For these questions you will need to refer to Section 1.5.2 to 1.5.4 of the course material on fatigue and fracture.

1) You have been given the following stress cycling history for a component:

Stress Range (MPa)	No. of Cycles per Year
5	2.00E+06
10	1.00E+06
30	4.00E+05
50	1.50E+04
100	5.00E+02
120	3.00E+02

a) Calculate the number of cycles to design failure for each stress range and plot the results as an S-N curve. You are told that A=0.431x10^{12} MPa^3 and m=3 for this material and configuration.

b) Calculate the annual fatigue damage for each stress range.

S-N Curves (Stress - Number of Cycles)

N to failure increases with decreasing load.

curve takes the form:

$$N = \frac{A}{S^m}$$

N = number of cycles to failure
S = Stres range
A and m are constants

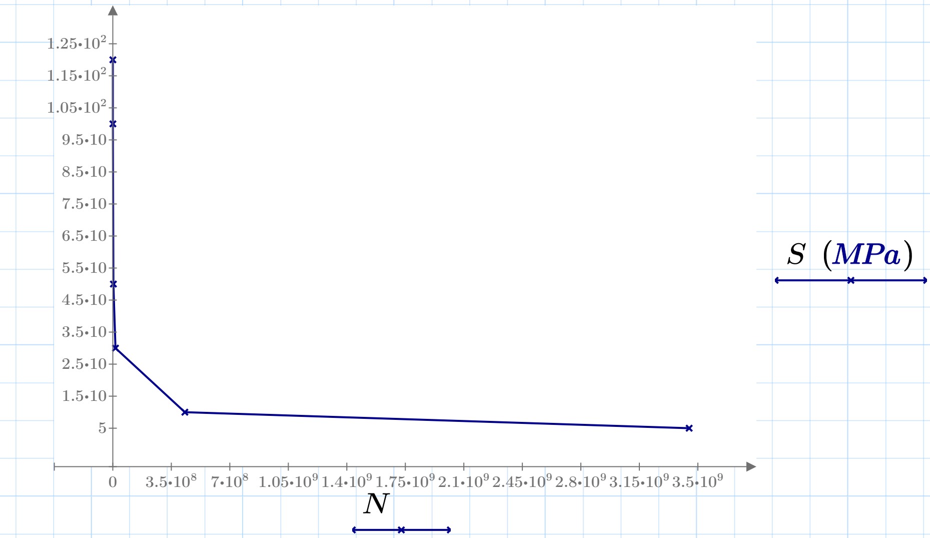
(for steel, m ranges 3 - 5)

$$S := \begin{bmatrix} 5 \\ 10 \\ 30 \\ 50 \\ 100 \\ 120 \end{bmatrix} MPa \qquad n := \begin{bmatrix} 2 \cdot 10^6 \\ 1 \cdot 10^6 \\ 4 \cdot 10^5 \\ 1.5 \cdot 10^4 \\ 5 \cdot 10^2 \\ 3 \cdot 10^2 \end{bmatrix} \cdot \frac{1}{yr}$$

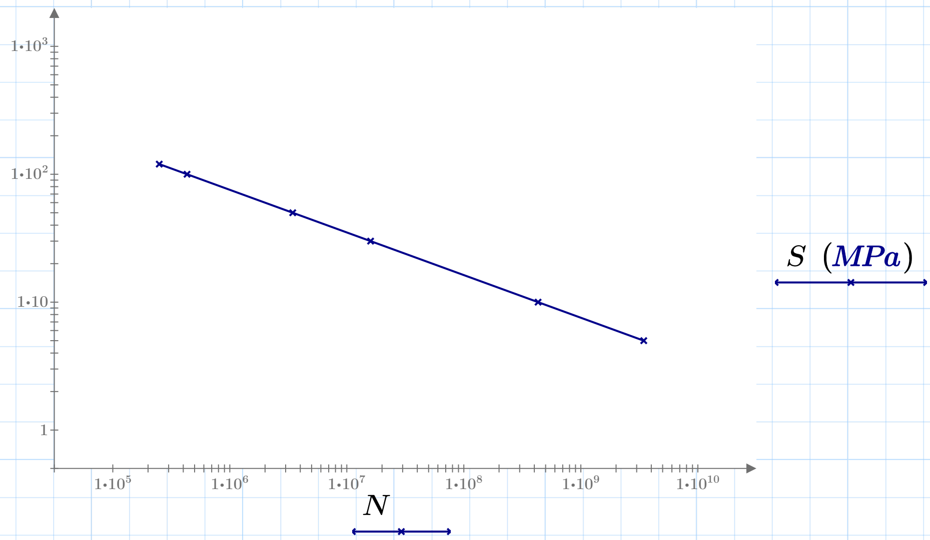
$$A := 0.431 \cdot 10^{12}\ MPa^3 \qquad m := 3 \qquad N := \frac{A}{S^m}$$

$$S = \begin{bmatrix} 5 \\ 10 \\ 30 \\ 50 \\ 100 \\ 120 \end{bmatrix} MPa \qquad N = \begin{bmatrix} 3.448 \cdot 10^9 \\ 4.31 \cdot 10^8 \\ 1.596 \cdot 10^7 \\ 3.448 \cdot 10^6 \\ 4.31 \cdot 10^5 \\ 2.494 \cdot 10^5 \end{bmatrix} \qquad i := 0 .. \text{last}(S)$$

