Prestressed or Postensioned Simple Span Beam

- Can be applied to both prestressed and postensioned simple span beams
  (with some engineering judgement and may be slightly different input data values)
- Service level limit stress and limit shear strength checks are included. Deflection and Limit fle
- Straight, 1 or 2-points draped and parabolic tendon layouts are included.
- Allows for concentrated loads, own weight and uniform load.
- Single stage construction is here contemplated.
- You do input in blue background cells and get output in the yellow background cells.

A further and not difficult development of this is to make an optimal cost fully automatical design

$l := 20 \cdot m$ span length

$q_D := 1.5 \cdot \frac{ton}{m}$ uniform dead load, **exclusive of** (i.e., not including) beam's weight

$q_L := 2 \cdot \frac{ton}{m}$ uniform live load

$CS_D := 1.6$ safety factor for the dead loads

$CS_L := 1.6$ safety factor for the uniform live loads

These are the common values in Spain, enter yours.

$CS_{LR} := 1.6$ safety factor for the point live loads, just in case if inferior to CS_L in account load reduction; for limit strength evaluation

Point Loads

If needed, use Mathcad to add rows to vector data

Dead Live Abscissa, from left bearing

$P_D := ton \cdot \begin{pmatrix} 2 \\ 2 \\ 2 \\ 0 \end{pmatrix}$ $P_L := ton \cdot \begin{pmatrix} 1 \\ 1 \\ 1 \\ 0 \end{pmatrix}$ $A := m \cdot \begin{pmatrix} 5 \\ 10 \\ 15 \\ 0 \end{pmatrix}$ $P_{TOT} := P_D + P_L$

$N_P := \text{length}(P_D)$ $N_P = 4$

Concrete

$f_c := 45 \cdot MPa$ specified strength $k_{ci} := 0.9$ $\gamma_c := 2400 \cdot \frac{kgf}{m^3}$

$f_{ci} := k_{ci} \cdot f_c$ $f_{ci} = 40.5\,MPa$ at transfer

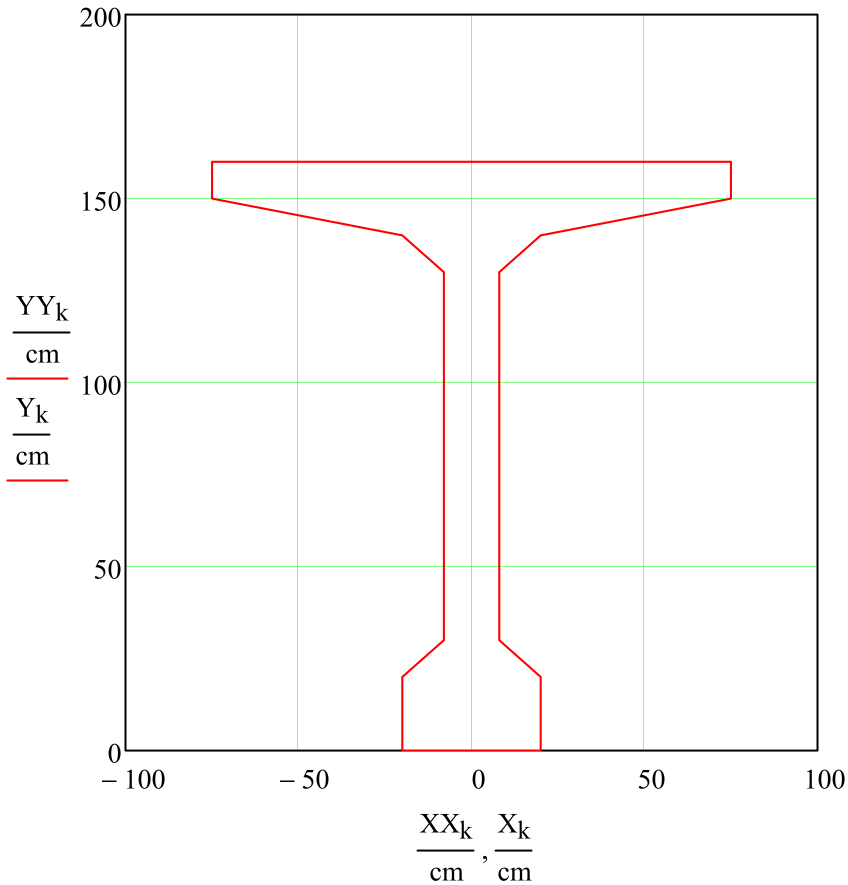
$\phi_s := 0.85$ shear strength reduction factor

