CHAPTER 8: Earth Retaining Structures

8.3 Anchored Bulkheads

Description

Anchored bulkheads are used at waterfront areas to prevent soil erosion and to provide areas suitable for docking. Bulkheads range from 2 inches or 3 inches thick tongue and groove timber sheet pile construction, to 12 inches deep steel "Z" piles. This application computes net pressures on the bulkhead, required depth of embedment of the sheet piles, maximum bending moment and the bending moment at the tie rod anchor point, the total active soil and water pressure, the net passive earth pressure reaction, and the tie rod anchor force using the "free earth" method.

The soil acting on the bulkhead must be divided into segments with linearly varying pressure within each segment. This application provides for entry of any number of segments between the top of the bulkhead and the design depth and for two segments below the design depth. The user must enter the height from the top of the bulkhead to the design depth, the depth of the soil segments, the active soil pressure coefficients for each segment, the passive soil pressure coefficients for the segments below the design depth, embedment increase for safety against toe failure, the dry or submerged unit weight of the soil within each segment, the unit weight of fresh or salt water, the water head above mean low water and a uniform surcharge load.

A summary of input and calculated values is shown on pages 11 and 12.

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**Input**

**Notation**

- **Uniform surcharge load:** \( q = 0 \cdot \text{psf} \)
- **Depths of soil segments above design depth:** \( y = [3 \ 4 \ 1 \ 2 \ 2] \cdot \text{ft} \)
- **Unit weight of soils (dry weights above water or submerged weights below water):** \( \gamma = [100 \ 100 \ 60 \ 60 \ 60 \ 60] \cdot \text{pcf} \)
- **Active soil pressure coefficients for all soil segments:** \( k_a = [0.30 \ 0.30 \ 0.30 \ 0.30 \ 0.30 \ 0.30] \cdot \text{pcf} \)
- **Definitions of n and range variable i:**
  \[
  n := \text{length} (y) \quad n = 5 \quad i := 0 \ldots n - 1
  \]

**Note** ⇒ Range variable \( i \) and variable \( n \) are computed from the number of segments.

**Input Variables**

- **Passive soil pressure coefficients for all soil segments:** \( k_p \)
Passive soil pressure coefficients below design depth:
\[ k_{p_n} : = 5.0 \quad k_{p_{n+1}} : = 5.0 \]

Tie rod depth below top of bulkhead:
\[ A : = 3 \cdot \text{ft} + 0 \cdot \text{in} \]

Allowance for scour and over dredging:
\[ \delta : = 1 \cdot \text{ft} \]

Unit weight of fresh or salt water:
\[ \gamma_w : = 64 \cdot \text{pcf} \]

Water head behind bulkhead at MLW:
\[ h : = 1 \cdot \text{ft} \]

Index at top of water head behind bulkhead:
\[ s : = 2 \]

Required percentage increase in embedment for safety against toe failure:
\[ Per_d : = 40\% \]

Depth of soil segments below design depth:
\[ y_n : = 3 \cdot \text{ft} \quad y_{n+1} : = 25 \cdot \text{ft} \]

Computed Variables

- \( H \): height of bulkhead above dredge line
- \( H_d \): height of bulkhead above design depth
- \( y_w \): depth from top of bulkhead to top of water behind bulkhead
- \( p \): net pressures on bulkhead
- \( d_0 \): depth from design depth to the point of zero net pressure on bulkhead
- \( P_a \): total net force on bulkhead due to active soil pressures and water head
- \( d_1 \): depth from the point of zero net pressure to develop passive soil reaction \( R \)
- \( d' \): required additional embedment depth for safety against toe failure
- \( D' \): minimum embedment depth to develop passive reaction force \( R \)
<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>D</td>
<td>required embedment depth below dredge line</td>
</tr>
<tr>
<td>R</td>
<td>net reaction force developed by passive soil pressures below the design depth</td>
</tr>
<tr>
<td>T</td>
<td>tie rod anchor force per unit length of bulkhead</td>
</tr>
<tr>
<td>MA</td>
<td>bending moment in the sheet piling at the tie rod</td>
</tr>
<tr>
<td>Yo</td>
<td>depth from top of bulkhead to point of zero shear and maximum moment</td>
</tr>
<tr>
<td>M_{max}</td>
<td>maximum bending moment in sheet piling</td>
</tr>
</tbody>
</table>
Calculations

Height of bulkhead above design depth:

\[ H_d := \sum_i y_i \quad H_d = 12 \text{ ft} \]

