Aim is to calculate basic large angle stability assessment for a simple 'box' ship. This sheet was my first experimentation at a single ship station. I have since modelled the same for a full ship i.e. the 3D shape not just 2D.

Key:
Green Highlights are input data
Blue Highlights are output data
D = Depth of Ship i.e. length between keel and sheer
B = Breadth of Ship i.e. length between ship sides
T = Draught i.e. distance between the keel and the waterline (on an even keel).
KG = Distance between keel and the centre of gravity
S = Station data i.e. the ordinates of the ship profile to Starboard
Sp = Station data to Port
w and ww = waterline data
y' = horizontal ordinate for centre of buoyancy
z' = vertical ordinate for centre of buoyancy

$D := 20\ m$ $B := 10\ m$ $T := 5\ m$ $KG := 8\ m$ $z := 0\ m, 0.1\ m .. 21\ m$

Station Data

$$S := \begin{bmatrix} 0\ m & 0\ m \\ B & 0.001\ m \\ B & D \\ 0\ m & D + 0.001\ m \end{bmatrix}$$

$$S_p := \begin{array}{l} s^{\langle 0 \rangle} \leftarrow -S^{\langle 0 \rangle} \\ s^{\langle 1 \rangle} \leftarrow S^{\langle 1 \rangle} \\ \text{return } s \end{array}$$

Interpretation of Station Data

$$S_t(z) := \text{linterp}\left(S^{\langle 1 \rangle}, S^{\langle 0 \rangle}, z\right) \qquad S_{tp}(z) := \text{linterp}\left(S_p^{\langle 1 \rangle}, S_p^{\langle 0 \rangle}, z\right)$$

Waterline Data & Interpretation function w deals with heel angle s from 0 to 90 degrees and function ww deals with angles from 90 to 180 degrees heel. NB due to the interpretation process the waterline always has to increase as it is interpretated vertically. Therefore 0, 90 and 180 degrees cannot be interpreted precisely as MC falls over itself.

$$w(T, \phi) := \begin{bmatrix} -B & T - \tan(\phi) \cdot B \\ B & T + \tan(\phi) \cdot B \end{bmatrix} \qquad ww(T, \phi) := \begin{bmatrix} B & T - \tan(\phi) \cdot B \\ -B & T + \tan(\phi) \cdot B \end{bmatrix}$$

$$WL(z, T, \phi) := \begin{array}{l} \text{if } z < w(T, \phi)_{0,1} \\ \quad S_{tp}(z) \\ \text{else} \\ \quad \text{linterp}\left(w(T, \phi)^{\langle 1 \rangle}, w(T, \phi)^{\langle 0 \rangle}, z\right) \end{array}$$

$$VWL(z, T, \phi) := \begin{array}{l} \text{if } z > ww(T, \phi)_{1,1} \\ \quad S_{tp}(z) \\ \text{else} \\ \quad \text{linterp}\left(ww(T, \phi)^{\langle 1 \rangle}, ww(T, \phi)^{\langle 0 \rangle}, z\right) \end{array}$$

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$$\text{linterp}(ww(T,\phi), ww(T,\phi), z)$$

Calculation of of area between the ship sides and up to the draught on an even keel i.e. no heel. This is used to compare the heeled waterlines and the un heeled waterlines. The purpose being to ensure that as the ship heels the waterline gives a constant displacement. I have found that if you do not do this and heel past a deck or keel emergance/ immersion the ship displaces furter which is not correct as no additional mass is being added during the heeling process.

$$a := \int_{0\,m}^{T} \int_{S_{tp}(z)}^{S_t(z)} 1 \, dy \, dz$$

The following solve blocks find the height at which to integrate to for each angle of heel i.e. the height at whcih the waterline intersects the starboard side of the ship.