Concrete Geometry

- each row means a stacked trapeze of concrete, s
  (you can freely add or reduce rows of data=s
- first column is width of bottom of trapeze
- second colum is width of top base of trapeze
- third colum is height of trapeze

$$d := cm \cdot \begin{pmatrix} 40 & 40 & 20 \\ 40 & 16 & 10 \\ 16 & 16 & 100 \\ 16 & 40 & 10 \\ 40 & 150 & 10 \\ 150 & 150 & 10 \end{pmatrix}$$

uncollapse to see calculation of properties



Properties of the defined beam

$h = 160\,cm$

$y_g = 102.03\,cm$

$Area = 0.54\,m^2$

$weight_{girder} = 1.3\,\frac{ton}{m}$

$I_x = 16816202.21\,cm^4$

$S_b = 164820.92\,cm^3$ elastic modulus of section at bottom face

$S_t = 290070.11\,cm^3$ elastic modulus of section at top face

Propertie
section c

Moment from exclusively weight, service level

$$M_w(x) := \frac{weight_{girder} \cdot x}{2} \cdot (l - x)$$

Moment, Shear of a point load P at abscissa ab

$$M_P(P, ab, x) := \begin{cases} P \cdot \frac{(l - ab)}{l} \cdot x & \text{if } x \le ab \\ P \cdot \frac{ab}{l} \cdot (l - x) & \text{otherwise} \end{cases}$$

$$V_P(P, ab, x) := \begin{cases} P \cdot \frac{(l - ab)}{l} & \text{if } x \le ab \\ -P \cdot \frac{ab}{l} & \text{otherwise} \end{cases}$$

Service Level

$$M(x) := \frac{\left(q_D + q_L + weight_{girder}\right) \cdot x}{2} \cdot (l - x) + \sum_{i=1}^{N_P} M_P\left(P_{D_i} + P_{L_i}, A_i, x\right)$$

$$V(x) := \left(q_D + q_L + weight_{girder}\right) \cdot \left(\frac{l}{2} - x\right) + \sum_{i=1}^{N_P} V_P\left(P_{D_i} + P_{L_i}, A_i, x\right)$$

Moment and Shear required at the factored (limit) level

$$M_u(x) := \frac{\left(CS_D \cdot q_D + CS_L \cdot q_L + CS_D \cdot weight_{girder}\right) \cdot x}{2} \cdot (1 - x) + \sum_{i=1}^{N_P} M_P\left(CS_D \cdot P_D\right.$$

$$V_u(x) := \left(CS_D \cdot q_D + CS_L \cdot q_L + CS_D \cdot weight_{girder}\right) \cdot \left(\frac{1}{2} - x\right) + \sum_{i=1}^{N_P} V_P\left[\left(CS_D \cdot P_D\right.\right.$$

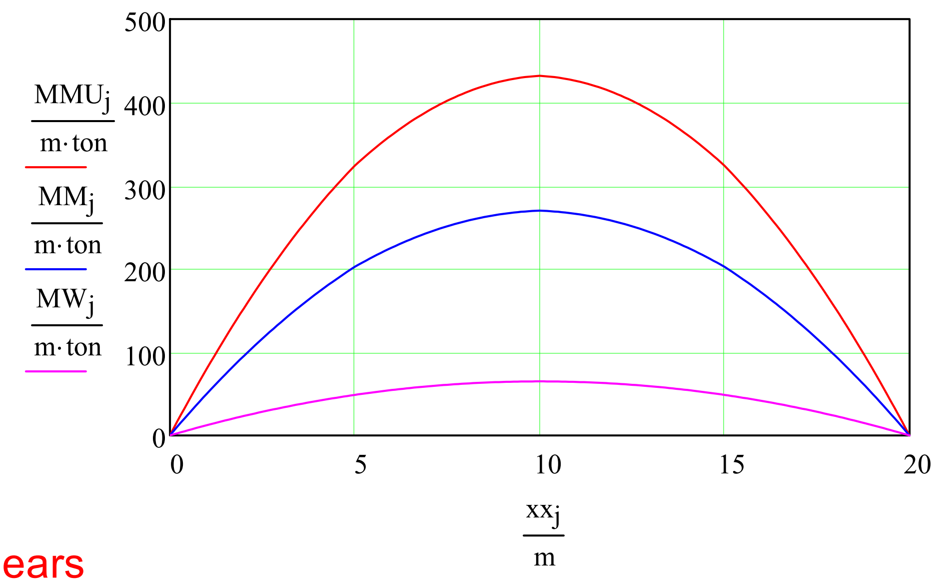
$Nparts := 200$ by setting to a irregular number you may avoid discontinuity problems in evaluation of slope when harped tendons are used