Height of bulkhead above dredge line:

\[ H := H_d - \delta \quad H = 11 \text{ ft} \]

Depth to top of water head:

\[ y_w := \sum_i \left((i < s) \cdot y_i\right) \quad y_w = 7 \text{ ft} \]

Net active soil pressures + water head acting on bulkhead above design depth:

\[ p_0 := k_a \cdot q \quad p_{i+1} := p_i + k_{a_i} \cdot \gamma_i \cdot y_i + (i = s) \cdot \gamma_w \cdot h \]

\[ p^T = [0 \ 90 \ 210 \ 292 \ 328 \ 364] \text{ psf} \]

Net passive pressure increases per unit of depth in first and second strata below design depth:

\[ w_{p1} := (k_{p_n} - k_{a_n}) \cdot \gamma_n \quad w_{p1} = 282 \frac{\text{psf}}{\text{ft}} \]

\[ w_{p2} := (k_{p_{n+1}} - k_{a_{n+1}}) \cdot \gamma_{n+1} \quad w_{p2} = 282 \frac{\text{psf}}{\text{ft}} \]

Net soil pressure at bottom of first strata below design depth (+ indicates a net active pressure and - indicates a net passive pressure):

\[ p_{n+1} := p_n - w_{p1} \cdot y_n \quad p_{n+1} = -482 \text{ psf} \]

**Embedment depth from design depth to point of zero pressure:**

\[ d_0 := \text{if} \left( w_{p1} \cdot y_n \geq p_n \cdot \frac{p_n}{w_{p1}} + y_n + \left( \frac{p_n - w_{p1} \cdot y_n}{w_{p2}} \right) \right) \quad d_0 = 1.291 \text{ ft} \]
Forces acting on each segment above design depth:

\[ F_{\text{top}_i} = \frac{1}{2} \cdot p_i \cdot y_i \cdot \text{ft} \]

\[ F_{\text{bot}_i} = \frac{1}{2} \cdot p_{i+1} \cdot y_i \cdot \text{ft} \]

\[ F := F_{\text{top}} + F_{\text{bot}} \]

\[ F_{\text{top}}^T = [0 \ 180 \ 105 \ 292 \ 328] \text{ lbf} \]

\[ F_{\text{bot}}^T = [135 \ 420 \ 146 \ 328 \ 364] \text{ lbf} \]

\[ F^T = [135 \ 600 \ 251 \ 620 \ 692] \text{ lbf} \]

Forces acting outward on the bulkhead below design depth:

Case 1: \( d_0 < y_n \):

\[ F_{\text{top}_n} = \frac{1}{2} \cdot p_n \cdot d_0 \cdot \text{ft}, \frac{1}{2} \cdot p_n \cdot y_n \cdot \text{ft} \]

\[ F_{\text{top}_n} = 234.922 \text{ lbf} \]

\[ F_{\text{bot}_n} = \frac{1}{2} \cdot p_{n+1} \cdot y_n \cdot \text{ft} \]

\[ F_{\text{bot}_n} = 0 \text{ lbf} \]

\[ F_n := F_{\text{top}_n} + F_{\text{bot}_n} \]

\[ F_n = 234.922 \text{ lbf} \]
\[
F_{n+1} := \begin{cases} 
    d_0 \leq y_n, & 0 \cdot lb, \frac{1}{2} \cdot p_{n+1} \cdot (d_0 - y_n) \cdot ft \\
    & \quad \text{if}
\end{cases}
F_{n+1} = 0 \text{ lb}
\]

Total net force on bulkhead due to active soil pressures and water pressure:

\[P_a := \sum F \quad P_a = 2533 \text{ lb}f\]

Distance from top of bulkhead to top of each segment above design depth:

\[k := 0 \ldots n \quad S_k := \sum_i (i < k) \cdot y_i \quad S^T = [0 \ 3 \ 7 \ 8 \ 10 \ 12] \text{ ft}\]

Summation of moments due to net active soil pressure forces about tie rod:

\[M_T := \sum_i \left( F_{top} \cdot \left( \frac{y_i}{3} + S_i - A \right) + F_{bot} \cdot \left( \frac{2}{3} \cdot y_i + S_i - A \right) \right) + \text{if} \left( d_0 \leq y_n, F_n \cdot \left( \frac{d_0}{3} + S_n - A \right), F_{top} \cdot \left( \frac{y_n}{3} + S_n - A \right) \right) + F_{bot} \cdot \left( \frac{2}{3} \cdot y_n \right)\]

\[M_T = 13857 \text{ lb} \cdot \text{ft}\]

