CHAPTER 8: Earth Retaining Structures

8.2 Reinforced Concrete Retaining Walls with Sloping Backhill

Description

Earth retaining walls are used in construction wherever a separation of grades at different levels is required. Retaining walls must be designed to resist the lateral pressures due to the retained soil.

This application determines required footing width, footing thickness, wall thickness and reinforcement areas for retaining walls with sloping backfill. Lateral earth pressures and the slope of the backfill may be specified by the user.

Intermediate values computed by this application include the lateral forces on a retaining wall wall due to earth pressure from a sloping backfill, the weight of the wall and the weight of the soil over the toe and heel of the base, the required toe and heel projections of the retaining wall base or footing, bearing length and base soil pressures at the toe and heel under service loads and factored loads, overturning moment and sliding force, resisting moment and forces resisting sliding, and wall and footing thicknesses required for flexure and shear. The Strength Design Method of ACI 318-89 is used.

The required input for this application includes the strength of the concrete and the reinforcement, unit weights of concrete, reinforced concrete and soil, the backfill slope, the internal angle of friction of the soil or the equivalent horizontal and vertical fluid pressures due to the soil, the coefficient of friction between footing and soil, the allowable soil bearing pressure at service load, the required safety factor against sliding, total wall height from top of wall to bottom of footing, depth from the lower grade level to the bottom of the footing, estimated wall and footing thicknesses and front and back face wall tapers (if applicable), preferred reinforcement ratio, rounding factors for base width and wall and footing thicknesses, clear concrete cover of wall, toe, and heel reinforcement, and the estimated reinforcing bar sizes for the wall, toe and heel.

This application uses Mathcad solve blocks to determine the smallest required base width for full bearing on the base with zero heel pressure under service loads (unless a wider base is required to avoid exceeding the permissible soil bearing pressure), to provide an adequate safety factor against sliding, or to obtain a ratio of resisting moment to overturning moment of two.

The application uses the estimated values for wall and footing thicknesses (unless the application determines that larger footing or wall thicknesses are required for shear or flexure). If it is necessary to revise the estimated values for wall or footing thicknesses, or bar sizes, the application may be rerun with the actual dimensions and bar sizes to confirm the initial results. Walls with known toe and heel dimensions may be checked by entering the toe and heel dimensions where indicated within this document.

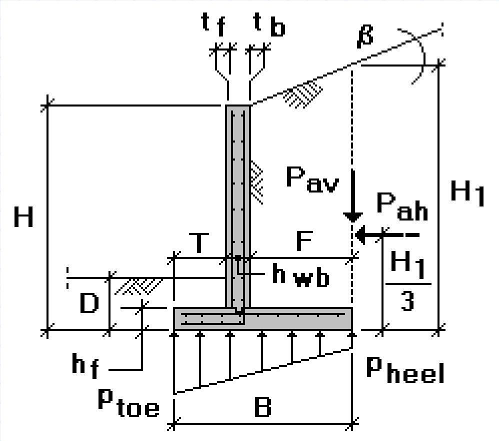
A summary of input and calculated values is shown on pages 22-26.

Reference:

- ACI 318-89 "Building Code Requirements for Reinforced Concrete." (Revised 1992)
- "Reinforced Concrete Fundamentals, 3rd Edition," by Phil M. Ferguson

Input

Notation



Input Variables

$$d_b := [0\ \ 0\ \ 0\ \ 0.375\ \ 0.5\ \ 0.625\ \ 0.75\ \ 0.875\ \ 1.00\ \ 1.128\ \ 1.27\ \ 1.41\ \ 0\ \ 0\ \ 1.693\ \ 0\ \ 0\ \ 0\ \ 2.257]^T \cdot in$$

Total height from top of wall to bottom of footing:	$H := 22\ ft$
Depth from lower grade level to bottom of footing:	$D := 0\ ft$
Estimated footing thickness:	$h_f := 18\ in$
Estimated wall thickness at top of footing:	$h_{wb} := 18\ in$
Front face wall taper:	$t_f := 0\ in$
Back face wall taper:	$t_b := 6\ in$
Unit weight of soil:	$\gamma := 100\ pcf$
Coefficient of friction between footing and soil:	$c_f := 0.55$
Allowable soil bearing pressure:	$p_s := 5\ ksf$
Wall reinforcing bar diameter:	$d_{bw} := d_{b_7}$
Toe reinforcing bar diameter:	$d_{bt} := d_{b_6}$
Heel reinforcing bar diameter:	$d_{bh} := d_{b_8}$

The variable d_b represents standard reinforcing bars No. 3 to No. 11, No. 14 and No. 18, defined below. The subscript number designates the specific bar size. Bar diameters may also be entered directly.

Enter the assumed angle of internal friction ϕ and the backfill slope angle β. β must be less than or equal to ϕ. Enter ϕ = 0 deg if k_{ah} and k_{av} are to be entered directly:

$$\phi := 33.67 \cdot deg \qquad \beta := \text{atan}\left(\frac{1}{2}\right) = 26.565\ deg$$

k_{ah} and k_{av} computed using Rankine Theory:

$$k_a := \cos(\beta) \cdot \left(\frac{\cos(\beta) - \sqrt{\cos(\beta)^2 - \cos(\phi)^2}}{\cos(\beta) + \sqrt{\cos(\beta)^2 - \cos(\phi)^2}} \cdot \gamma \right) = 41.48\ \frac{psf}{ft}$$

$$k_{ah} := k_a \cdot \cos(\beta) = 37.1\ \frac{psf}{ft}$$

$$k_{av} := k_a \cdot \sin(\beta) = 18.55\ \frac{psf}{ft}$$

Initially, the values for wall and footing thicknesses, wall tapers, and bar sizes must be estimated. If the final selected values differ significantly form those initially assumed this application should be repeated using the final values.

