

For these questions you will need to refer to Section 1.5.2 to 1.5.4 of the course material on fatigue and fracture.

1) You have been given the following stress cycling history for a component:

Stress Range (MPa)	No. of Cycles per Year
5	2.00E+06
10	1.00E+06
30	4.00E+05
50	1.50E+04
100	5.00E+02
120	3.00E+02

a) Calculate the number of cycles to design failure for each stress range and plot the results as an S-N curve. You are told that $A = 0.431 \times 10^{12} \text{ MPa}^3$ and $m = 3$ for this material and configuration.

b) Calculate the annual fatigue damage for each stress range.

S-N Curves (Stress - Number of Cycles)

N to failure increases with decreasing load.

curve takes the form:

$$N = \frac{A}{S^m}$$

N = number of cycles to failure

S = Stress range

A and m are constants

(for steel, m ranges 3 - 5)

$$S := \begin{bmatrix} 5 \\ 10 \\ 30 \\ 50 \\ 100 \\ 120 \end{bmatrix} \text{ MPa}$$

$$n := \begin{bmatrix} 2 \cdot 10^6 \\ 1 \cdot 10^6 \\ 4 \cdot 10^5 \\ 1.5 \cdot 10^4 \\ 5 \cdot 10^2 \\ 3 \cdot 10^2 \end{bmatrix} \cdot \frac{1}{\text{yr}}$$

$$A := 0.431 \cdot 10^{12} \text{ MPa}^3$$

$$m := 3$$

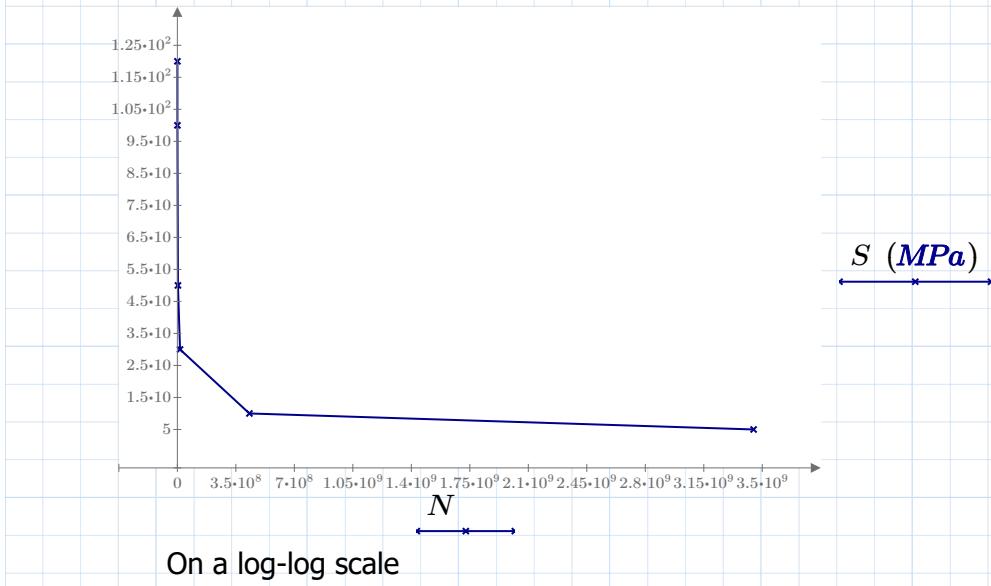
$$N := \frac{A}{S^m}$$

$$S = \begin{bmatrix} 5 \\ 10 \\ 30 \\ 50 \\ 100 \\ 120 \end{bmatrix} \text{ MPa}$$