Constraints Values

$$z := 6\ m$$
$$S_t(z) = WL(z, T, \phi)$$
$$Z(T,\phi) := \text{find}(z)$$

Constraints Values

$$z := 6\ m$$
$$S_t(z) = WWL(z, T, \phi)$$
$$Zz(T,\phi) := \text{find}(z)$$

The following calculate the area under each heeled waterline and the ship side. Due to the programme above this integrates between the port side and the starboard side up to the waterline. This is the key issue I was trying to resolve in the thread on the community. A deals with angles to 90 and AA to 180.

$$A(T,\phi) := \int_{0\,m}^{Z(T,\phi)} \int_{WL(z,T,\phi)}^{S_t(z)} 1 \, dy \, dz \qquad AA(T,\phi) := \int_{Zz(T,\phi)}^{S_{3,1}} \int_{WWL(z,T,\phi)}^{S_t(z)} 1 \, dy \, dz$$

The following solve blocks find a draught whereby the original unheeled waterline and the heeled waterlines give an equal area. i.e. to prevent further displacement as the ship is heeled.

Constraints Values

$$T := 5\ m$$
$$A(T,\phi) = a$$
$$TT(\phi) := \text{find}(T)$$

Constraints Values

$$T := 12\ m$$
$$AA(T,\phi) = a$$
$$Tt(\phi) := \text{find}(T)$$

This inversing function allows interpretation of waterlines between 90 and 180 degrees as what I am actually doing is caluclating the inverse waterline of between 0 to 90 degrees.

$$\phi_{inv}(\phi) := -(\phi - 180\ deg)$$

The following find the y and z orginates of the centre of buoyancy (area under waterline) for angles to 90 degrees.

$$y'(\phi) := \frac{\int_{0\,m}^{Z(TT(\phi),\phi)} \int_{WL(z,TT(\phi),\phi)}^{S_t(z)} y \, dy \, dz}{\int^{Z(TT(\phi),\phi)} \int^{S_t(z)} 1 \, dy \, dz} \qquad z'(\phi) := \frac{\int_{0\,m}^{Z(TT(\phi),\phi)} \int_{WL(z,TT(\phi),\phi)}^{S_t(z)} z \, dy \, dz}{\int^{Z(TT(\phi),\phi)} \int^{S_t(z)} 1 \, dy \, dz}$$

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The following do the same from 90 degrees to 180 degrees.

$$yy'(\phi) := \frac{\int_{Zz(Tt(\phi),\phi)}^{S_{3,1}} \int_{WWL(z,Tt(\phi),\phi)}^{S_t(z)} y \, dy \; dz}{\int_{Zz(Tt(\phi),\phi)}^{S_{3,1}} \int_{WWL(z,Tt(\phi),\phi)}^{S_t(z)} 1 \, dy \; dz}$$

$$zz'(\phi) := \frac{\int_{Zz(Tt(\phi),\phi)}^{S_{3,1}} \int_{WWL(z,Tt(\phi),\phi)}^{S_t(z)} z \, dy \; dz}{\int_{Zz(Tt(\phi),\phi)}^{S_{3,1}} \int_{WWL(z,Tt(\phi),\phi)}^{S_t(z)} 1 \, dy \; dz}$$

These programmes return either ordinates and waterplanes dependant on the angle of heel being investigated.

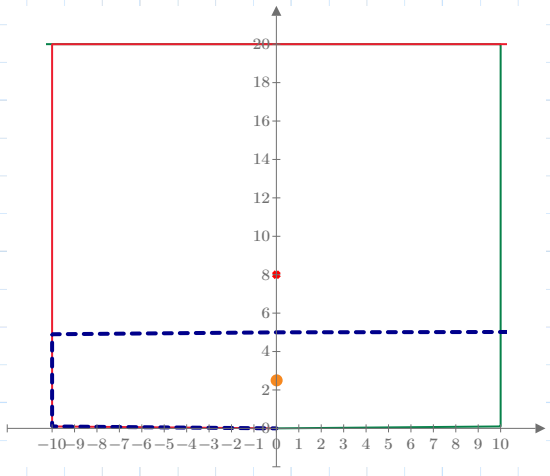
$$Y'(\phi) := \begin{cases} yy'(\phi_{inv}(\phi)) & \text{if } \phi > 90 \ \textit{deg} \\ y'(\phi) & \text{else} \end{cases}$$

$$Z'(\phi) := \begin{cases} zz'(\phi_{inv}(\phi)) & \text{if } \phi > 90 \ \textit{deg} \\ z'(\phi) & \text{else} \end{cases}$$

$$W(z,\phi) := \begin{cases} \text{if } \phi > 90 \ \textit{deg} \\ \quad WWL(z, Tt(\phi_{inv}(\phi)), \phi_{inv}(\phi)) \\ \text{else if } \phi < 90 \ \textit{deg} \\ \quad WL(z, TT(\phi), \phi) \end{cases}$$