Computed Variables

H_1	height from bottom of footing at heel to top of sloping grade
B	minimum required width of footing
T	minimum required toe projection from face of wall at top of footing
F	minimum required heel projection from face of wall at top of footing
P_{ah}	horizontal force due to soil pressure
P_{av}	vertical force due to soil pressure
W_R	total dead load
M_{ot}	overturning moment due to active soil pressure and surcharge
M_R	resisting moment due to dead load
p_{toe}	soil bearing pressure at toe of footing at service load
p_{heel}	soil bearing pressure at heel of footing at service loads
L_b	length of bearing on base at service load
p_{toe_f}	soil bearing pressure at toe of footing at factored load
p_{heel_f}	soil bearing pressure at heel of footing at service loads
L_{b_f}	length of bearing on base at service load
h_f	minimum required footing thickness
h_{wb}	minimum required wall thickness at top of footing
t_f	front face wall taper corresponding to required wall thickness
t_b	back face wall taper corresponding to required wall thickness
A_{s_toe}	required area of toe reinforcement
A_{s_heel}	required area of heel reinforcement
A_{s_wall}	required areas of wall reinforcement at specified points

Material Properties and Constants

Enter f'_c, f_y, w_c, w_{rc}, k_v and k_w if different from that shown.

Specified compressive strength of concrete: $f'_c := 4\ ksi$

Specified yield strength of reinforcement (fy may not exceed 60 ksi, ACI 318 11.5.2): $f_y := 60\ ksi$

Unit weight of concrete:	$w_c := 145\ pcf$
Weight of reinforced concrete:	$w_{rc} := 150\ pcf$
Shear strength reduction factor for lightweight concrete kv = 1 for normal weight, 0.75 for all-lightweight and 0.85 for sand-lightweight concrete (ACI 318, 11.2.1.2.):	$k_v := 1$
Weight factor for increasing development and splice lengths kw =1 for normal weight and 1.3 for lightweight aggregate concrete (ACI 318, 12.2.4.2):	$k_w := 1$
Modulus of elasticity of reinforcement (ACI 318, 8.5.2):	$E_s := 29000\ ksi$
Strain in concrete at compression failure (ACI 318, 10.3.2):	$\varepsilon_c := 0.003$
Strength reduction factor for flexure (ACI 318, 9.3.2.1):	$\phi_f := 0.9$
Strength reduction factor for shear (ACI 318, 9.3.2.3):	$\phi_v := 0.85$
Sizing factor for rounding wall and footing thicknesses:	$SzF := 2\ in$
Sizing factor for rounding base width:	$SzB := 3\ in$
Concrete cover of bottom toe reinforcement:	$cl_t := 3\ in$
Concrete cover of top heel reinforcement:	$cl_h := 1.5\ in$
Concrete cover of wall reinforcement:	$cl_w := 1.5\ in$
Sliding factor of safety:	$SF := 1.5$

The safety factor for the base sliding on soil should not be less than 1.5 unless other means are provided to prevent sliding.

Reinforcing bar number designations, diameters and areas:

$$No := [0\ \ 1\ \ 2\ \ 3\ \ 4\ \ 5\ \ 6\ \ 7\ \ 8\ \ 9\ \ 10\ \ 11\ \ 12\ \ 13\ \ 14\ \ 15\ \ 16\ \ 17\ \ 18]^T$$

$$d_b := [0\ \ 0\ \ 0\ \ 0.375\ \ 0.5\ \ 0.625\ \ 0.75\ \ 0.875\ \ 1.00\ \ 1.128\ \ 1.27\ \ 1.41\ \ 0\ \ 0\ \ 1.693\ \ 0\ \ 0\ \ 0\ \ 2.257]^T \cdot in$$

$$A_b := [0\ \ 0\ \ 0\ \ 0.11\ \ 0.20\ \ 0.31\ \ 0.44\ \ 0.60\ \ 0.79\ \ 1.00\ \ 1.27\ \ 1.56\ \ 0\ \ 0\ \ 2.25\ \ 0\ \ 0\ \ 0\ \ 4.00]^T \cdot in^2$$

Bar numbers, diameters and areas are in the vector rows (or columns in the transposed vectors shown) corresponding to the bar numbers. Individual bar numbers, diameters, areas and development lengths and splices of a specific bar can be referred to by using the vector subscripts as shown in the example below.

Example: $No_5 = 5$ $d_{b_5} = 0.625\ in$ $A_{b_5} = 0.31\ in^2$

Limit the value of f'c for computing shear and development lengths to 10 ksi by substituting f'_{c_max} for f'_c in formulas for computing shear (ACI 318, 11.1.2, 12.1.2):

$$f'_{c\_max} := \mathbf{if}\left(f'_c > 10 \cdot ksi, 10 \cdot ksi, f'_c\right)$$

The following values are computed from the entered material properties.

Nominal "one way" shear strength per unit area in concrete (ACI 318, 11.3.1.1, Eq. (11-3), 11.5.4.3):

$$v_c := k_v \cdot 2 \cdot \sqrt{\frac{f'_{c\_max}}{psi}} \cdot psi = 126.491\ psi$$

Strain in reinforcement at yield stress:

$$\varepsilon_y := \frac{f_y}{E_s} = 0.00207$$

Factor used to calculate depth of equivalent rectangular stress block (ACI 318, 10.2.7.3):

$$\beta_1 := \mathbf{if}\left(\left(f'_c \geq 4 \cdot ksi\right) \cdot \left(f'_c \leq 8 \cdot ksi\right), 0.85 - 0.05 \cdot \frac{f'_c - 4 \cdot ksi}{ksi}, \mathbf{if}\left(\left(f'_c \leq 4 \cdot ksi\right), 0.85, 0.65\right)\right) = 0.85$$

Reinforcement ratio producing balanced strain conditions (ACI 318, 10.3.2):

$$\rho_b := \frac{\beta_1 \cdot 0.85 \cdot f'_c}{f_y} \cdot \frac{E_s \cdot \varepsilon_c}{E_s \cdot \varepsilon_c + f_y} = 2.851\%$$

Maximum reinforcement ratio (ACI 318, 10.3.3):

$$\rho_{max} := \frac{3}{4} \cdot \rho_b = 2.138\%$$

Minimum reinforcement ratio for beams (ACI 318, 10.5.1, (10-3)):

$$\rho_{min} := \frac{200}{f_y} \cdot \frac{lbf}{in^2} = 0.333\%$$

Shrinkage and temperature reinforcement ratio (ACI 318, 7.12.2.1):

$$\rho_{temp} := \begin{cases} \text{if } f_y \le 50\ ksi & 0.002 \\ \text{else if } f_y \le 60\ ksi & 0.002 - \dfrac{f_y}{60\ ksi} \cdot 0.0002 \\ \text{else if } \dfrac{0.0018 \cdot (60\ ksi)}{f_y} \ge 0.0014 & \dfrac{0.0018 \cdot (60\ ksi)}{f_y} \\ \text{else} & 0.0014 \end{cases} = 0.18\%$$