$j := 1 .. Nparts + 1$ $\quad xx_j := \frac{1}{Nparts} \cdot (j - 1)$ $\quad MM_j := M(xx_j)$ $\quad MMU_j$

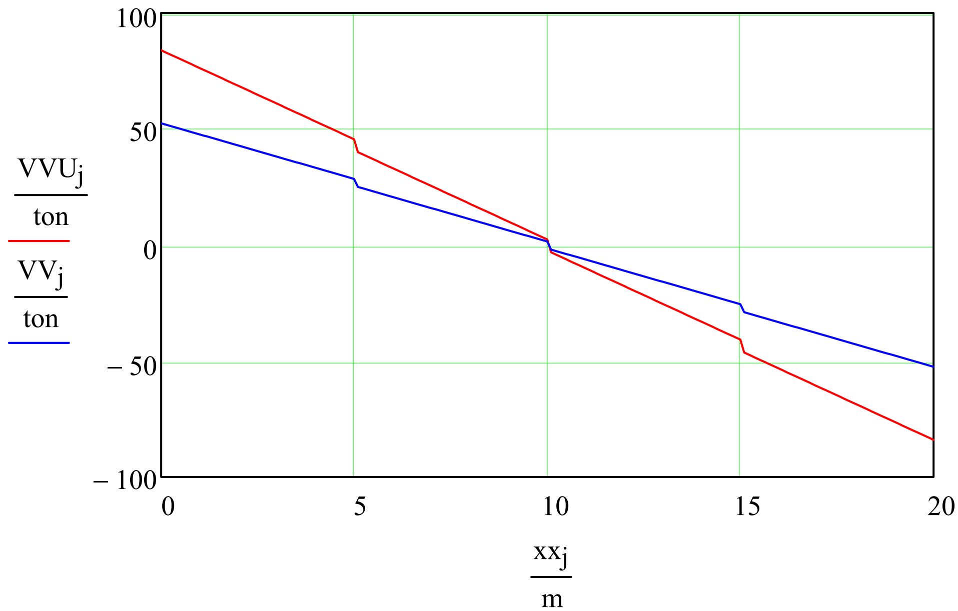
$MW_j := M_w(xx_j)$

In red the factored
In blue service level.
In magenta from dead weight only.

Moments



Shears



Tendon profile

$Choice := 3$

3 if Parabolic
2 if harped at 2 points
1 if harped at center
0 if straight

$k_L := 0.4$

Drape poir
will have u

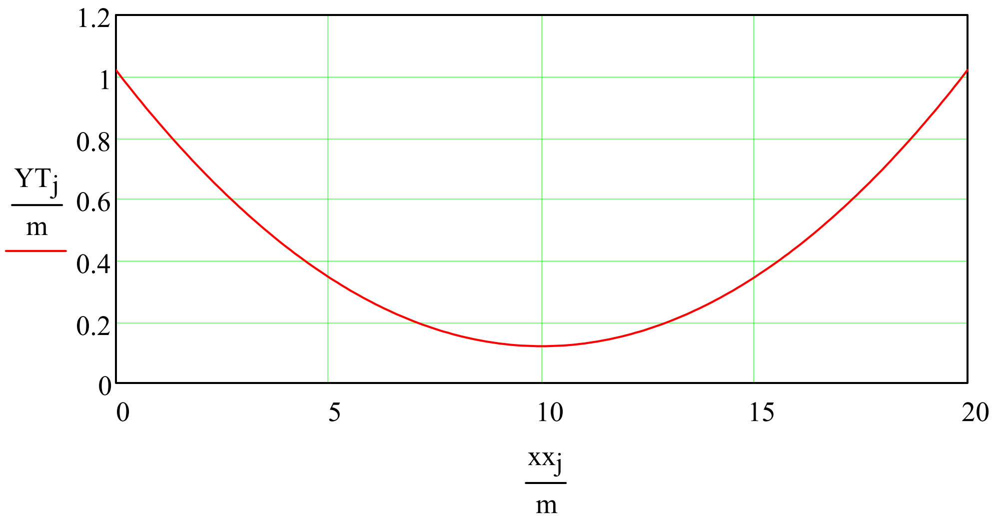
$y_e := y_g$ height of tendon at end

from bottom

$y_c := 12 \cdot cm$ height of tendon at center

Chart Profile

tendon



Prestress Force

$N_{strands} := 18$ $A_{p\_1} := 0.217 \cdot in^2$ $\phi := 0.6 \cdot in$ $f_{pu} := 270$