Visual representation of each angle investigated.

$$ang_a := 0.1 \ \textit{deg}$$



$z \ (m)$

$z \ (m)$

$z \ (m)$

$Z'(ang_a) \ (m)$

$KG \ (m)$

$S_t(z) \ (m)$

$S_{tp}(z) \ (m)$

$W(z, ang_a) \ (m)$

$Y'(ang_a) \ (m)$

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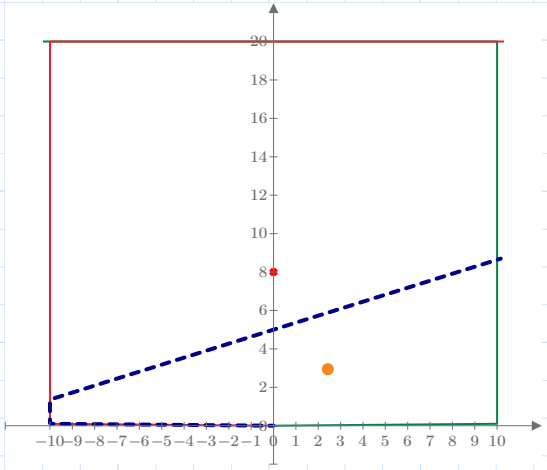
$Y'(ang_a)\ \ (m)$

$0\ m\ \ (m)$

$ang_b := 20\ deg$



$z\ \ (m)$

$z\ \ (m)$

$z\ \ (m)$

$Z'(ang_b)\ \ (m)$

$KG\ \ (m)$

$S_t(z)\ \ (m)$

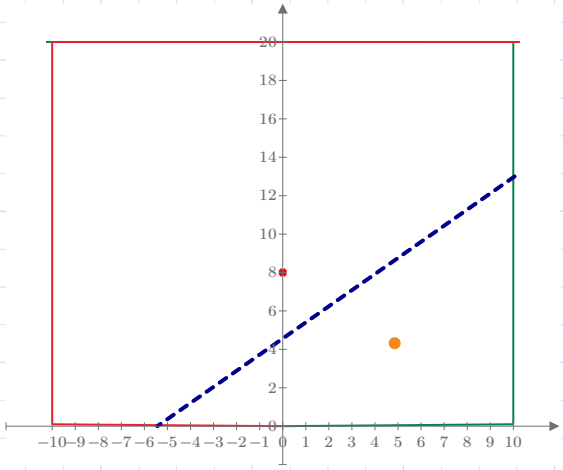
$S_{tp}(z)\ \ (m)$

$W(z, ang_b)\ \ (m)$

$Y'(ang_b)\ \ (m)$

$0\ m\ \ (m)$

$ang_c := 40\ deg$



$z\ \ (m)$

$z\ \ (m)$

$z\ \ (m)$

$Z'(ang_c)\ \ (m)$

$KG\ \ (m)$

$S_t(z)\ \ (m)$

$S_{tp}(z)\ \ (m)$

$W(z, ang_c)\ \ (m)$

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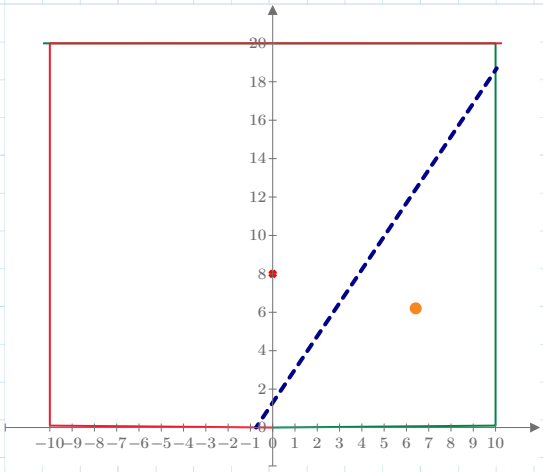
$Y'(ang_c)$ (m)