Preferred reinforcement ratio:

$$\rho_{pref} := \frac{1}{2} \cdot \rho_{max} = 1.069\%$$

Flexural coefficient K, for rectangular beams or slabs, as a function of ρ (ACI 318, 10.2):
(Moment capacity $\phi M_n = K(\rho)F$, where $F = bd^2$)

$$K(\rho) := \phi_f \cdot \rho \cdot \left(1 - \frac{\rho \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y$$

Calculations

Thickness at top of wall:

$$h_{wt} := h_{wb} - t_f - t_b = 12\ \mathit{in}$$

Stem wall height:

$$C := H - h_f = 20.5\ \mathit{ft}$$

Depth of fill over toe:

$$E := \textbf{if}\left(\left(D - h_f\right) \ge 0 \cdot \mathit{ft}, D - h_f, 0 \cdot \mathit{ft}\right) = 0\ \mathit{ft}$$

Footing width B as a function of F and T:

$$B(F, T) := T + h_{wb} + F$$

Height H_1 as a function of F:

$$H_1(F) := H + \left(F + t_b\right) \cdot \tan(\beta)$$

Soil pressure resultants P_{ah} and P_{av} as a function of F:

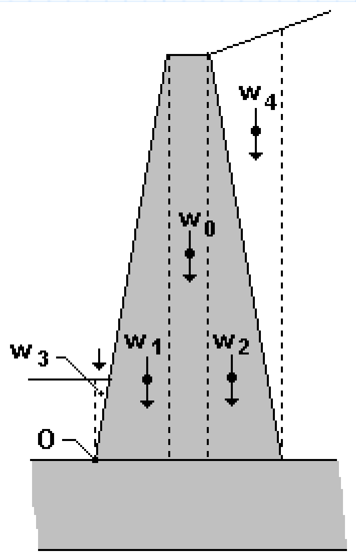
$$P_{ah}(F) := k_{ah} \cdot \frac{H_1(F)^2}{2} \cdot \mathit{ft} \qquad P_{av}(F) := k_{av} \cdot \frac{H_1(F)^2}{2} \cdot \mathit{ft}$$

Overturning moment M_{ot} as a function of F:

$$M_{ot}(F) := P_{ah}(F) \cdot \frac{H_1(F)}{3}$$

Weights of wall segments and soil segments over the wall tapers per unit length of wall:

$$w := \begin{bmatrix} h_{wt} \cdot C \cdot w_{rc} \\ \frac{t_f}{2} \cdot C \cdot w_{rc} \\ \frac{t_b}{2} \cdot C \cdot w_{rc} \\ \frac{t_f \cdot E^2}{2 \cdot C} \cdot \gamma \\ \frac{t_b \cdot \left(C + t_b \cdot \tan(\beta)\right)}{2} \cdot \gamma \end{bmatrix}$$



$$w^{\mathrm{T}} = [3.075\ \ 0\ \ 0.769\ \ 0\ \ 0.519]\ \frac{\mathit{kip}}{\mathit{ft}}$$

Distances from the area centroids of segments to the face of the wall at top of footing:

$$x := \left[t_f + \frac{h_{wt}}{2} \;\; \frac{2 \cdot t_f}{3} \;\; t_f + h_{wt} + \frac{t_b}{3} \;\; \frac{E \cdot t_f}{3 \cdot C} \;\; h_{wb} - \frac{t_b}{3} \right]^{\mathrm{T}}$$

$$x^{\mathrm{T}} = [6 \;\; 0 \;\; 14 \;\; 0 \;\; 16] \; in$$

Total dead load as a function of F and T:

$$w_R(F,T) := P_{av}(F) + \left(\sum w + \left(B(F,T) \cdot h_f \cdot w_{rc} + \left(\left(T \cdot E + F \cdot \left(C + \left(\frac{F}{2} + t_b \right) \cdot \tan(\beta) \right) \right) \right) \cdot \gamma \right) \right) \cdot ft$$

Distance from top of wall to top of sloping backfill at back face of wall at top of footing:

$$X1 := t_b \cdot \tan(\beta) = 3 \; in$$

Distance from top of backfill at back face of wall at top of footing to top of backfill at edge of heel as a function of F:

$$X2(F) := \frac{F \cdot \tan(\beta)}{2}$$

Dead load resisting moment as a function of F and T:

$$M_R(F,T) := \left\| \begin{array}{l} B \leftarrow B(F,T) \\ P_{av}(F) \cdot B + \left(\sum \left(\overrightarrow{w \cdot (x+T)} \right) + \left(\frac{B^2 \cdot h_f}{2} \cdot w_{rc} + \left(\frac{T^2}{2} \cdot E + F \cdot \left((C + X1) \cdot \left(B - \frac{F}{2} \right) + X2(F) \cdot \left(B - \frac{F}{3} \right) \right) \right) \cdot \gamma \right) \right) \end{array} \right.$$

Location of dead load resultant from the edge of the toe as a function of F an T:

$$x_R(F,T) := \frac{M_R(F,T) - M_{ot}(F)}{w_R(F,T)}$$

Eccentricity of the dead load resultant from the footing midpoint as a function of F and T:

$$e(F,T) := \frac{B(F,T)}{2} - \frac{M_R(F,T) - M_{ot}(F)}{w_R(F,T)}$$

Base contact bearing length as a function of F and T:

$$L_B(F,T) := \textbf{if}\left(e(F,T) \le \frac{B(F,T)}{6}, B(F,T), 3 \cdot x_R(F,T) \right)$$

Concrete volume of the wall per unit length as a function of F and T:

$$Vol(F,T) := \left(\left(h_{wt} + \frac{t_f}{2} + \frac{t_b}{2}\right) \cdot C + B(F,T) \cdot h_f\right) \cdot ft$$

Soil bearing pressure at toe of base as a function of F and T:

$$p_{toe}(F,T) := \text{if}\left(e(F,T) \ge \frac{B(F,T)}{6}, \frac{2 \cdot w_R(F,T)}{L_B(F,T)}, \frac{w_R(F,T)}{B(F,T)} + \frac{6 \cdot e(F,T) \cdot (w_R(F,T))}{B(F,T)^2}\right) \cdot \frac{1}{ft}$$