Initial

$k_i := 0.75$ locked fraction of fpu $f_{pi} := f_{pu} \cdot k_i$ $f_{pi} = 202.5\,ksi$

$P_i := N_{strands} \cdot A_{p\_1} \cdot f_{pi}$ $P_i = 358.78\,ton$

Final

$loss := 0.2$ per one, long term

$k_{pe} := k_i \cdot (1 - loss)$ $k_{pe} = 0.6$ $f_{pe} := k_{pe} \cdot f_{pu}$ $f_{pe} = 162\,ksi$

$P_e := N_{strands} \cdot A_{p\_1} \cdot f_{pe}$ $P_e = 287.02\,ton$

Transfer length

We take it 50 diameters (we are assuming strand built up tendons)

$lt := 50 \cdot \phi$ $\qquad$ $lt = 76.2\,cm$

Transfer length deflation of prestress at ends

We surmiss a linear decay from the full value of prestress at the age towards ends, hence prestress forces at each section have to be redefined...

Initial

$$PI_j := \begin{cases} P_i \cdot \dfrac{xx_j}{lt} & \text{if } xx_j \le lt \\ \text{otherwise} & \begin{cases} P_i & \text{if } AND2(xx_j > lt, xx_j \le l - lt) \\ P_i \cdot \dfrac{l - xx_j}{lt} & \text{otherwise} \end{cases} \end{cases}$$

Final or effective

$$PE_j := \begin{cases} P_e \cdot \dfrac{xx_j}{lt} & \text{if } xx_j \le lt \\ \text{otherwise} & \begin{cases} P_e & \text{if } AND2(x \\ P_e \cdot \dfrac{l - xx_j}{lt} & \text{othe} \end{cases} \end{cases}$$

Stresses

Compression figures positive $\qquad$ t stands for top face and b for bottom face

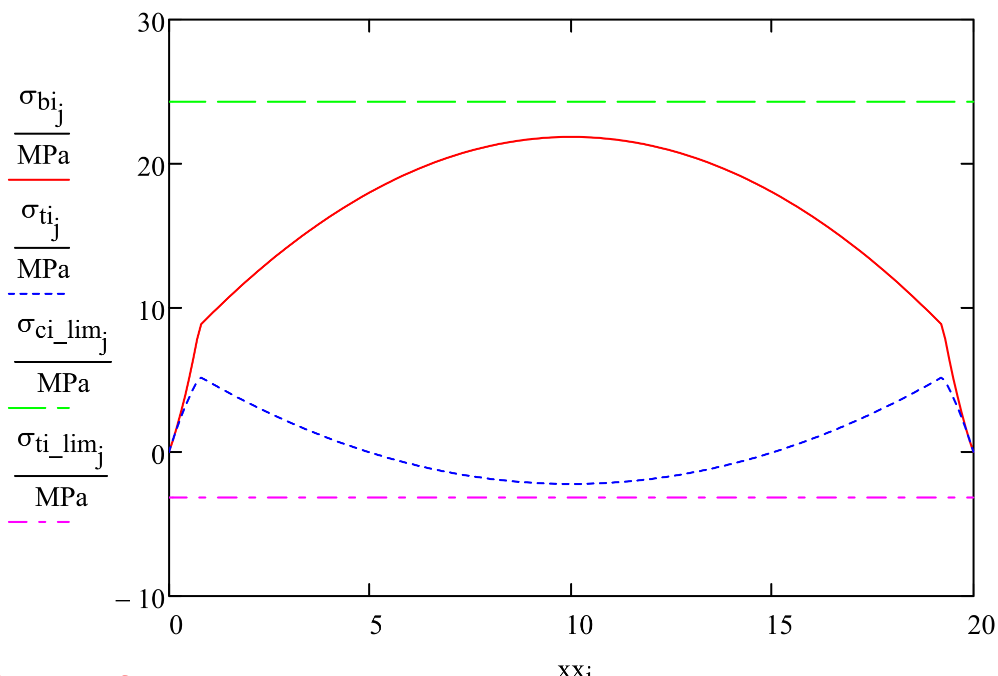
Check initial stresses at service level

Caused by weight plus highest ever prestress

$$\sigma_{bi_j} := \frac{PI_j}{Area} + \frac{PI_j \cdot \left(y_g - y_t\left(xx_j\right)\right)}{S_b} - \frac{M_w\left(xx_j\right)}{S_b}$$

$$\sigma_{ti_j} := \frac{PI_j}{Area} - \frac{PI_j \cdot \left(y_g - y_t\left(xx_j\right)\right)}{S_t} + \frac{M_w\left(xx_j\right)}{S_t}$$

$$\sigma_{ci\_lim_j} := 0.6 \cdot f_{ci} \qquad \sigma_{ti\_lim_j} := -6 \cdot \sqrt{\frac{f_{ci}}{psi}} \cdot psi$$