**Embedment \(d_1\) depth below point of zero net pressure to develop reaction force \(R\)**

- Case 1: depth \(d_1\) entirely within the first strata below design depth
- Case 3: depth \(d_1\) entirely within the second strata below design depth

![Diagram](image)

Case 2: top of first soil strata below design depth within depth \(d_1\):

![Diagram](image)

Depth \(d_1\) for Case 1 or Case 2 as a function of \(w_p\):

\[\text{Case 1 or Case 2: } d_1 = \ldots \text{ ft}\]
Guess value of \( d_1 \):

\[
d_1 := 2 \cdot d_0 \quad d_1 = 2.582 \text{ ft}
\]

\[
f (w_p) := \sqrt{\frac{1}{2} \cdot w_p \cdot d_1^2 \cdot ft \cdot \left( H_d - A + d_0 + \frac{2}{3} \cdot d_1 \right) - M_T, d_1}
\]

Embedment depth \( e_1 \): (\( e_1 \) is zero for Case 1 or 3)

\[
e_1 := \begin{cases} 0 & (d_0 \geq y_n) \end{cases} e_1 = 1.709 \text{ ft}
\]

Embedment depth \( e_2 \) for Case 2:

Guess value of \( e_2 \):

\[
e_2 := 2 \cdot d_0 - e_1 \quad e_2 = 0.872 \text{ ft}
\]

\[
g (e_2) := \frac{1}{2} \cdot \left( w_{p1} \cdot e_1^2 \cdot ft \cdot \left( H_d - A + d_0 + \frac{2}{3} \cdot e_1 \right) \right) + w_{p1} \cdot e_1 \cdot e_2 \cdot ft \cdot \left( H_d - A + d_0 + e_1 + \frac{1}{3} \cdot e_2 \right) + (w_{p1} \cdot e_1 + w_{p2} \cdot e_2) \cdot e_2
\]

\[
e_2 := \text{root} (g (e_2), e_2) \quad e_2 = 1.131 \text{ ft}
\]

Embedment depth \( d_1 \):

\[
d_1 := \begin{cases} y_n & (d_0 \geq y_n) \end{cases} \left( d_0 + f (w_{p1}) \leq y_n, f (w_{p1}), e_1 + e_2 \right) = 2.84 \text{ ft}
\]

\[
d_1 = 2.84 \text{ ft}
\]

Embedment depth to develop passive reaction force \( R \):

\[
D' := \delta + d_0 + d_1 \quad D' = 5.131 \text{ ft}
\]

Additional embedment depth required for safety against toe failure:

\[
d' := \text{Per}_d \cdot (d_0 + d_1) \quad d' = 1.652 \text{ ft}
\]

Embedment depth below dredge line:

\[
D := \delta + d_0 + d_1 + d' \quad D = 6.783 \text{ ft}
\]

Net reaction force developed by passive soil pressures below design depth:

\[
R := \frac{1}{2} \cdot \begin{cases} 0 & (d_0 \geq y_n) \end{cases} \left( w_{p2} \cdot d_1^2, w_{p1} \cdot e_1^2 \right) \cdot \text{ft}
\]

\[
R = 1137 \text{ lbf}
\]

Anchor rod tension for a unit length of the bulkhead:

\[
T = R \cdot D
\]
\[ T := P_a - R \]  
\[ T = 1396 \text{ lbf} \]

Index at top of segment containing the tie rod:
\[ t_0 := \text{match} (A, S) \]
\[ t := \begin{cases} \lfloor t_0 \rfloor & t = 1 \end{cases} \]

Net soil pressure at top of segment containing the tie rod:
\[ p_{\text{top}} := p_t \]
\[ p_{\text{top}} = 90 \text{ psf} \]

Net pressure at tie rod (if the tie rod is at a segment \( p_{\text{top}} = p_{\text{bot}} \)):
\[ p_{\text{bot}} := p_{\text{top}} + k_a \cdot \gamma \cdot \left( A - S \right) \]
\[ p_{\text{bot}} = 90 \text{ psf} \]

Bending moment in the sheet piling at the tie rod:
\[
M_A := \frac{1}{2} \left( \sum_i \left( (i < t) \cdot \left( p_i \cdot y_i \cdot \left( \frac{2}{3} \cdot y_i + A - S \right)_{i+1} \right) + p_{i+1} \cdot y_{i+1} \cdot \left( \frac{1}{3} \cdot y_{i+1} + A - S \right)_{i+1} \right) \right) + \left( \frac{2 \cdot p_{\text{top}} + p_{\text{bot}}}{3} \cdot (A - S)^2 \right) \cdot \text{ft} \\
M_A = 135 \text{ lbf} \cdot \text{ft} 
\]

Location of point of zero shear and maximum moment from top of bulkhead.

