

Continuous Beam Analysis

Get this out of the way.

$\text{ORIGIN} := 1$

Define an analysis element stiffness for a beam element

$$K_{b\delta\theta}(L, I_z, E) := \begin{pmatrix} \frac{12 \cdot E \cdot I_z}{L^3} & \frac{6 \cdot E \cdot I_z}{L^2} & -\frac{12 \cdot E \cdot I_z}{L^3} & \frac{6 \cdot E \cdot I_z}{L^2} \\ \frac{6 \cdot E \cdot I_z}{L^2} & \frac{4 \cdot E \cdot I_z}{L} & -\frac{6 \cdot E \cdot I_z}{L^2} & \frac{2 \cdot E \cdot I_z}{L} \\ -\frac{12 \cdot E \cdot I_z}{L^3} & -\frac{6 \cdot E \cdot I_z}{L^2} & \frac{12 \cdot E \cdot I_z}{L^3} & -\frac{6 \cdot E \cdot I_z}{L^2} \\ \frac{6 \cdot E \cdot I_z}{L^2} & \frac{2 \cdot E \cdot I_z}{L} & -\frac{6 \cdot E \cdot I_z}{L^2} & \frac{4 \cdot E \cdot I_z}{L} \end{pmatrix}$$

Define the problem.

Number of elements [not spans]
as you can break a beam span into
a number of analysis elements

$L_s := 6$

Define the element section property types. Make it a matrix one row for each type

length	I_z	E
2000	$\frac{325^3 \cdot 250}{12}$	32000
3500	$\frac{400^3 \cdot 250}{12}$	32000

Assign these section types to all the analysis elements via a column vector

$$\text{nc} := \begin{pmatrix} 1 \\ 1 \\ 2 \\ 2 \\ 1 \\ 1 \end{pmatrix}$$

Define the restraints
Use a 1 to fix component
Use a 0 to keep component free.

Node #	Fix δ	Fix θ
1	1	1
3	1	0
5	1	0
7	1	1

Define the loads

Node #	Load Case	Applied Force	Applied Moment
2	1	-25000	0
4	1	-25000	0
6	1	-25000	0
4	2	-75000	0

Do the analysis subject to the definitions above

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Do_It_All(Ls, EI, nc, res, lc) := "Define the number of nodes"
    nn <- Ls + 1
    "create the section nested matrix"
    for i ∈ 1 .. rows(EI)
        Kmbi <- Kbδθ(EIi, 1, EIi, 2, EIi, 3)
    "create the stiffness matrix size"
    Ksgnn·2, nn·2 <- 0.0
    "Fill out the stiffness matrix"
    for k ∈ 1 .. Ls
        for i ∈ 1 .. 4
            for j ∈ 1 .. 4
                Ksg(k-1)·2+i, (k-1)·2+j <- Ksg(k-1)·2+i, (k-1)·2+j + [Kmb(nck)]i,j
    "Define the fixed support spring value"
    Kre <- max(Ksg) · 1 · 1010
    "Apply the restraints on the diagonal defined in "res" res=0 adds zero, if res = 1 adds Kre"
    for i ∈ 1 .. rows(res)
        Ksg(resi, 1-1-1)·2+1, (resi, 1-1-1)·2+1 <- Ksg(resi, 1-1-1)·2+1, (resi, 1-1-1)·2+1 + Kre · resi, 2
        Ksg(resi, 1-1-1)·2+2, (resi, 1-1-1)·2+2 <- Ksg(resi, 1-1-1)·2+2, (resi, 1-1-1)·2+2 + Kre · resi, 3
    "Create the externally applied force matrix - allow for multiple load cases"
    nlc <- max(lc(2))
    "Create load matrix "f""
    fnn·2, nlc <- 0.0
    "fill "f" out as defined by "lc""
    for i ∈ 1 .. rows(lc)
        f(lci, 1-1-1)·2+1, lci, 2 <- lci, 3
        f(lci, 1-1-1)·2+2, lci, 2 <- lci, 4
    "Solve the equation"
    δ <- Ksg-1 · f
    "Calculate the internal member forces"
    for i ∈ 1 .. Ls
        imfi <- Kmb(nci) · submatrix[δ, (i - 1) · 2 + 1, (i - 1) · 2 + 4, 1, nlc]
    return 
$$\begin{pmatrix} \delta \\ \text{imf} \end{pmatrix}$$


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$$\text{Result} := \text{Do_It_All}(L_s, EI, nc, res, lc) = \begin{pmatrix} \{14,2\} \\ \{6,1\} \end{pmatrix}$$

Nested return matrices

Element 1 is the deflections:

one row per analysis node

one column per load case

Element 2 is itself a nested matrix of Analysis Element End

Forces one matrix per analysis element.

	1	2
1	0	0
2	$-9.681 \cdot 10^{-15}$	$2.195 \cdot 10^{-14}$
3	-0.23	0.935
4	$6.682 \cdot 10^{-5}$	$4.677 \cdot 10^{-4}$
5	0	0
6	$-2.673 \cdot 10^{-4}$	$-1.871 \cdot 10^{-3}$
7	-1.514	-6.414
8	0	0
9	0	0
10	$2.673 \cdot 10^{-4}$	$1.871 \cdot 10^{-3}$
11	-0.23	0.935
12	$-6.682 \cdot 10^{-5}$	$-4.677 \cdot 10^{-4}$
13	0	0
14	$9.681 \cdot 10^{-15}$	$-2.195 \cdot 10^{-14}$

Load Case 1 has been verified against other software as correct

$$\text{Result}_2 = \begin{pmatrix} \{4,2\} \\ \{4,2\} \\ \{4,2\} \\ \{4,2\} \\ \{4,2\} \\ \{4,2\} \end{pmatrix}$$

For the first analysis element

$$\text{Result}_2 = \begin{pmatrix} \text{Load Case 1} & \text{Load Case 2} \\ \begin{pmatrix} 1.021 \times 10^4 & -1.606 \times 10^4 \\ 9.442 \times 10^6 & -2.141 \times 10^7 \\ -1.021 \times 10^4 & 1.606 \times 10^4 \\ 1.097 \times 10^7 & -1.07 \times 10^7 \end{pmatrix} & \begin{matrix} \delta_1 \\ \theta_1 \\ \delta_2 \\ \theta_2 \end{matrix} \end{pmatrix}$$

Load Case 1 has been verified against other software as correct