B\_f} \leq T - d_w, 0 \cdot ksf, p_{heel\_f} + X4 \cdot \left(\Delta p\_f\right)\right)\right)$$

Shear on toe as a function of footing thickness, using 0.9 factor for opposing dead load:

$$V_{u\_toe}(h1) := \left(\frac{p_{toe\_f} + p_{d\_f}(h1)}{2} - 0.9 \cdot \left(h_f \cdot w_{rc} + E \cdot \gamma\right)\right) \cdot X5(h1) \cdot ft$$

Solve for footing thickness h1 required for shear on toe:

Guess value of h1 = hf, the initial assumed footing thickness:

$$h1 := h_f = 18\ in$$

$$f1(h1) := \phi_v \cdot v_c \cdot ft \cdot \left(h1 - cl_t - \frac{1}{2} \cdot d_{bt}\right) - V_{u\_toe}(h1)$$

$$h1 := \mathbf{root}(f1(h1), h1) \qquad h1 = 0.275\ m$$

$$V_{u\_toe}(h1) = \left(4.271 \cdot 10^4\right) \frac{kg \cdot m}{s^2}$$

Bending moment on toe using 0.9 factor for opposing dead load:

$$M_{u\_toe} := \left(\left(\frac{p_{toe\_f}}{3} + \frac{p_{front\_f}}{6}\right) \cdot T^2 - 0.9 \cdot \left(h_f \cdot w_{rc} + \left(D - h_f\right) \cdot \gamma\right) \cdot \frac{T^2}{2}\right) \cdot ft$$

$$M_{u\_toe} = \left(2.247 \cdot 10^4\right) \frac{kg \cdot m^2}{s^2}$$

Toe thickness required for flexure:

$$h2 := \sqrt{\frac{M_{u\_toe}}{K\left(\rho_{pref}\right) \cdot ft}} + cl_t + \frac{1}{2} \cdot d_{bt} \qquad h2 = 0.229\ m$$

The larger toe thickness required for shear or flexure:

$$h_{toe} := \mathbf{if}(h1 \geq h2, h1, h2) \qquad h_{toe} = 0.275\ m$$

Intermediate values used to calculate shear and bending moment on heel:

$$Y1 := cl_w + \frac{d_{bw}}{2} = 1.938\ in$$

$$Y2 := L_{B\_f} - T - d_w = 6.579\ ft$$

$$Y3 := \frac{F}{2} + Y1 = 4.411\ ft$$

Shear V_u and bending moment M_u on heel using a 1.7 load factor for surcharge load q_w:

$$V_{u\_heel} := \left(\left(1.4 \cdot \left(C \cdot \gamma + h_f \cdot w_{rc}\right) + 1.7 \cdot q_w\right) \cdot F - \frac{p_{back\_f} + p_{heel\_f}}{2} \cdot (Y2) \right) \cdot ft = 14.35\ kip$$

$$M_{u\_heel} := \left(\left(1.4 \cdot \left(C \cdot \gamma + h_f \cdot w_{rc}\right) + 1.7 \cdot q_w\right) \cdot F \cdot (Y3) - \left(\frac{p_{back\_f}}{6} + \frac{p_{heel\_f}}{3} \right) \cdot Y2^2 \right) \cdot ft = 90.207\ kip \cdot ft$$

Heel thickness required for shear:

$$h3 := \frac{V_{u\_heel}}{\phi_v \cdot v_c \cdot ft} + cl_h + \frac{1}{2} \cdot d_{bh} = 13.122\ in$$

Heel thickness required for flexure:

$$h4 := \sqrt{\frac{M_{u\_heel}}{K\left(\rho_{pref}\right) \cdot ft}} + cl_h + \frac{1}{2} \cdot d_{bh} = 15.136\ in$$

The larger heel thickness required for shear or flexure:

$$h_{heel} := \mathbf{if}\left(h3 \geq h4, h3, h4\right) = 15.136\ in$$

Minimum required footing thickness as the larger thickness required for the toe or heel:

$$h_{f1} := \mathbf{if}\left(h_{toe} \geq h_{heel}, h_{toe}, h_{heel}\right) = 15.136\ in$$

Minimum required footing thickness

Minimum footing thickness rounded up to the nearest multiple of SzF:

$$h'_f := SzF \cdot \mathrm{ceil}\left(\frac{h_{f1}}{SzF}\right) = 16\ in$$

Effective footing depths to centroids of reinforcement using the larger of the entered thickness or the calculated footing thickness:

$$d_{e\_toe} := \mathbf{if}\left(h_f > h'_f, h_f, h'_f\right) - cl_t - \frac{1}{2} \cdot d_{bt} = 14.625\ in$$

$$d_{e\_heel} := \mathbf{if}\left(h_f > h'_f, h_f, h'_f\right) - \mathbf{if}\left(M_{u\_heel} < 0 \cdot kip \cdot ft, cl_t, cl_h\right) - \frac{1}{2} \cdot d_{bh} = 16\ in$$

$$d_{e\_heel} := \text{if}\left(h_f > h'_f, h_f, h'_f\right) - \text{if}\left(M_{u\_heel} < 0 \cdot \boldsymbol{kip} \cdot \boldsymbol{ft}, cl_t, cl_h\right) - \frac{1}{2} \cdot d_{bh} = 16\ \boldsymbol{in}$$