For standard scale:



On a log-log scale



Miner's Rule states:

Failure will occure when:

$$\frac{n_1}{N_1} + \frac{n_2}{N_2} + \frac{n_3}{N_3} = \sum \frac{n_i}{N_i} = 1$$

Where:
ni =number of cycles of stresses in a band si
Ni = number of cycles to failure for stress si
Proportion of life used of damage caused by ni cycles of si =

$$D_i := \frac{n_i}{N_i} \qquad D = \begin{bmatrix} 5.8 \cdot 10^{-4} \\ 0.002 \\ 0.025 \end{bmatrix} \; 1$$

$$_i \quad N_i \qquad D = \begin{bmatrix} 0.025 \\ 0.004 \\ 0.001 \\ 0.001 \end{bmatrix} \frac{1}{yr}$$

2) Using the stress cycling history detailed in Question 1 use the crack propagation procedure shown in Section 1.5.2 and Paris Law to calculate the crack size every year for the next 30 years.

You are told that the stresses occur in a random sequence and therefore you should use the equivalent stress procedure to calculate the weighted average stress that produces the same average crack growth rate/cycle.

Plot your results on a graph of crack size (mm) against time (years).

The Paris Law constants for this material and detail are: m=3, C=12.5x10^{-12} and Y=1.5. The initial crack size is 0.0005m.

Crack growth calculations

The procedure for a crack growth calculation is:

a. Estimate initial crack length, a.

b. Determine the stress range causing the crack to grow.
In an offshore structure the stresses usually have a variable amplitude and then it is usually reasonable to use an equivalent stress range σ_{eq} that gives the same crack propagation rate as the actual distribution of stress ranges: σ_r. If however threshold or low cycle fatigue effects are to be taken into account it will be necessary to perform the calculation with representative groups of stress cycles of different sizes.
If σ_r has N values then:

$$\sigma_{eq} = \sqrt[m]{\frac{1}{N}\sum \sigma_{r_i}{}^m} \tag{1.20}$$

In this equation m is the material constant for the Paris equation.

In principle the growing crack leads to a reduction in stiffness and a redistribution of stress away from the crack. This would require the acting stress to be recalculated for different crack lengths. However, this effect is not found to be important, so the acting stress is calculated for the uncracked structure only.

c. Calculate $\delta K = Y\sigma_{eq}\sqrt{(\pi a)}$ at each stage of crack growth allowing for the crack size. Y is determined from published literature (e.g. Rooke and Cartwright, 1976) or δK can be determined directly from FE analysis. This application of finite element methods may seem laborious, but sub-structuring techniques (see Chapter 6) can be used so that each re-run with a new crack length is very quick.

d. δN is selected to give a small change in crack length.

f. a is increased from the previous value to $a + \delta a$.

g. Steps c to f are repeated. Hence a graph of crack length v. cycles or time may be plotted, see Figure 1.25.

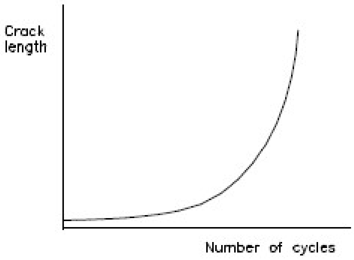


Figure 1.25 Crack growth with cycles of applied stress

a. initial crack length is given:

$a := 0.0005\ m$

b. equivilent stress range is defined using formula 1.20:

$$\sigma_{eq} = \sqrt[m]{\frac{1}{N}\sum \sigma_r^{\ m}}$$

It is assumed that the 6 stress ranges are as in the first problem and that:

$$\sigma_r := S = \begin{bmatrix} 5 \\ 10 \\ 30 \\ 50 \\ 100 \\ 120 \end{bmatrix} MPa \qquad N_1 := 6 \qquad \sigma_{eq} := \sqrt[m]{\frac{1}{N_1}\sum \sigma_r^{\ m}}$$