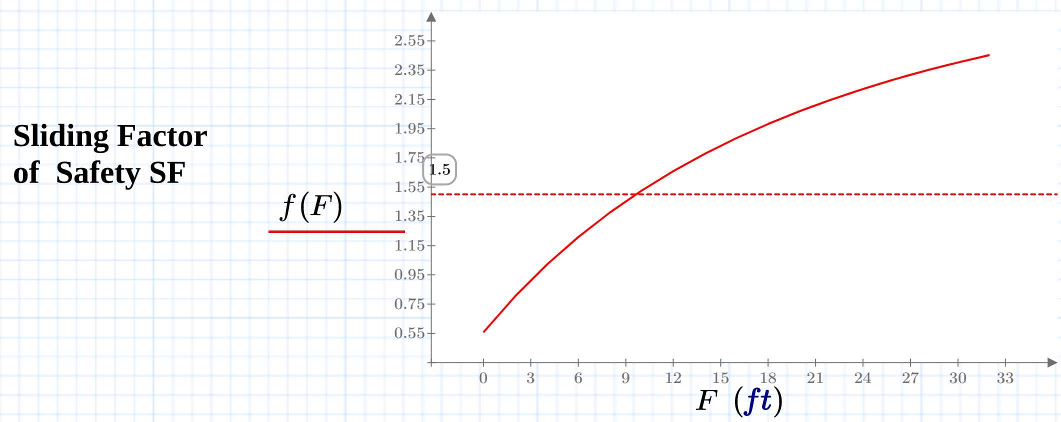
Soil bearing pressure at heel of base as a function of F and T:

$$p_{heel}(F,T) := \text{if}\left(e(F,T) \ge \frac{B(F,T)}{6}, 0 \cdot \frac{kip}{ft}, \frac{w_R(F,T)}{B(F,T)} - \frac{6 \cdot e(F,T) \cdot w_R(F,T)}{B(F,T)^2}\right) \cdot \frac{1}{ft}$$

Check feasibility of proposed design

For some walls with a steep sloping backfill and a low sliding resistance, there may be no feasible base length. A plot of the factor of safety against sliding with T = F/2, versus heel length F will show if a practical design is feasible:

$$F := 0 \cdot ft, 2 \cdot ft .. 1.5 \cdot H \qquad f(F) := \frac{c_f \cdot w_R\left(F, \frac{F}{2}\right)}{P_{ah}(F)}$$



Heel Length F in feet

If the required factor of safety for sliding cannot be provided with a practical heel length the backfill slope must be reduced, or the sliding resistance must be increased. If necessary, a base shear lug can be used to raise the coefficient of sliding from the value for concrete on soil to the value for soil on soil (approximately 25%). The placement of base shear lugs requires inspection. If the permissible soil bearing pressure is very low it is possible that there is no base width that meets all of the criteria. In this case the Mathcad solve block will give the message "did not find solution".

Calculations to determine required toe and heel dimensions T and F

Guess values of F and T: $F := H$ $T := 0 \cdot ft$

Estimated value of heel projection F calculated assuming the toe projection T is equal to zero, and that the weight of the wall and footing, and the soil over the heel, is just sufficient to provide the required safety factor against sliding:

$F := \mathbf{root}\left(c_f \cdot w_R(F,T) - SF \cdot \left(P_{ah}(F)\right), F\right)$ $F = 10.286\ ft$

$\dfrac{M_R(F,T)}{M_{ot}(F)} = 2.082$ $\dfrac{c_f \cdot \left(w_R(F,T)\right)}{P_{ah}(F)} = 1.5$

$p_{toe}(F,T) = 6.984\ ksf$ $p_{heel}(F,T) = 0\ ksf$

$L_B(F,T) = 10.872\ ft$ $B(F,T) = 11.786\ ft$

F and T with w_R equal to SF x Pah(F), the resisting moment greater than 2 times the overturning moment, full contact bearing on the soil, and toe and heel pressures less than or equal to the allowable p_s:

Guess Values

$F = 10.286\ ft$ $T = 0\ ft$

Constraints

$\dfrac{c_f \cdot w_R(F,T)}{P_{ah}(F)} = SF$ $L_B(F,T) = B(F,T)$

$p_{toe}(F,T) \le p_s$ $p_{heel}(F,T) \le p_s$

Solver

$\begin{bmatrix} F \\ T \end{bmatrix} := \mathbf{Find}(F,T) = \begin{bmatrix} 10.088 \\ 1.481 \end{bmatrix} ft$

$\dfrac{M_R(F,T)}{M_{ot}(F)} = 2.48$ $\dfrac{c_f \cdot w_R(F,T)}{P_{ah}(F)} = 1.5$

$p_{toe}(F,T) = 5\ ksf$ $p_{heel}(F,T) = 0.768\ ksf$

$L_B(F,T) = 13.068\ ft$ $B(F,T) = 13.068\ ft$

$F = 10.088\ ft$ $T = 1.481\ ft$

$B_1 := F + T + h_{wb}$ $B_1 = 13.068\ ft$

Required base width, toe and heel projections

Final footing width B and toe projection T by rounding B_1 up to the nearest multiple of SzB, and rounding the resulting value of T up to the nearest multiple of SzF:

$SzB = 3\ in$ $\qquad$ $SzF = 2\ in$

$$B := \text{ceil}\left(\frac{B_1}{SzB}\right) \cdot SzB = 13.25\ ft$$

$$T := \textbf{if}\left(\text{ceil}\left(\frac{T-(B-B_1)}{SzF}\right) \cdot SzF > 0 \cdot ft, \text{ceil}\left(\frac{T-(B-B_1)}{SzF}\right) \cdot SzF, 0 \cdot ft\right) = 1.333\ ft$$

$$F := B - T - h_{wb} = 10.417\ ft$$

Note ⇒ Walls may be checked at this point by defining known footing dimensions T and F instead of using calculated values.

$$B := T + F + h_{wb} = 13.25\ ft$$

Values of total dead load, horizontal and vertical forces due to soil pressure, overturning moment, resisting moment, toe and heel pressures, bearing length and concrete volume with the final selected values of the heel and toe projections, defined as simple variables:

Toe projection:		$F = 10.417\ ft$
Heel projection:		$T = 1.333\ ft$
Total weight of soil and concrete:	$w_R := w_R(F, T)$	$w_R = 38.664\ kip$
Horizontal force due to soil pressure:	$P_{ah} := P_{ah}(F)$	$P_{ah} = 13.986\ kip$
Vertical force due to soil pressure:	$P_{av} := P_{av}(F)$	$P_{av} = 6.993\ kip$
Overturning moment:	$M_{ot} := M_{ot}(F)$	$M_{ot} = 128.012\ kip \cdot ft$
Resisting moment:	$M_R := M_R(F, T)$	$M_R = 321.693\ kip \cdot ft$
Soil bearing pressure at toe:	$p_{toe} := p_{toe}(F, T)$	$p_{toe} = 5.053\ ksf$
Soil bearing pressure at heel:	$p_{heel} := p_{heel}(F, T)$	$p_{heel} = 0.783\ ksf$

Bearing length: $L_B := L_B(F, T)$ $L_B = 13.25\ ft$

Concrete volume
per unit length of wall: $Vol := Vol(F, T)$ $Vol = 45.5\ ft^3$

Calculations to determine soil bearing pressures at factored load

Factored dead load w_{R_f} and resisting moment M_{R_f} using an 0.9 factor for wall and footing dead loads, and a 1.4 factor for soil and surcharge weights for computing soil bearing pressures at factored loads.
(See: "Reinforced Concrete Fundamentals" by Phil M. Ferguson):

Load factors for weights w: $f := [0.9\ \ 0.9\ \ 0.9\ \ 1.4\ \ 1.4]^T$

Distance from top of wall to top of sloping backfill at back face of wall at top of footing:

$$X1 := t_b \cdot \tan(\beta) = 3\ in$$

Distance from top of backfill at back face of wall at top of footing to top of backfill at edge of heel, as a function of F:

$$X2 := \frac{F \cdot \tan(\beta)}{2} = 2.604\ ft$$

Factored dead load:

$$w_{R\_f} := 1.7 \cdot P_{av} + \left(\sum \overrightarrow{w \cdot f} + 0.9 \cdot \left(B \cdot h_f \cdot w_{rc}\right) + 1.4 \cdot \left(\left(T \cdot E + F \cdot \left(C + \left(\frac{F}{2} + t_b \right) \cdot \tan(\beta) \right) \right) \cdot \gamma \right) \right) \cdot ft = 52.815\ kip$$

Factored dead load resisting moment:

$$M_{R\_f} := 1.7 \cdot P_{av} \cdot B + \left(\sum \overrightarrow{(w \cdot f) \cdot (x + T)} + 0.9 \cdot \left(\frac{B^2 \cdot h_f}{2} \cdot w_{rc} \right) + 1.4 \cdot \left(\frac{T^2}{2} \cdot E \cdot \gamma + \left(F \cdot (C + X1) \cdot \left(\frac{F}{2} + T + h_{wb} \right) + F \cdot X2 \right.\right.\right.$$

$$M_{R\_f} = 464.512\ kip \cdot ft$$

Location of factored dead load resultant from toe:

$$x_f := \frac{M_{R\_f} - 1.7 \cdot M_{ot}}{w_{R\_f}} = 4.675\ ft$$

Eccentricity of dead load resultant from base midpoint at factored load:

$$e_f := \frac{B}{2} - \frac{M_{R\_f} - 1.7 \cdot M_{ot}}{w_{R\_f}} = 23.404\ in$$

Bearing length on base at factored load:

$$L_{B\_f} := \text{if}\left(e_f \le \frac{B}{6}, B, 3 \cdot x_f\right) = 13.25\ ft$$

Soil bearing pressures at factored loads:

$$p_{toe\_f} := \text{if}\left(e_f \ge \frac{B}{6}, \frac{2 \cdot w_{R\_f}}{L_{B\_f}}, \left(\frac{w_{R\_f}}{B} + \frac{6 \cdot e_f \cdot w_{R\_f}}{B^2}\right)\right) \cdot \frac{1}{ft} = 7.506\ ksf$$

$$p_{heel\_f} := \text{if}\left(e_f \ge \frac{B}{6}, 0 \cdot \frac{kip}{ft}, \left(\frac{w_{R\_f}}{B} - \frac{6 \cdot e_f \cdot w_{R\_f}}{B^2}\right)\right) \cdot \frac{1}{ft} = 0.466\ ksf$$

Wall Design

Shear on stem wall as a function of distance y above base:

$$V_{uw}(y) := 1.7 \cdot k_{ah} \cdot (C - y) \cdot \left(\frac{C - y}{2}\right) \cdot ft$$

Bending moment on stem wall as a function of distance y above base:

$$M_{uw}(y) := 1.7 \cdot k_{ah} \cdot (C - y)^2 \cdot \left(\frac{C - y}{6}\right) \cdot ft$$

Minimum required wall thickness for shear:

$$h_{shear} := \frac{V_{uw}(0 \cdot ft)}{\phi_v \cdot v_c \cdot ft} + cl_w + \frac{1}{2} \cdot d_{bw} = 12.209\ in$$

Minimum required wall thickness for flexure:

$$h_{flex} := \sqrt{\frac{M_{uw}(0 \cdot ft)}{K(\rho_{pref}) \cdot ft}} + cl_w + \frac{1}{2} \cdot d_{bw} = 15.099\ in$$

Minimum required wall thickness, compared to thickness entered:

$$h_1 := \text{if}\left(h_{flex} \ge h_{shear}, h_{flex}, h_{shear}\right) = 15.099\ in$$

$$h_{wb} = 18\ in$$

Minimum required wall thickness

Minimum required wall thickness h'wb at base and corresponding wall tapers (minimum values are indicated by prime marks ') rounded up to the nearest even inch:

$$h'_{wb} := 2 \cdot in \cdot \text{ceil}\left(\frac{h_1}{2 \cdot in}\right) = 16\ in$$

$$t'_b := t_b - \frac{t_b}{t_f + t_b} \cdot \left(h_{wb} - h'_{wb}\right) = 4\ in$$

$$t'_f := t_f - \frac{t_f}{t_f + t_b} \cdot \left(h_{wb} - h'_{wb}\right) = 0\ in$$

Wall thickness and effective depth as functions of distance y above top of footing using the larger of the entered wall thickness or the calculated wall thickness:

$$h_w(y) := \textbf{if}\left(h_{wb} > h'_{wb}, h_{wb}, h'_{wb}\right) - \frac{y}{C} \cdot \textbf{if}\left(h_{wb} > h'_{wb}, \left(t_f + t_b\right), \left(t'_f + t'_b\right)\right)$$

$$d_{ew}(y) := h_w(y) - cl_w - \frac{d_{bw}}{2}$$

Wall thickness and effective depth at top of footing, and corresponding footing thickness:

$h_w(0 \cdot ft) = 18\ in$ $\quad d_{ew}(0 \cdot ft) = 16.063\ in$ $\quad h_f = 18\ in$

Distance y from top of footing at 1 ft intervals and at top of wall:

$i1 := 0 .. \frac{C}{ft}$ $\quad y_{i1} := i1 \cdot ft$ $\quad N := \text{ceil}\left(\frac{C}{ft}\right)$ $\quad y_N := C$ $\quad i := 0 .. N$

Theoretical calculated reinforcement ratio required for flexure as a function of y:

$$\rho(y) := \overrightarrow{\left(1 - \left(\sqrt{1 - \frac{2 \cdot M_{uw}(y)}{\phi_f \cdot ft \cdot d_{ew}(y)^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y}}$$

Minimum required reinforcement ratio: $\rho_{min} = 0.333\%$

Preferred maximum reinforcement ratio: $\rho_{pref} = 1.069\%$

Maximum reinforcement ratio: $\max(\rho(y)) = 0.007$

Maximum permissible reinforcement ratio: $\rho_{max} = 2.138\%$

Required reinforcement area, the larger of the calculated reinforcement or the minimum required reinforcement:

$$A_{s\_wall_i} := \textbf{if}\left(\rho\left(y_i\right) \ge \rho_{min}, \rho\left(y_i\right) \cdot ft \cdot d_{ew}\left(y_i\right), \rho_{min} \cdot ft \cdot d_{ew}\left(y_i\right)\right)$$

Maximum and minimum reinforcement areas:

$\max\left(A_{s\_wall}\right) = 1.334\ in^2$ $\quad min\left(A_{s\_wall}\right) = 0.403\ in^2$

All calculated values of y, and A_{s_wall} are displayed in graphical and tabular form in the **Summary**.

Footing Design

Difference between factored toe and heel soil bearing pressures, and the effective wall depth d_w to the centroid of the wall reinforcement at top of footing:

$$\Delta p\_f := p_{toe\_f} - p_{heel\_f} = 7.041\ ksf$$

$$d_w := d_{ew}(0 \cdot ft) = 16.063\ in$$

Intermediate values used in calculating soil bearing pressures on footing

$$X1 := \frac{L_{B\_f} - T}{L_{B\_f}} = 0.899$$

$$X3 := \frac{L_{B\_f} - T - d_w}{L_{B\_f}} = 0.798$$

$$X2 := \frac{B - T}{B} = 0.899$$

$$X4 := \frac{B - T - d_w}{B} = 0.798$$

Location of critical section for shear on the toe from front face of wall as a function of required toe thickness for shear:

$$X5(h1) := T - \left(h1 - cl_t - \frac{1}{2} \cdot d_{bt}\right)$$

Ratio of the distance from the point of minimum bearing pressure on the footing to the critical section for shear on the toe to the bearing length on the footing as a function of required toe thickness for shear:

$$X6(h1) := \frac{L_{B\_f} - X5(h1)}{L_{B\_f}}$$

Ratio of the distance from the heel to the critical section for shear on the toe to the bearing length on the footing as a function of required toe thickness for shear:

$$X7(h1) := \frac{B - X5(h1)}{B}$$

Soil bearing pressures due to factored loads at distance from front face of wall to the critical section for shear on the toe as a function of required toe thickness for shear:

$$p_{d\_f}(h1) := \textbf{if}\left(\left(\left(L_{B\_f} \ge X5(h1)\right) + \left(L_{B\_f} \le B\right), X6(h1) \cdot p_{toe\_f}, \textbf{if}\left(\left(L_{B\_f} \le X5(h1)\right), 0 \cdot ksf, p_{heel\_f} + X7(h1) \cdot (\Delta p\_f)\right)\right)\right)$$

Soil bearing pressures on footing at front and back face of wall:

$$p_{front\_f} := \textbf{if}\left(\left(\left(L_{B\_f} \ge T\right) + \left(L_{B\_f} \le B\right), X1 \cdot p_{toe\_f}, \textbf{if}\left(L_{B\_f} \le T, 0 \cdot ksf, p_{heel\_f} + X2 \cdot (\Delta p\_f)\right)\right)\right)$$

$$p_{back\_f} := \textbf{if}\left(\left(\left(L_{B\_f} \ge T + d_{ew}(0 \cdot ft)\right) + \left(L_{B\_f} \le B\right), X3 \cdot p_{toe\_f}, \textbf{if}\left(L_{B\_f} \le T - d_{ew}(0 \cdot ft), 0 \cdot ksf, p_{heel\_f} + X4 \cdot (\Delta p\_f)\right)\right)\right)$$

Shear on toe as a function of footing thickness, using 0.9 factor for opposing dead load:

$$V_{u\_toe}(h1) := \left(\frac{p_{toe\_f} + p_{d\_f}(h1)}{2} - 0.9 \cdot \left(h_f \cdot w_{rc} + E \cdot \gamma\right)\right) \cdot X5(h1) \cdot ft$$

Solve for footing thickness h1 required for shear on toe:

Guess value of h1 = hf, the initial assumed footing thickness:

$h1 := h_f$ $\quad h_f = 18\ in$

$$f(h1) := \phi_v \cdot v_c \cdot ft \cdot \left(h1 - cl_t - \frac{1}{2} \cdot d_{bt}\right) - V_{u\_toe}(h1)$$

$$h1 := \mathbf{root}(f(h1), h1) = 8.378\ in$$

Shear on toe:

$$V_{u\_toe}(h1) = 6.455\ kip$$

Bending moment on toe using 0.9 factor for opposing dead load:

$$M_{u\_toe} := \left(\left(\frac{p_{toe\_f}}{3} + \frac{p_{front\_f}}{6}\right) \cdot T^2 - 0.9 \cdot \left(h_f \cdot w_{rc} + \left(D - h_f\right) \cdot \gamma\right) \cdot \frac{T^2}{2}\right) \cdot ft$$

$$M_{u\_toe} = 6.389\ kip \cdot ft$$

Toe thickness required for flexure:

$h2 := \sqrt{\frac{M_{u\_toe}}{K\left(\rho_{pref}\right) \cdot ft}} + cl_t + \frac{1}{2} \cdot d_{bt}$ $\quad h2 = 6.871\ in$

The larger toe thickness required for shear or flexure:

$h_{toe} := \mathbf{if}(h1 \geq h2, h1, h2)$ $\quad h_{toe} = 8.378\ in$

Factored pressures on heel due to vertical component of active soil pressure:

$H_1 := H_1(F)$ $\quad H_1 = 27.458\ ft$

Pressure p1 at back face of wall:

$p1 := 1.7 \cdot \left(k_{av} \cdot \left(H_1 - h_f - F \cdot \tan(\beta)\right) \cdot \tan(\beta)\right)$ $\quad p1 = 0.327\ ksf$

Pressure p2 at edge of heel:

$p2 := 1.7 \cdot \left(k_{av} \cdot \left(H_1 - h_f\right) \cdot \tan(\beta)\right)$ $\quad p2 = 0.409\ ksf$

Intermediate values used to calculate shear and bending moment on heel:

$$Y1 := cl_w + \frac{d_{bw}}{2} \qquad Y1 = 1.938\ in$$

$$Y2 := L_{B\_f} - T - d_w \qquad Y2 = 10.578\ ft$$

$$Y3 := \frac{F}{2} + Y1 \qquad Y3 = 5.37\ ft$$

Shear Vu and bending moment Mu on heel:

$$V_{u\_heel} := \left(\left(1.4 \cdot \left(C \cdot \gamma + h_f \cdot w_{rc}\right) + \frac{p1 + p2}{2}\right) \cdot F - \frac{p_{back\_f} + p_{heel\_f}}{2} \cdot (Y2)\right) \cdot ft$$

$$V_{u\_heel} = 2.854\ kip$$

$$M_{u\_heel} := \left(\left(1.4 \cdot \left(C \cdot \gamma + h_f \cdot w_{rc}\right) \cdot F \cdot (Y3) + \frac{F}{2} \cdot \left(p1 \cdot \left(\frac{F}{3} + Y1\right) + p2 \cdot \left(\frac{2 \cdot F}{3} + Y1\right)\right)\right) - \left(\frac{p_{back\_f}}{6} + \frac{p_{heel\_f}}{3}\right) \cdot Y2^2\right) \cdot ft$$

$$M_{u\_heel} = 70.364\ kip \cdot ft$$

If M_{u_heel} is negative, the reinforcement of the heel is on the bottom.

Heel thickness required for shear:

$$h3 := \frac{\left|V_{u\_heel}\right|}{\phi_v \cdot v_c \cdot ft} + cl_h + \frac{1}{2} \cdot d_{bh} = 4.212\ in$$

Heel thickness required for flexure:

$$h4 := \sqrt{\frac{\left|M_{u\_heel}\right|}{K\left(\rho_{pref}\right) \cdot ft}} + cl_h + \frac{1}{2} \cdot d_{bh} = 13.601\ in$$

The larger heel thickness required for shear or flexure:

$$h_{heel} := \text{if}\left(h3 \ge h4, h3, h4\right) = 13.601\ in$$

Minimum required footing thickness as the larger thickness required for the toe or heel:

$$h_{f1} := \text{if}\left(h_{toe} \ge h_{heel}, h_{toe}, h_{heel}\right) \qquad h_{f1} = 13.601\ in$$

Minimum required footing thickness

Minimum footing thickness rounded up to the nearest multiple of SzF:

$$h'_f := SzF \cdot \text{ceil}\left(\frac{h_{f1}}{SzF}\right) = 14\ in$$

Effective footing depths to centroids of reinforcement using the larger of the entered thickness or the calculated footing thickness:

$$d_{e\_toe} := \textbf{if}\left(h_f > h'_f, h_f, h'_f\right) - cl_t - \frac{1}{2} \cdot d_{bt}$$

$$d_{e\_toe} = 14.625\ in$$

$$d_{e\_heel} := \textbf{if}\left(h_f > h'_f, h_f, h'_f\right) - \textbf{if}\left(M_{u\_heel} < 0 \cdot kip \cdot ft, cl_t, cl_h\right) - \frac{1}{2} \cdot d_{bh}$$

$$d_{e\_heel} = 16\ in$$

Minimum toe and heel Reinforcement

Toe reinforcement:

$$\rho_{toe} := \left(1 - \left(\sqrt{1 - \frac{2 \cdot M_{u\_toe}}{\phi_f \cdot ft \cdot {d_{e\_toe}}^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y} = 0.056\%$$

$$A_{s\_toe} := \textbf{if}\left(\rho_{toe} \geq \rho_{min}, \rho_{toe} \cdot ft \cdot d_{e\_toe}, \rho_{min} \cdot ft \cdot d_{e\_toe}\right) = 0.585\ in^2$$

Heel reinforcement:

$$\rho_{heel} := \left(1 - \left(\sqrt{1 - \frac{2 \cdot \left|M_{u\_heel}\right|}{\phi_f \cdot ft \cdot {d_{e\_heel}}^2 \cdot 0.85 \cdot f'_c}}\right)\right) \cdot \frac{0.85 \cdot f'_c}{f_y} = 0.534\%$$

$$A_{s\_heel} := \textbf{if}\left(\rho_{heel} \geq \rho_{min}, \rho_{heel} \cdot ft \cdot d_{e\_heel}, \rho_{min} \cdot ft \cdot d_{e\_heel}\right) = 1.026\ in^2$$

Compare factored shear and moment on toe and heel with useable capacities:

$$\phi V_{n\_toe} := \phi_v \cdot v_c \cdot ft \cdot d_{e\_toe} = 18.869\ kip$$

$$V_{u\_toe}\left(h_f\right) = 0.833\ kip$$

$$\phi M_{n\_toe} := \left(\phi_f \cdot \rho_{toe} \cdot \left(\left(1 - \frac{\rho_{toe} \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y\right)\right) \cdot ft \cdot {d_{e\_toe}}^2$$

$$\phi M_{n\_toe} = 6.389\ kip \cdot ft \qquad M_{u\_toe} = 6.389\ kip \cdot ft$$

$$\phi V_{n\_heel} := \phi_v \cdot v_c \cdot ft \cdot d_{e\_heel}$$

$$\phi V_{n\_heel} = 20.643\ kip \qquad V_{u\_heel} = 2.854\ kip$$

$$\phi M_{n\_heel} := \left(\phi_f \cdot \rho_{heel} \cdot \left(1 - \frac{\rho_{heel} \cdot f_y}{2 \cdot 0.85 \cdot f'_c}\right) \cdot f_y\right) \cdot ft \cdot {d_{e\_heel}}^2$$