Minimum required toe and heel reinforcement

Toe reinforcement:

$$\rho_{toe} := \left(1 - \left(\sqrt{1 - \frac{2 \cdot M_{u\_toe}}{\phi_f \cdot \boldsymbol{ft} \cdot {d_{e\_toe}}^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y}$$

$$\rho_{toe} = 0.001$$

$$A_{s\_toe} := \text{if}\left(\rho_{toe} \ge \rho_{min}, \rho_{toe} \cdot \boldsymbol{ft} \cdot d_{e\_toe}, \rho_{min} \cdot \boldsymbol{ft} \cdot d_{e\_toe}\right)$$

$$A_{s\_toe} = \left(3.774 \cdot 10^{-4}\right)\ \boldsymbol{m}^2$$

Heel reinforcement:

$$\rho_{heel} := \left(1 - \left(\sqrt{1 - \frac{2 \cdot M_{u\_heel}}{\phi_f \cdot \boldsymbol{ft} \cdot {d_{e\_heel}}^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y}$$

$$\rho_{heel} = 0.007$$

$$A_{s\_heel} := \text{if}\left(\rho_{heel} \ge \rho_{min}, \rho_{heel} \cdot \boldsymbol{ft} \cdot d_{e\_heel}, \rho_{min} \cdot \boldsymbol{ft} \cdot d_{e\_heel}\right)$$

$$A_{s\_heel} = 1.335\ \boldsymbol{in}^2$$

Compare factored shear and moment on toe and heel with useable capacities:

$$\phi V_{n\_toe} := \phi_v \cdot v_c \cdot \boldsymbol{ft} \cdot d_{e\_toe}$$

$$\phi V_{n\_toe} = \left(8.393 \cdot 10^4\right)\ \frac{\boldsymbol{kg} \cdot \boldsymbol{m}}{\boldsymbol{s}^2} \qquad V_{u\_toe}\left(h_f\right) = \left(3.007 \cdot 10^4\right)\ \frac{\boldsymbol{kg} \cdot \boldsymbol{m}}{\boldsymbol{s}^2}$$

$$\phi M_{n\_toe} := \left(\phi_f \cdot \rho_{toe} \cdot \left(\left(1 - \frac{\rho_{toe} \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y\right)\right) \cdot \boldsymbol{ft} \cdot {d_{e\_toe}}^2$$

$$\phi M_{n\_toe} = \left(2.247 \cdot 10^4\right)\ \frac{\boldsymbol{kg} \cdot \boldsymbol{m}^2}{\boldsymbol{s}^2} \qquad M_{u\_toe} = \left(2.247 \cdot 10^4\right)\ \frac{\boldsymbol{kg} \cdot \boldsymbol{m}^2}{\boldsymbol{s}^2}$$

$$\phi V_{n\_heel} := \phi_v \cdot v_c \cdot \boldsymbol{ft} \cdot d_{e\_heel}$$

$$\phi V_{n\_heel} = \left(9.183 \cdot 10^4\right)\ \frac{\boldsymbol{kg} \cdot \boldsymbol{m}}{\boldsymbol{s}^2} \qquad V_{u\_heel} = \left(6.383 \cdot 10^4\right)\ \frac{\boldsymbol{kg} \cdot \boldsymbol{m}}{\boldsymbol{s}^2}$$

$$\phi M_{n\_heel} := \left(\phi_f \cdot \rho_{heel} \cdot \left(1 - \frac{\rho_{heel} \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y\right) \cdot \boldsymbol{ft} \cdot {d_{e\_heel}}^2$$

$$2 \cdot 0.85 \cdot f'_c$$

$$\phi M_{n\_heel} = \left(1.223 \cdot 10^5\right) \frac{kg \cdot m^2}{s^2} \qquad M_{u\_heel} = \left(1.223 \cdot 10^5\right) \frac{kg \cdot m^2}{s^2}$$

Summary

Material Properties and Constants

Sizing factor for rounding base width: $SzB = 0.076\ m$

Sizing factor for rounding toe projection and wall and footing thicknesses: $SzF = 0.051\ m$