$\sigma_{eq} = 78.308\ MPa$

c. Calculation of dK (change in stress intensity factor)

$$\delta K = Y \cdot \sigma_{eq} \cdot \sqrt{\pi \cdot a} \qquad Y := 1.5 \qquad \delta K := Y \cdot \sigma_{eq} \cdot \sqrt{\pi \cdot a} = \left(4.655377 \cdot 10^{6}\right) \frac{kg}{m^{\frac{1}{2}} \cdot s^{2}}$$

d. Select the number of cycles to give a small crack length increase

$cycle := 1 \qquad \delta N := 10000\ cycle$

e. Apply the Paris' Law to get change in crack length (and define the other Paris' Law constants)

$$C := 12.5 \cdot 10^{-12} \frac{m}{cycle \cdot \left(MPa \cdot m^{0.5}\right)^{m}} \qquad \delta a := C \cdot (\delta K)^{m} \cdot \delta N$$

This gives the first change in crack length growth after dN cycles

$\delta a = \left(1.261 \cdot 10^{-5}\right)\ m$

f. da and a are added to give the new crack length

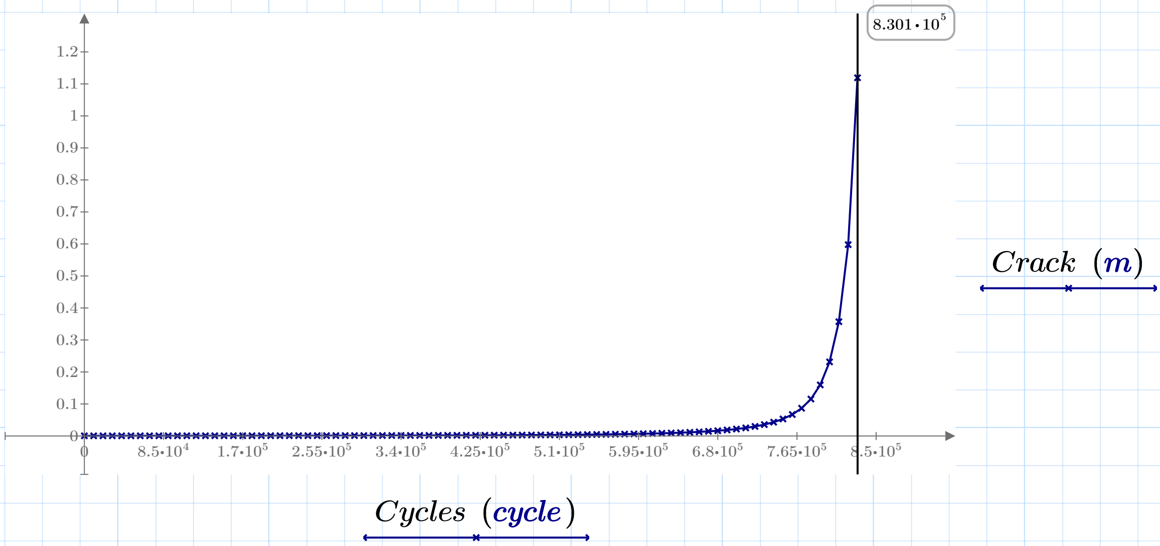
$$a_1 := a + \delta a = \left(512.611724 \cdot 10^{-6}\right)\ m$$

g. steps c to f are repeated in an iterative process to give a series of values for a which can then be plotted against the number of cumulative cycles and converted to time by referencing the number of cycles per year given in the first part of the question.

$$Crack := \left\| \begin{array}{l} i \leftarrow 0 \\ \delta N \leftarrow 10000\ cycle \\ a_i \leftarrow 0.0005\ m \\ \text{while } a_i < 1\ m \\ \quad \left\| \begin{array}{l} i \leftarrow i+1 \\ \delta K_i \leftarrow Y \cdot \sigma_{eq} \cdot \sqrt{\pi \cdot a_{i-1}} \\ \delta a_i \leftarrow C \cdot \left(\delta K_i\right)^m \cdot \delta N \\ a_i \leftarrow a_{i-1} + \delta a_i \end{array} \right. \\ a \leftarrow \text{augment}(a) \end{array} \right.$$

$$ii := \text{rows}(Crack)$$

$$Cycles := \left\| \begin{array}{l} i \leftarrow 0 \\ \delta N \leftarrow 10000\ cycle \\ N_i \leftarrow 0\ cycle \\ \text{for } ii \in 0..\text{last}(Crack) \\ \quad \left\| \begin{array}{l} i \leftarrow i+1 \\ N_i \leftarrow N_{i-1} + \delta N \end{array} \right. \\ N \leftarrow \text{augment}(N) \end{array} \right.$$



$$\sum n = \left(3.416 \cdot 10^6\right) \frac{1}{yr}$$