$0\ m$ (m)

$ang_d := 60\ deg$



z (m)

z (m)

z (m)

$Z'(ang_d)$ (m)

KG (m)

$S_t(z)$ (m)

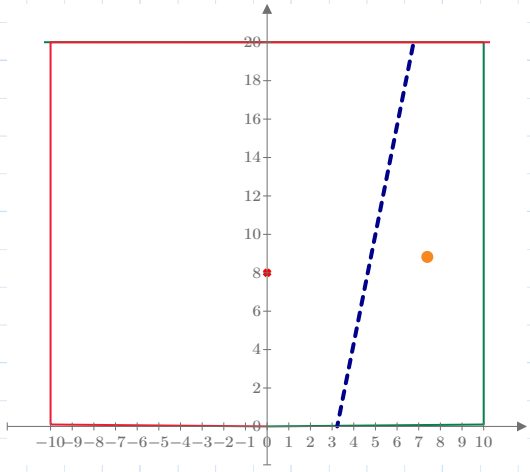
$S_{tp}(z)$ (m)

$W(z, ang_d)$ (m)

$Y'(ang_d)$ (m)

$0\ m$ (m)

$ang_e := 80\ deg$



z (m)

z (m)

z (m)

$Z'(ang_e)$ (m)

KG (m)

$S_t(z)$ (m)