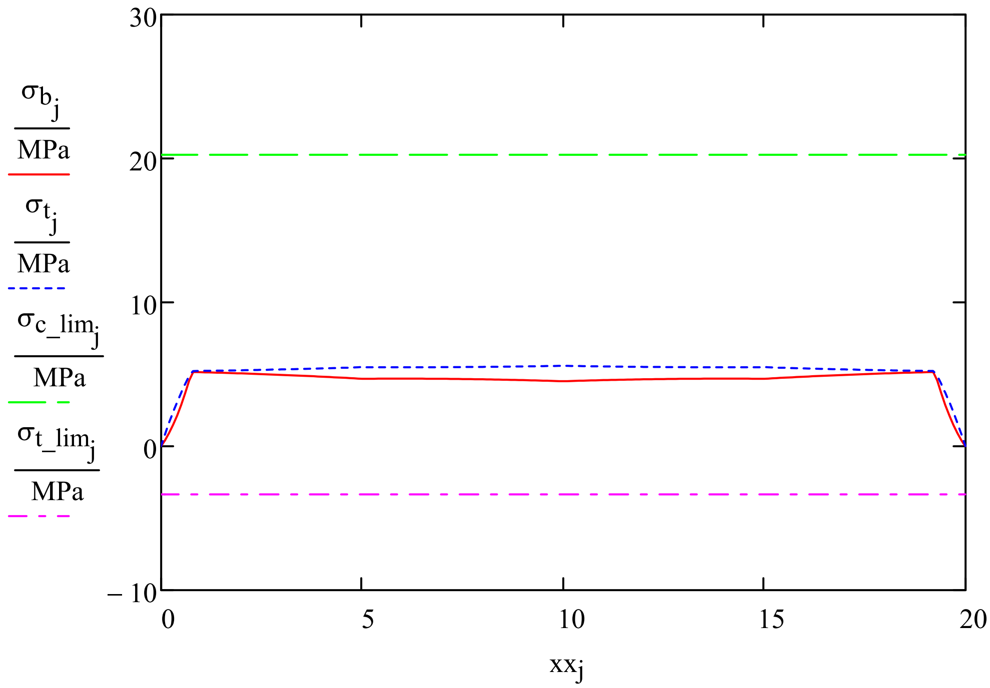
Check final stresses at service level

Caused by weight plus highest ever prestress

$$\sigma_{b_j} := \frac{PE_j}{Area} + \frac{PE_j \cdot \left(y_g - y_t\left(xx_j\right)\right)}{S_b} - \frac{M\left(xx_j\right)}{S_b}$$

$$\sigma_{t_j} := \frac{PE_j}{Area} - \frac{PE_j \cdot \left(y_g - y_t\left(xx_j\right)\right)}{S_t} + \frac{M\left(xx_j\right)}{S_t}$$

$$\sigma_{c\_lim_j} := 0.45 \cdot f_c \qquad \sigma_{t\_lim_j} := -6 \cdot \sqrt{\frac{f_c}{psi}} \cdot psi$$



Note how for the example the full service level condition is very favourable, since the full beam remains compressed around the average 5 MPa level. However the beam has to have too much depth for our taste, due to uncommon loading.

$$f_{pc} := \frac{P_e}{Area} \qquad f_{pc} = 5.2\,MPa$$

Change the choice of tendon profile and see the stresses vary.

Checking Limit Strength

Logic for this we don't include here but do with another Mathcad sheet, **fx124b.mcd**

We only will check center point which here takes maximum moment

$Mmax_{factored} := max(MMU)$ $Mmax_{factored} = 431.87\, m \cdot ton$

It is usual to provide bonded reinforcement at 4 per thousand of area between cgc and bottom face, distributed cgc down.
When no tensile stresses appear long term (like here) or they are moderate, no bonded reinforcement is required per ACI 18.9.3.1. Still it is a good practice.