Index of segment where shear passes through zero:
\[
\text{Sum}_F := \sum_i (i < k) \cdot F_i \\
\]
\[
u := \begin{cases} j & j \leftarrow 0 \\ \text{while } T > \text{Sum}_F_j \\ j \leftarrow j + 1 \\ \text{return } j - 1 \end{cases} \\
u = 3 
\]

Shear at top of segment where shear passes through zero:
\[
V_u := T - \text{Sum}_F_u \quad V_u = 409.647 \text{ lbf} 
\]

Shear as a function of distance \( y' \) from top of segment where shear passes through zero:
\[
f(y') := V_u - \left( p_u \cdot y' + \frac{1}{2} \cdot k_a \cdot \gamma \cdot y'^2 \right) \cdot \text{ft} 
\]

Guess value of \( y' \):
\[
u' := \frac{y}{u} \quad u' = 1 \text{ ft} \]
Guess value of $y'$:

$$y' = \frac{1}{2}, \quad y' = 1 \text{ ft}$$

$$y' = \sqrt{f(y', y')} \quad y' = 1.347 \text{ ft}$$

Check (should approach zero):

$$V - p_u \cdot y' \cdot ft - \frac{1}{2} \cdot k_u \cdot \gamma_u \cdot y'^2 \cdot ft = -1.054 \cdot 10^{-12} \text{ lbf}$$

Net soil pressure at point of zero shear:

$$p_o = p_u + k_u \cdot \gamma_u \cdot y' \quad p_o = 316.246 \text{ psf}$$

Distance $Y_o$ from top of bulkhead to point of zero shear and maximum moment:

$$Y_o = S_u + y' \quad Y_o = 9.347 \text{ ft}$$

$$M_{max} = \left\{ T \cdot (Y_o - A) \sum_{i} \left\{ \begin{array}{l} \begin{array}{l} i < u \end{array} \end{array} \right\} \left\{ \begin{array}{l} F_{top} \cdot \left(S + \frac{y}{3} - Y_o\right) + F_{bot} \cdot \left(S + \frac{2}{3} \cdot y - Y_o\right) \right\} \right\} - \frac{1}{6} \cdot y'^2 \cdot \left(2 \cdot p_u + p_o\right) \cdot ft$$

$$M_{max} = 4689 \text{ lbf} \cdot ft$$

The maximum moment may be reduced for flexible bulkheads penetrating medium compact and compact coarse-grained soils (Rowe's Moment Reduction). The calculation of the possible moment reduction is not within the scope of this application.
Uniform surcharge load: \( q = 0 \text{ psf} \)

Water head behind bulkhead at mean low water: \( h = 1 \text{ ft} \)

Unit weight of fresh or salt water: \( \gamma_w = 64 \text{ pcf} \)

Depth below dredge line to allow for scour and over dredging: \( \delta = 1 \text{ ft} \)

Specified percentage increase in embedment depth for safety against toe failure: \( Per_d = 40 \% \)

Depth of tie rod below top of bulkhead: \( A = 3 \text{ ft} \)

Segment depths: \( y^T = [3\ 4\ 1\ 2\ 2\ 3\ 25] \text{ ft} \)

Unit weights of soil: \( \gamma^T = [100\ 100\ 60\ 60\ 60\ 60\ 60] \text{ pcf} \)

Active soil pressure coefficients: \( k_a^T = [0.3\ 0.3\ 0.3\ 0.3\ 0.3\ 0.3\ 0.3] \)
Computed Variables

Height of bulkhead above dredge line: \( H = 11 \text{ ft} \)

Required embedment depth below dredge line: \( D = 6.78 \text{ ft} \)

Tie rod tension per unit length of bulkhead: \( T = 1396 \text{ lbf} \)

Bending moment at tie rod: \( M_A = 135 \text{ lbf } \cdot \text{ft} \)

Maximum bending moment: \( M_{max} = 4689 \text{ lbf } \cdot \text{ft} \)