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$KG \ (m)$

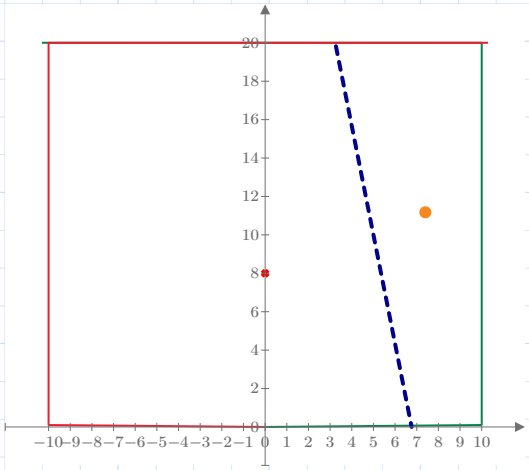
$S_{tp}(z) \ (m)$

$W(z, ang_e) \ (m)$

$Y'(ang_e) \ (m)$

$0 \ m \ (m)$

$ang_f := 100 \ deg$



$z \ (m)$

$z \ (m)$

$z \ (m)$

$Z'(ang_f) \ (m)$

$KG \ (m)$

$S_t(z) \ (m)$

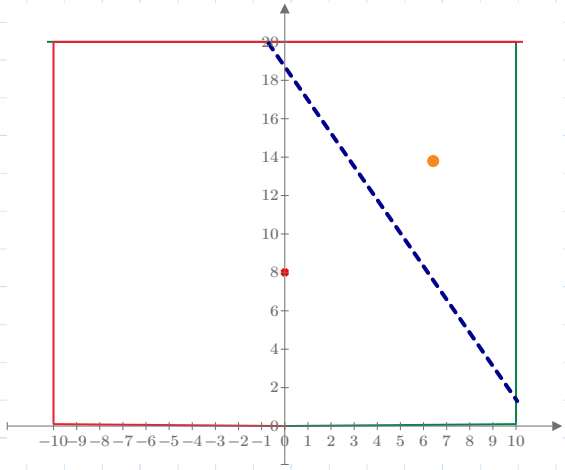
$S_{tp}(z) \ (m)$

$W(z, ang_f) \ (m)$

$Y'(ang_f) \ (m)$

$0 \ m \ (m)$

$ang_g := 120 \ deg$



$z \ (m)$

$z \ (m)$

$z \ (m)$

$Z'(ang_g) \ (m)$

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$S_t(z)\ \ (m)$

$S_{tp}(z)\ \ (m)$

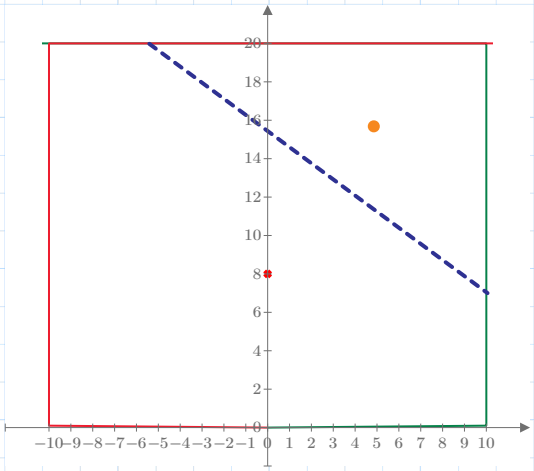
$W(z, ang_g)\ \ (m)$

$Y'(ang_g)\ \ (m)$

$0\ m\ \ (m)$

$Z'(ang_g)\ \ (m)$

$KG\ \ (m)$



$z\ \ (m)$

$z\ \ (m)$

$Z'(140\ deg)\ \ (m)$

$KG\ \ (m)$

$z\ \ (m)$

$S_t(z)\ \ (m)$

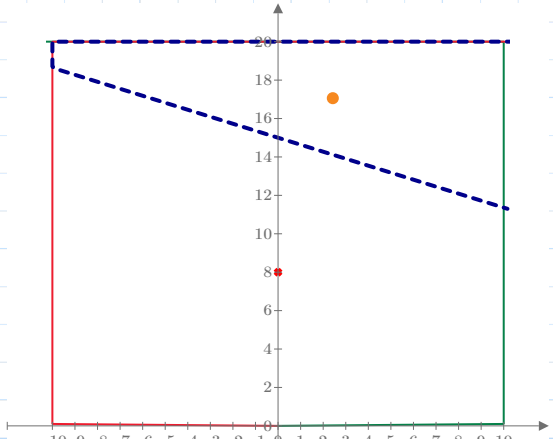
$S_{tp}(z)\ \ (m)$

$Y'(140\ deg)\ \ (m)$

$0\ m\ \ (m)$

$W(z, 140\ deg)\ \ (m)$

$ang_i := 160\ deg$



$z\ \ (m)$

$z\ \ (m)$

$z\ \ (m)$

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$S_t(z)\ \ (m)$

$S_{tp}(z)\ \ (m)$

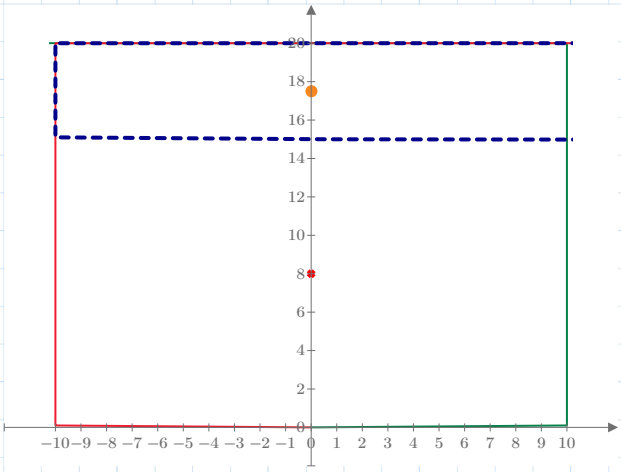
$W\langle z, ang_i\rangle\ \ (m)$

$Y'\langle ang_i\rangle\ \ (m)$

$0\ m\ \ (m)$

$Z'\langle ang_i\rangle\ \ (m)$

$KG\ \ (m)$



$S_t(z)\ \ (m)$

$S_{tp}(z)\ \ (m)$

$0\ m\ \ (m)$

$Y'(179.9\ deg)\ \ (m)$

$W(z, 179.9\ deg)\ \ (m)$

$z\ \ (m)$

$z\ \ (m)$

$KG\ \ (m)$

$Z'(179.9\ deg)\ \ (m)$

$z\ \ (m)$

Derivation og GZ (the ship's righting lever at each angle investigated. Including graphing of GZ and interpretation for other angles.