$A_{s\_recommended} := 0.004 \cdot Area_{cgc\_down}$ $A_{s\_recommended} = 8.93\, cm^2$

We check limit strength with **fx124b.mcd** and 6 ϕ16 mm passive rebar (more than recommended) close to the bottom of the beam and we see that at the center we have $\phi \cdot$Mn=429.15 m·ton capacity, which is close enough for the factored condition (any arbitrary or based in engineering practice assumed value can cause such difference in flexural capacity) and we decide to approve flexural strength at least for this theoretical case. Other sections would need to be checked.

The compatibility of deformations analysis based in a realistic asessment of stress-strain laws for both concrete and steels discovers that the moment strength attains its maximum *for this section* when all the materials remain practically elastic, whereas further progression in the inelastic behaviour for them can only be made with some accompanying reduction of the capacity attained at such maximum.

Raising eyebrows? Take this then. Rare as it may sound to you, it may be happening that in some bridge decks where concrete is of much lower strength than the precast beams below, the deck adds nothing to limit bending strength; it is no more than butter (and flexurally a nonuseful burden) to the stiff members below. The overall maximum flexural strength never will be more than that of the strong supporting precast members.

This can also happen for composite decks on steel stringers. Once the compressed part of the steel shape attains plasticity, the axial rigidity of the deck is (for such cases) unable to refrain it. Many composite beam checks can be promptly dismissed if you prove such is the case, since the deck is then more than anything another superimposed dead load.

Checking shear

First we also state mean compression at every point taking into account transfer length

$$F_{pc_j} := \frac{PE_j}{Area}$$

$Slope_t(x) := \frac{d}{dx} y_t(x)$ $SLOPE_j := Slope_t(xx_j)$ the harped tendons may re other treatment due to met discontinuity

Since prestress is always favourably opposing shearing action, we can consider it always positive.

$V_{p_j} := |PE_j \cdot SLOPE_j|$ vertical component of prestress force

$$Depth_j := max\left(\begin{pmatrix} 0.8 \cdot h \\ h - y_t(xx_j) \end{pmatrix}\right)$$

We identify width of web as the lesser of the stated widths $b_{w1} := min\left(d^{\langle 1 \rangle}\right)$ b

$b_w = 16\,cm$

$$V_{cw_j} := \left(3.5 \cdot \sqrt{\frac{f_c}{psi}} \cdot psi + 0.3 \cdot F_{pc_j}\right) \cdot b_w \cdot Depth_j + V_{p_j}$$

Web shear cracking capacity at the investigated points

$$\sigma_{b\_p_j} := \frac{PE_j}{Area} + \frac{PE_j \cdot (y_g - y_t(xx_j))}{S_b}$$

$$M_{cr_j} := \left(\sigma_{b\_p_j} + \left|\sigma_{t\_lim_j}\right|\right)\cdot S_b$$

Moment at which upon decompression by flexion action and further bending causing tension, the bottom face cracks

$$j := 2 .. Nparts$$

to avoid divide by moment zero

$$V_{ci_j} := \max\left[\left[\begin{matrix} 0.6\cdot\sqrt{\frac{f_c}{psi}}\cdot psi\cdot b_w\cdot Depth_j + \frac{\left|V\left(xx_j\right)\right|}{M\left(xx_j\right)}\cdot M_{cr_j} \\ 1.7\cdot\left(\sqrt{\frac{f_c}{psi}}\cdot psi\cdot b_w\cdot Depth_j\right)\end{matrix}\right]\right]$$

since in a quotient facto unfactored moments if c don't mind.

$$j := 1 \qquad V_{ci_j} := 0.6\cdot\sqrt{\frac{f_c}{psi}}\cdot psi\cdot b_w\cdot Depth_j$$

mind not

$$j := Nparts + 1 \qquad V_{ci_j} := 0.6\cdot\sqrt{\frac{f_c}{psi}}\cdot psi\cdot b_w\cdot Depth_j$$