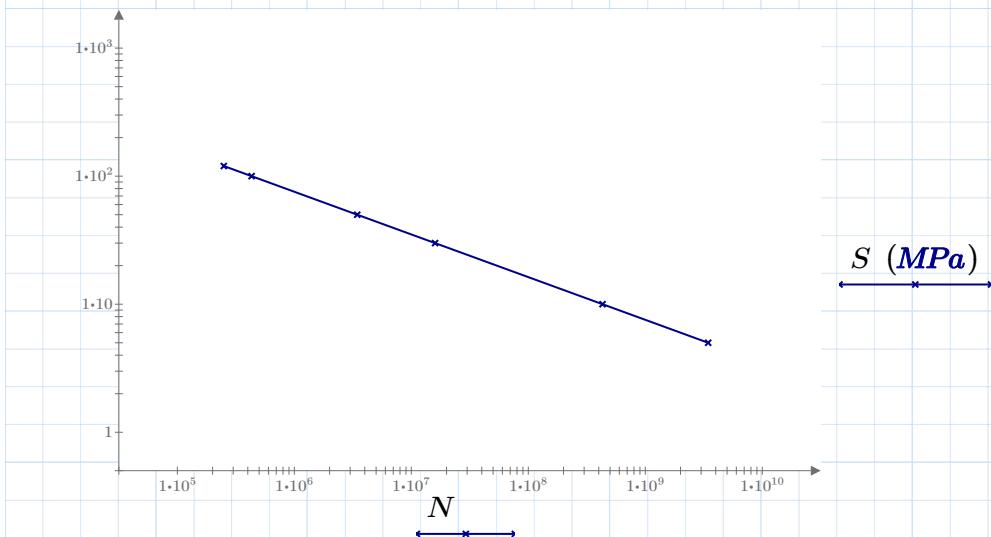
$$N = \begin{bmatrix} 3.448 \cdot 10^9 \\ 4.31 \cdot 10^8 \\ 1.596 \cdot 10^7 \\ 3.448 \cdot 10^6 \\ 4.31 \cdot 10^5 \\ 2.494 \cdot 10^5 \end{bmatrix}$$

$$i := 0 \dots \text{last}(S)$$

For raw data:



On a log-log scale



Miner's Rule states:

Failure will occur when:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = \sum \frac{n_i}{N_i} = 1$$

Where:

n_i = number of cycles of stresses in a band s_i

N_i = number of cycles to failure for stress s_i

Proportion of life used of damage caused by n_i cycles of s_i =

$$D_i := \frac{n_i}{N} = \begin{bmatrix} 5.8 \cdot 10^{-4} \\ 0.002 \\ 0.025 \end{bmatrix} \quad | \quad 1$$

$$D = \begin{bmatrix} 0.025 \\ 0.004 \\ 0.001 \\ 0.001 \end{bmatrix} \frac{1}{\text{yr}}$$

Using the stress cycling history detailed in Question 1 use the crack propagation procedure shown in Section 1.5.2 and Paris Law to calculate the crack size every year for the next 30 years.

You are told that the stresses occur in a random sequence and therefore you should use the equivalent stress procedure to calculate the weighted average stress that produces the same average crack growth rate/cycle.

Plot your results on a graph of crack size (mm) against time (years).

The Paris Law constants for this material and detail are: $m=3$, $C=12.5 \times 10^{-12}$ and $Y=1.5$. The initial crack size is 0.0005 m .

$$a := 0.0005 \text{ m} \quad Y := 1.5$$

In the supporting literature it claims that the units for m is MPa and units for C are m/cycle. However, some googling claims m is unitless and C is $\text{m}/(\text{cycle MPa m}^{0.5})$. Any thoughts on this and links to reference material?

Additionally, the material I have does not talk about units for dK but again the internet tells me this in in MPa $\text{m}^{0.5}$

$$\sigma_{eq} := \sqrt[m]{\frac{1}{N_1} \sum \sigma_r^m} \quad \sigma_r := S \quad N_1 := \text{last}(S) + 1$$

$$\sigma_{eq} = 78.308 \text{ MPa} \quad \text{cycle} := 1$$

$$\delta K := Y \cdot \sigma_{eq} \cdot \sqrt{\pi \cdot a} = 4.655 \text{ MPa} \cdot \text{m}^{0.5}$$

$$\delta N := \sum n \cdot \text{cycle} \quad C := 12.5 \cdot 10^{-12} \frac{\text{m}}{\text{cycle} \cdot \text{MPa} \cdot \text{m}^{0.5}}$$

$$\delta a := C \cdot (\delta K)^m \cdot \delta N$$

$$\delta a = 136.512 \frac{\text{kg}^2}{\text{m} \cdot \text{s}^5} \cdot \text{m}$$

I would expect the crack growth da to be in m/yr?