$$GZ(\phi) := Y'(\phi)\cdot\cos(\phi) + (Z'(\phi) - KG)\cdot\sin(\phi)$$

$$r := \begin{bmatrix} 0.1 \\ 20 \\ 40 \\ 60 \\ 80 \\ 100 \\ 120 \end{bmatrix} deg \qquad i := 0..\text{last}(r) \qquad Gz_i := GZ\left(r_i\right)$$

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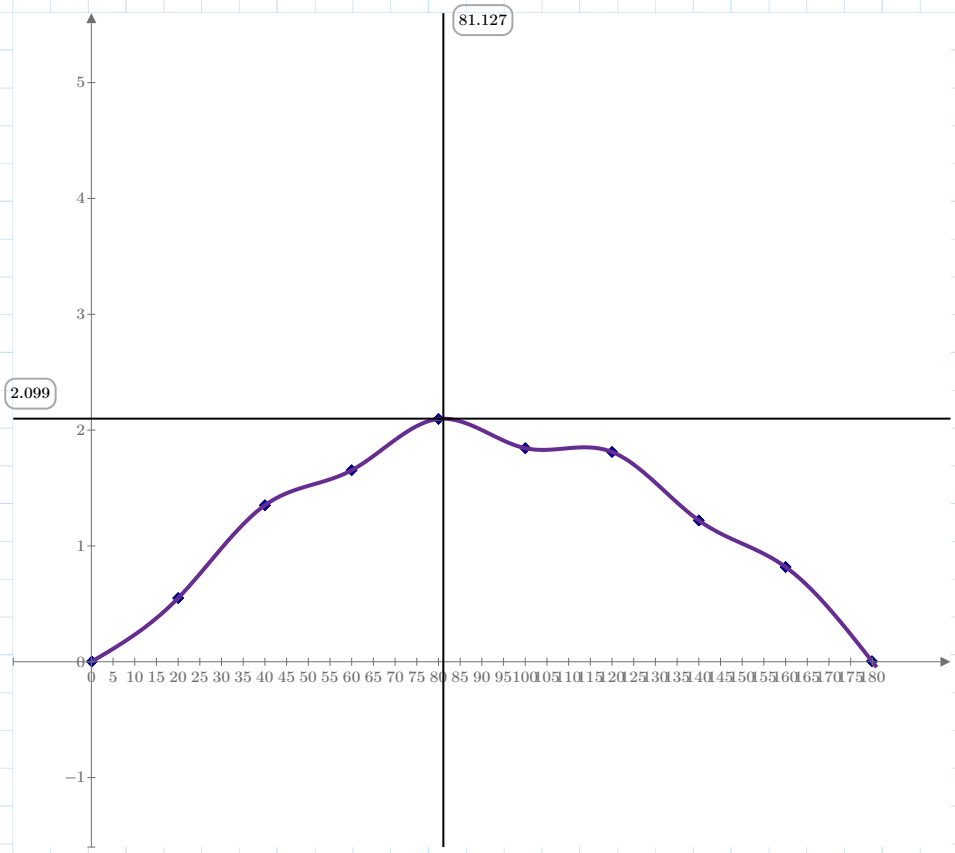
$$\begin{bmatrix} 120 \\ 140 \\ 160 \\ 179.9 \end{bmatrix}$$

$Plot := \text{lspline}(r, Gz)$ $f(x) := \text{interp}(Plot, r, Gz, x)$ $\phi_x := 0\ \textit{deg}, 1\ \textit{deg} .. \text{last}(r)$



Given
$\phi_x := 80\ \textit{deg}$
$\frac{d}{d\phi_x} f(\phi_x) = 0$
Solver
$max := \text{find}(\phi_x) = 81.127\ \textit{deg}$

$GZ_{max} := f(max) = 2.099\ \textit{m}$

Given
$\phi_x := 0\ \textit{deg}$
$f(\phi_x) = 0\ m$
Solver
$\phi_e := \text{find}(\phi_x) = 0.008\ \textit{deg}$

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Values

$\phi_x := 180\ deg$

Guess

$f(\phi_x) = 0\ m$

Constraints

$\phi_v := \text{find}(\phi_x) = 180.004\ deg$

$Range := \phi_v - \phi_e = 179.996\ deg$

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