mind not

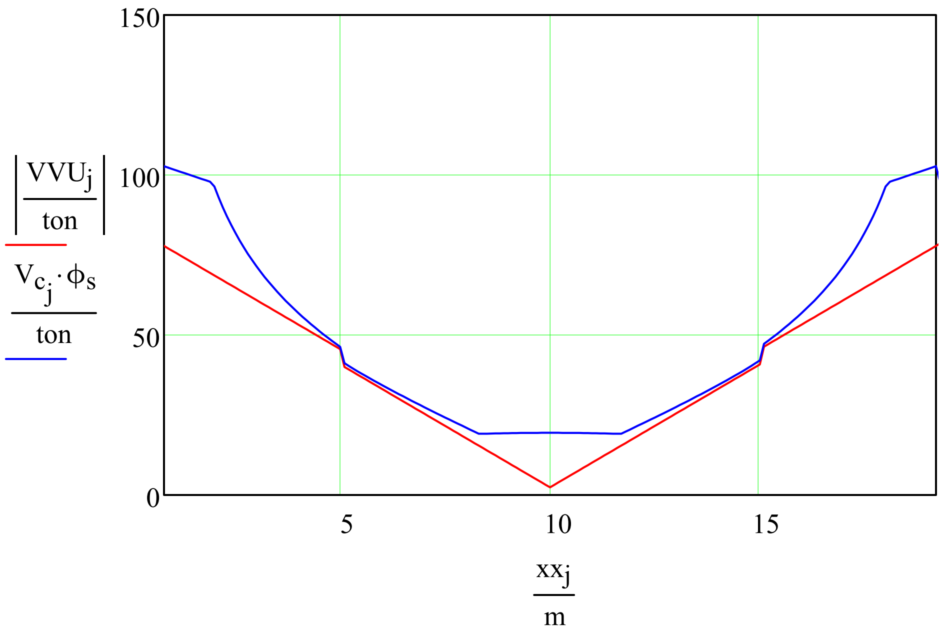
$$j := 1 .. Nparts + 1$$

restoring full set

$$V_{c_j} := \min\left(\left(\begin{matrix} V_{cw_j} \\ V_{ci_j}\end{matrix}\right)\right)$$

Shear capacity (of concrete plus vertical projection of tend

(Factored loads level) Shear demand in this beam



- The calculated shear capacity is the minimum of that limit for
  web shear cracking
  or
  flexure-shear cracking
  since both would amount to unwanted cracks at the being checked factored level.
- It is seen that in our case we remain safe by calculation against shear failure even without resc stirrups.
- We always dispose anyway minimum shear reinforcement.
- The chart is curtailed at 0.5 h from both ends as pertains to shear checks for prestressed bear the beam themselves are not being shown.
- Note that the shear strength reduction factor is already applied in the chart.
- The proper level of shear strength requirement is established by the amplification of dead and li nothing to do with the shear strength reduction factor.
- Where the red line exceeds the blue line, stirrups or other shear reinforcement are required. The required to be delivered by stirrups after $\phi_{sfactor}$ reduction is given by such excess.
- It is interesting to observe how the shear capacity is lesser in the regions of maximum moment implied Shear-Moment interaction in the formulation of shear capacity, even we don't use to thi concrete beams.

See where we stand in top smeared shear capacity

$V_{cmax} := \max(V_c)$ $v_{cmax} := \frac{V_{cmax}}{b_w \cdot 0.9 \cdot h}$ $v_{cmax} = 5.14\,MPa$ maximum smeared c
only concrete **plus in**

Stirrups reinforcement

$$\phi V_{s_j} := \begin{cases} \max\left(\begin{pmatrix} |VVU_j| - V_{c_j} \cdot \phi_s \\ 0 \cdot ton \end{pmatrix}\right) & \text{if } AND2(xx_j \geq 0.5 \cdot h, xx_j \leq l - 0.5 \cdot h) \\ 0 \cdot ton & \text{otherwise} \end{cases}$$

$$V_{s_j} := \frac{\phi V_{s_j}}{\phi_s}$$

Note that we exclude from any stirrups need the sections closer than 0.5h to supports, as proper for prestressed beams. This is not to say we won't be placing stirrups there.

$Check_{size_j} := V_{s_j} - 4 \cdot V_{c_j}$ $Check_{size\_max} := \max(Check_{size})$

$Section_{Size} :=$ | "is valid anywhere from the viewpoint of size required for the assume
| "you need enlarge the shear section since it does not meet the shear si

$Section_{Size} =$ "is valid anywhere from the viewpoint of size required for the assumed

Will reinforce in this sheet in the classical approach where the θ angle formed by the concrete struts with longitudinal axis is not a factor. This is not consistent but is a conservative assumption for any prestressed beam. Note however that prestress effect is at least partially accounted for in the evaluation of the web shear and shear-flexural strength capacities above.

Let's define a tentative stirrups arrangement.
Required separations will be calculated.