Factor of safety against sliding: $SF = 1.5$

Specified compressive strength of concrete: $f'_c = 4\ ksi$

Specified yield strength of reinforcement: $f_y = 60\ ksi$

Unit weight of concrete: $w_c = 0.084\ \dfrac{lbf}{in^3}$

Concrete cover of wall reinforcement: $cl_w = 0.038\ m$

Concrete cover of bottom toe reinforcement: $cl_t = 0.076\ m$

Concrete cover of top heel reinforcement: $cl_h = 0.038\ m$

Preferred reinforcement ratio: $\rho_{pref} = 0.011$

Shear Strength reduction factor for lightweight concrete: $k_v = 1$

Unit weight of reinforced concrete: $w_{rc} = 0.087\ \dfrac{psi}{in}$

Input

Total height from top of wall to bottom of footing: $H = 6.553\ m$

Depth from lower grade level to bottom of footing: $D = 0\ m$

Surcharge on backfill: $q = 2.083\ psi$

Surcharge weight over the heel: $q_w = 0\ psi$

Estimated footing thickness: $h_f = 0.457\ m$

Estimated wall thickness at top of footing: $h_{wb} = 0.457\ m$

Front face wall taper: $t_f = 0\ m$

Front face wall taper: $t_f = 0\ m$

Back face wall taper: $t_b = 0.152\ m$

Unit weight of soil: $\gamma = 0.694\ \frac{psi}{ft}$

Equivalent horizontal fluid pressure due to soil: $k_h = \left(1.736 \cdot 10^{-5}\right)\ \frac{ksi}{in}$

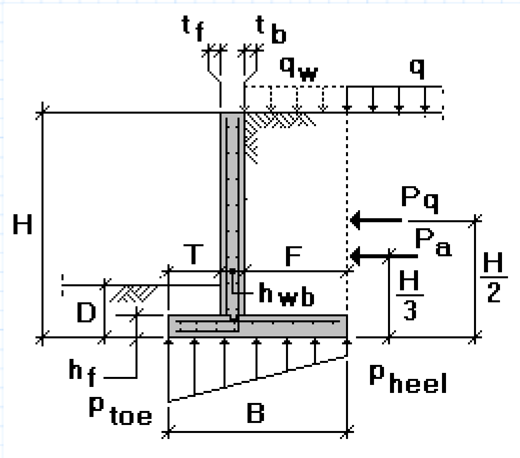
Coefficient of friction between footing and soil: $c_f = 0.55$

Allowable soil bearing pressure: $p_s = 0.028\ ksi$

Wall reinforcing bar diameter: $d_{bw} = 0.875\ in$

Toe reinforcing bar diameter: $d_{bt} = 0.75\ in$

Heel reinforcing bar diameter: $d_{bh} = 1\ in$



Wall Dimensions

Toe projection: $T = 0.762\ m$

Footing width: $B = 3.81\ m$

Heel Projection: $F = 2.591\ m$

Minimum required wall thickness: $h'_{wb} = 0.406\ m$

Minimum required footing thickness: $h'_f = 0.406\ m$

Front wall taper corresponding to minimum required wall thickness: $t'_f = 0\ m$

Back wall taper corresponding to minimum required wall thickness: $t'_b = 0.102\ m$

The larger of the entered or calculated wall and footing thicknesses are used for calculating shear and required reinforcement. If the calculated values for wall and footing thickness are larger than those entered, the application may be re-run with the calculated values as a final check.

Overturning and Resisting Forces and Moments

Horizontal force due to active soil pressure: $P_a = 6.934\ kip$

Horizontal force due to surcharge: $P_q = 1.935\ kip$

Overturning Moment: $M_{ot} = 70.493\ kip \cdot ft$

Total dead load: $w_R = 24.063\ kip$

Ratio of frictional sliding resistance to horizontal sliding forces: $\dfrac{c_f \cdot w_R}{P_a + P_q} = 1.492$

Dead load resisting moment: $M_R = 171.495\ kip \cdot ft$

Ratios of resisting dead load moment to overturning moment: $\dfrac{M_R}{M_{ot}} = 2.433$

Service and Factored Load Bearing Pressures on Base

Service load toe pressure: $p_{toe} = 0.027\ ksi$

Service load heel pressure: $p_{heel} = \left(1.977 \cdot 10^{-4}\right)\ ksi$

Base bearing length at service load: $L_B = 12.5\ ft$

Factored load toe pressure: $p_{toe\_f} = 0.041\ ksi$

Factored load heel pressure: $p_{heel\_f} = 0\ \dfrac{kg}{m \cdot s^2}$

Base bearing length at factored load: $L_{B\_f} = 3.175\ m$

Required Toe and Heel Reinforcement

Required toe reinforcement per unit length of wall: $A_{s\_toe} = \left(3.774 \cdot 10^{-4}\right)\ m^2$

Required toe reinforcement per unit length of wall: $A_{s\_heel} = \left(8.611 \cdot 10^{-4}\right)\ m^2$

Concrete volume per lineal foot of wall: $Vol = 1.239\ m^3$

Calculated and minimum required reinforcement area for stem wall versus height y from top of footing to top of wall:

$$y_i = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \end{bmatrix} ft \qquad A_{s\_wall_i} = \begin{bmatrix} 1.462 \\ 1.289 \\ 1.129 \\ 0.983 \\ 0.849 \\ 0.727 \\ 0.617 \\ 0.559 \\ 0.547 \\ 0.535 \\ 0.523 \\ 0.511 \\ 0.499 \\ 0.487 \\ 0.475 \\ 0.463 \\ 0.451 \\ 0.439 \\ 0.427 \\ 0.415 \\ 0.403 \end{bmatrix} in^2$$