$\phi M_{n\_heel} = 70.364\ kip \cdot ft$

$M_{u\_heel} = 70.364\ kip \cdot ft$

Summary

Constants

Sizing factor for rounding base width:	$SzB = 3\ in$
Sizing factor for rounding toe projection and wall and footing thicknesses:	$SzF = 2\ in$
Factor of safety against sliding:	$SF = 1.5$
Concrete cover of wall reinforcement:	$cl_w = 1.5\ in$
Concrete cover of bottom toe reinforcement:	$cl_t = 3\ in$
Concrete cover of top heel reinforcement:	$cl_h = 1.5\ in$
Preferred reinforcement ratio:	$\rho_{pref} = 1.069\%$

Material Properties

Specified compressive strength of concrete:	$f'_c = 4\ ksi$
Specified yield strength of reinforcement:	$f_y = 60\ ksi$
Unit weight of concrete:	$w_c = 145\ pcf$
Shear Strength reduction factor for lightweight concrete (ACI 318, 11.2.1.2.).	$k_v = 1$
Unit weight of reinforced concrete:	$w_{rc} = 150\ pcf$

Input

Total height from top of wall to bottom of footing:	$H = 22\ ft$
Depth from lower grade level to bottom of footing:	$D = 0\ ft$
Estimated footing thickness:	$h_f = 18\ in$
Estimated wall thickness at top of footing:	$h_{wb} = 18\ in$
Front face wall taper:	$t_f = 0\ in$

Back face wall taper: $t_b = 6\ in$

Unit weight of soil: $\gamma = 100\ pcf$

Coefficient of friction between footing and soil: $c_f := 0.55$

Angle of internal friction: $\phi = 33.67\ deg$

Backfill slope angle: $\beta = 26.565\ deg$

Equivalent horizontal fluid pressure due to soil: $k_{ah} = 37.1\ \frac{psf}{ft}$

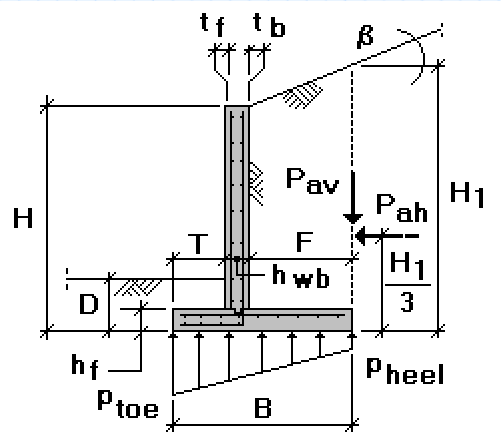
Equivalent vertical fluid pressure due to soil: $k_{av} = 18.55\ \frac{psf}{ft}$

Allowable soil bearing pressure: $p_s = 5\ ksf$

Wall reinforcing bar diameter: $d_{bw} = 0.875\ in$

Toe reinforcing bar diameter: $d_{bt} = 0.75\ in$

Heel reinforcing bar diameter: $d_{bh} = 1\ in$



Wall Dimensions

Toe projection: $T = 1.333\ ft$

Footing width: $B = 13.25\ ft$

Heel Projection: $F = 10.417\ ft$

Minimum required wall thickness: $h'_{wb} = 16\ in$

Minimum required footing thickness: $h'_f = 14\ in$

Front wall taper corresponding to minimum required wall thickness: $t'_f = 0\ in$

Back wall taper corresponding to minimum required wall thickness: $t'_b = 4\ in$

The larger of the entered or calculated wall and footing thicknesses are used for calculating shear and required reinforcement. If the calculated values for wall and footing thickness are larger than those entered, the application may be rerun with the calculated values as a final check.

Overturning and Resisting Forces and Moments

Horizontal force due to active soil pressure: $P_{ah} = 13.986\ kip$

Vertical force due to surcharge: $P_{av} = 6.993\ kip$

Overturning Moment: $M_{ot} = 128.012\ kip \cdot ft$

Total dead load: $w_R = 38.664\ kip$

Ratio of frictional sliding resistance to horizontal sliding force: $\frac{c_f \cdot w_R}{P_{ah}} = 1.52$

Dead load resisting moment: $M_R = 321.693\ kip \cdot ft$

Ratios of resisting dead load moment to overturning moment: $\frac{M_R}{M_{ot}} = 2.513$

Service and Factored Load Bearing Pressures on Base

Service load toe pressure: $p_{toe} = 5.053\ ksf$

Factored load toe pressure: $p_{toe\_f} = 7.506\ ksf$

Service load heel pressure: $p_{heel} = 0.783\ ksf$

Factored load heel pressure: $p_{heel\_f} = 0.466\ ksf$

Base bearing length at service load: $L_B = 13.25\ ft$

Base bearing length at factored load: $L_{B\_f} = 13.25\ ft$

Required Toe and Heel Reinforcement

Required toe reinforcement per unit length of wall: $A_{s\_toe} = 0.585\ in^2$

Required toe reinforcement per unit length of wall: $A_{s\_heel} = 1.026\ in^2$

Concrete volume per lineal foot of wall: $Vol = 45.5\ ft^3$

Calculated and minimum required reinforcement area for stem wall versus height y from top of footing to top of wall:

$$y_i = \begin{bmatrix} 0 \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \\ 10 \\ 11 \\ 12 \\ 13 \\ 14 \\ 15 \\ 16 \\ 17 \\ 18 \\ 19 \\ 20 \\ 20.5 \end{bmatrix} ft \qquad A_{s\_wall_i} = \begin{bmatrix} 1.334 \\ 1.161 \\ 1.003 \\ 0.86 \\ 0.731 \\ 0.615 \\ 0.572 \\ 0.561 \\ 0.549 \\ 0.537 \\ 0.525 \\ 0.514 \\ 0.502 \\ 0.49 \\ 0.479 \\ 0.467 \\ 0.455 \\ 0.443 \\ 0.432 \\ 0.42 \\ 0.408 \\ 0.403 \end{bmatrix} in^2$$