$f_y := 400 \cdot MPa$ stirrup's steel

$N_{leg\_e} := 2$ number of legs per plane of stirrups at beams ends

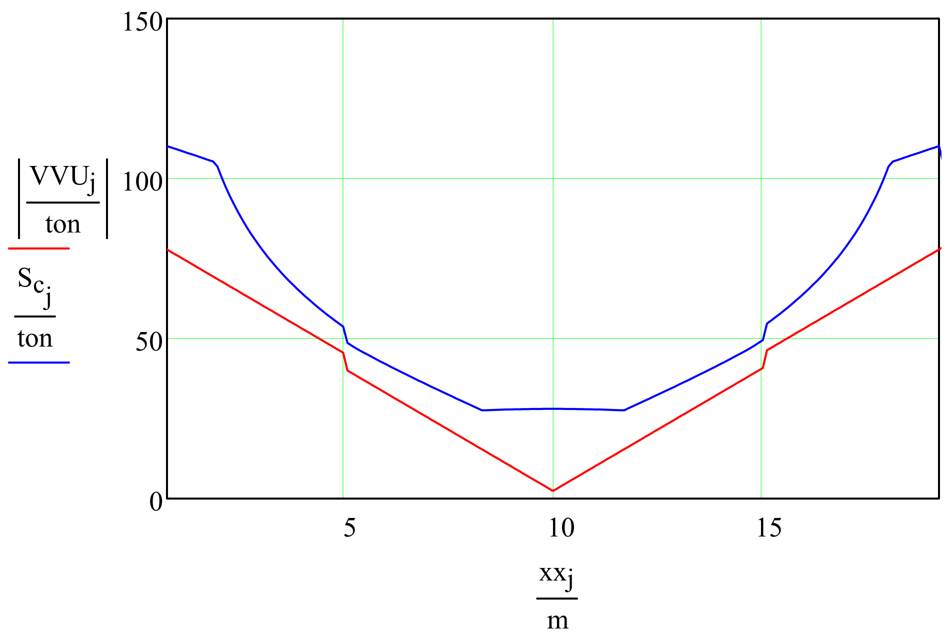
$\phi_e := 8 \cdot mm$

$k_e := 0.3$ fraction of l
stirrup arrar

$N_{leg\_c} := 2$ number of legs per plane of stirrups at remaining center $\phi_c := 8 \cdot mm$

$s_e = 60\,cm$ $s_c = 60\,cm$

Plot shear strength check with the input stirrups (at the given separation) contribution



Whatever the result, I wouldn't reinforce in shear with stirrups farther apart than $2 \cdot b_w$ for thin web beams.

Manuel Oliveros Martínez, Architect, 1999
Comments & Corrections e-mail

mom@arrakis.es

exural strength checks are not.

on the same basis.

of live

$$P_{TOT} = \begin{pmatrix} 3 \\ 3 \\ 3 \\ 0 \end{pmatrix} ton$$

starting from bottom trapeze
tacked trapezes, need not be of same vertex at interface)

es are of brute
of concrete

$\cdot_i + CS_{LR} \cdot P_{L_i}, A_i, x\Big)$

$\Big)_i + CS_{LR} \cdot P_{L_i}, A_i, x\Big]$

$:= M_u\left(xx_j\right) \quad VV_j := V\left(xx_j\right) \quad VVU_j := V_u\left(xx_j\right)$

ıt inf fraction of span, only
ıse if harped at 2 points

⋅ksi

$f_{pi} = 1396.21\,MPa$

$f_{pe} = 1116.97\,MPa$

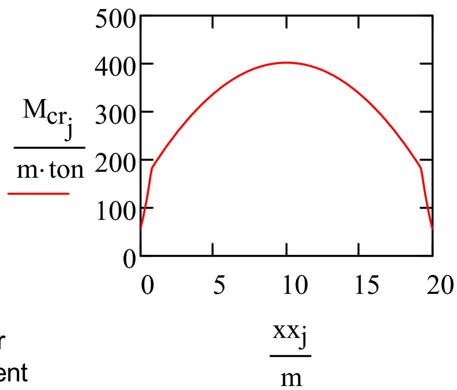
$xx_j > lt, xx_j \leq 1 - lt)$

erwise

quire some

$_{w2} := \min\left(d^{\langle 2 \rangle}\right)$ $b_w := \min\left(\begin{pmatrix} b_{w1} \\ b_{w2} \end{pmatrix}\right)$

Note we are not curtailing to
maximum shear 100 psi so beware
very high concrete strengths



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ıclination of tendon if any

$\phi V_{smax} := \max(\phi V_s)$ $\phi V_{smax} = 0\,\text{ton}$

:d factored shear" if $Check_{size\_max} \le 0 \cdot ton$

ze requirement at some point" otherwise

factored shear"

ength from end of this
ıgement